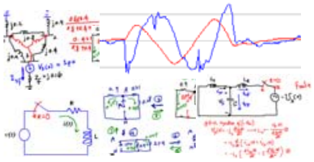
Source: www.mwftr.com

Charles Kim, "Lecture Notes on Fault Detection and Location in Distribution Systems," 2010.

FAULT DETECTION AND LOCATION IN DISTRIBUTION SYSTEMS

2. Faulted Power System Analysis : Review



Charles Kim

June 2010

Faulted Power System Analysis

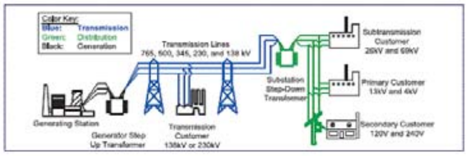
- **Faulted Power System Review**
  - Connection of Power Variables and Physics
  - Introduction of Asymmetrical Fault Analysis
    →Symmetry from Asymmetry
  - Review with MathCad Tutorial

Faulted Power System

- Power System Variables: ***V***, ***I***, and $\boldsymbol{\theta}$
- Impedance and Its Expression in Per-Unit (pu)
- Symmetrical Fault Current Calculation
- Systematic Fault Calculation with Bus Matrix
- Asymmetrical Fault Calculation Approach
  - Symmetrical Component (Sequence Component)
- Sequence Component Application to Asymmetrical Faults
- Fault Current Distribution

3-phase Power System

- Network of Circuit Elements in branches radiated from nodes
- One of the most complex networks
- AC over DC
- 1-P vs. 3-P
- 60Hz vs 50Hz
    - Rotor speed, pole, ***f*** relationship
    - Lighting performance
- 400Hz
    - Transformers and motors for 400 Hz are much smaller and lighter than at 50 or 60 Hz, which is an advantage in aircraft and ships.



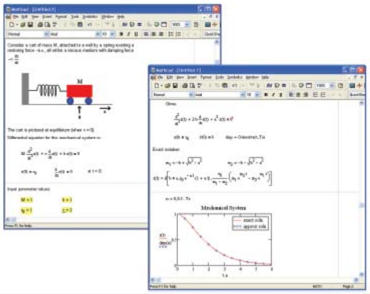
Mathcad

- Engineering calculation software
- Unique visual format
- Scratchpad interface
- Text, figure, and graph in a single worksheet
- Easy to learn and use

- 30-day trial version
- If you have not installed it on your laptop.

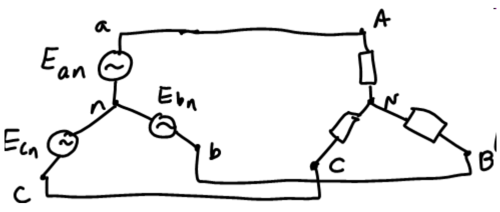
  http://www.ptc.com/products/mathcad/

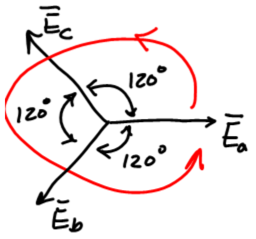
- Academic Version: ~ 100 Euros



3-Phase System

- 3-phase System
  - 3 alternating driving voltages sources
  - Same magnitude with 120 degrees apart – “balanced”
  - Constant Instantaneous power





$a = e^{j2\pi/3}$

$\bar{E}_b = a^2 \bar{E}_a$, $\bar{E}_c = a \bar{E}_a$

Mathcad Tutorial with 3-Phase Instantaneous Power

$va(t) := Vm \cdot \cos(w \cdot t)$

$vb(t) := Vm \cdot \cos\left(w \cdot t - \frac{2\pi}{3}\right)$

$vc(t) := Vm \cdot \cos\left(w \cdot t + \frac{2\pi}{3}\right)$

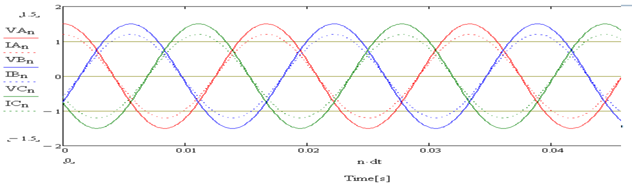
$ia(t) := Im \cdot \cos(w \cdot t + \theta)$

$ib(t) := Im \cdot \cos\left(w \cdot t - \frac{2\pi}{3} + \theta\right)$

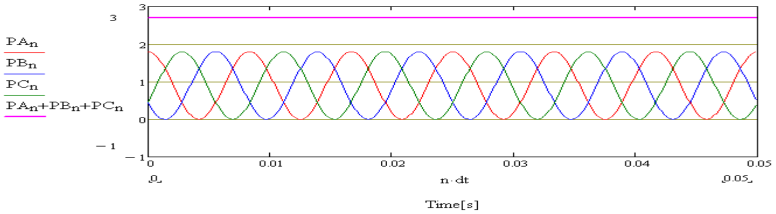
$ic(t) := Im \cdot \cos\left(w \cdot t + \frac{2\pi}{3} + \theta\right)$

Three_Phase_Power_1.xmcd

$\theta := 0$

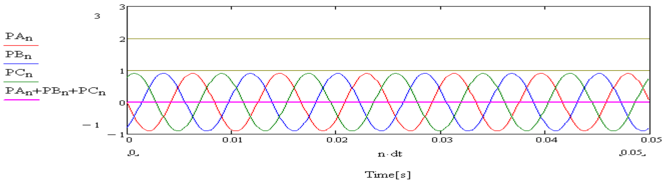


Three_Phase_Power_2.xmcd

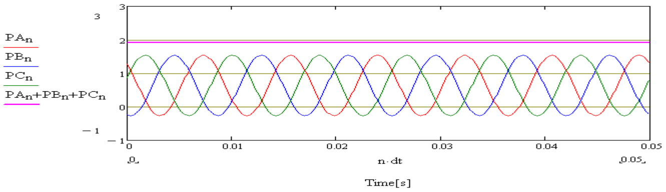


Phase Angle Influence

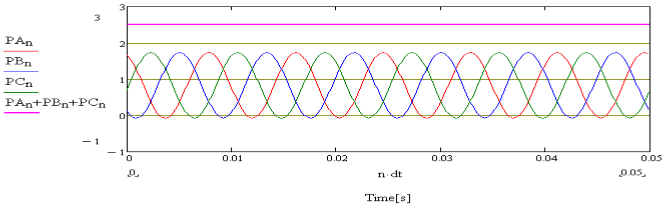
$\theta := \frac{\pi}{2}$



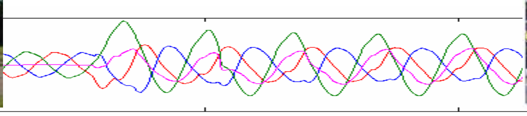
$\theta := \frac{\pi}{4}$



$\theta := \frac{\pi}{8}$



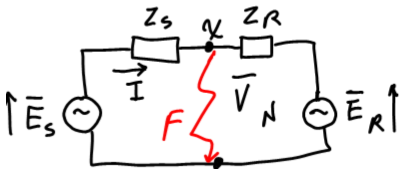
Fault Conditions



- **Fault:**
  - A sudden abnormal alteration to the normal circuit arrangement.
  - The circuit will pass through a **transient state** to a **steady state**.
  - In the transient state, the **initial magnitude of the fault current** will depend upon the point on the voltage wave at which the fault occurs.
  - The **decay of the transient condition**, until it merges into steady state, is a function of the parameters of the circuit elements.
- **Symmetrical Faults:**
  - three-phase fault, which involves all three phases equally at the same location
  - Analysis with single –phase network (per-phase analysis)
- **Asymmetrical Faults**
  - Single-phase and Double-Phase faults
  - Analysis with Sequence Components

3-Phase Fault Current Calculation - basis

- Per-Phase Analysis with Injection & Thevenin & superposition

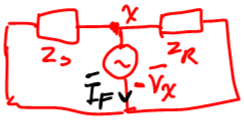


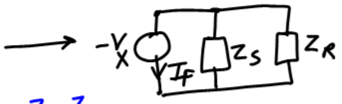
$\bar{V} = \bar{E}_s - \bar{I} \cdot Z_s$

$= \bar{I} \cdot \bar{Z}_R + \bar{E}_R$

(Normal)

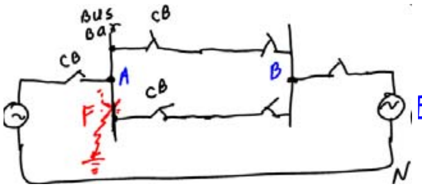
Fault : $v_{XN} = 0$ (or Supply $-\bar{v}_X$ @ x)

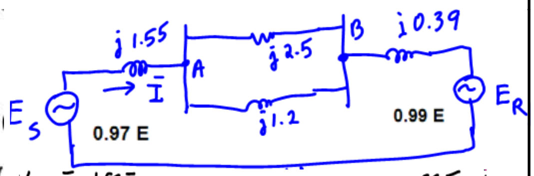




$$I_F = \bar{V}_X \frac{Z_S \cdot Z_R}{Z_S + Z_R}$$

Practical Example





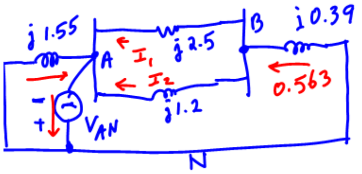
$$V_{AN} = \bar{E}_S - 1.55\bar{I}$$

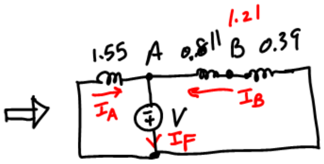
$$= E_R + \left(0.39 + \frac{(2.5)(1.2)}{2.5+1.2}\right) I$$

Since $E_S >> 1.55\bar{I}$

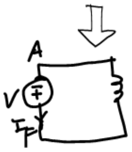
& $E_R >> 1.207\bar{I}$

$V_{AN} \approx E_S \approx E_R$





Assume that I_F = 1.0 pu



$$\frac{(1.55)(1.21)}{1.55+1.21} = 0.68$$

$$I_1 = (0.563)\frac{1.2}{1.2+2.5} = 0.183\,pu$$

$$I_2 = (0.563)\frac{2.5}{1.2+2.5} = 0.38\,pu$$

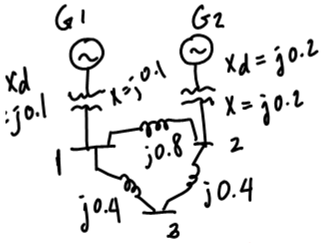
$$\therefore I_A = \frac{1.21}{2.76}pu = 0.437pu$$

$$I_B = \frac{1.55}{2.76}pu = 0.563pu$$

$$I_F = \frac{V_F}{0.68} \Rightarrow 1pu \quad 2.78$$

Another Example of Fault Calculation

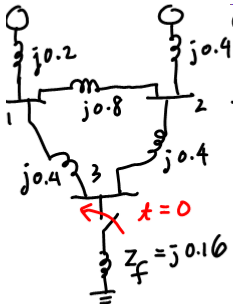
- First, Let's start with an example problem.

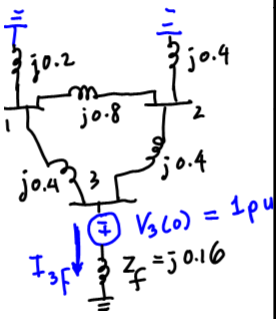


* Fault at Bus 3 with fault impedance of $Z_f = j0.16$.

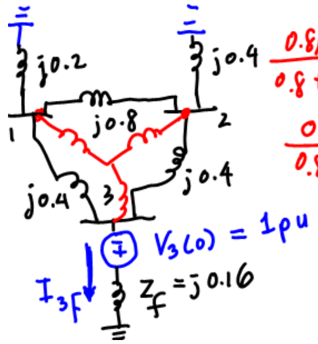
Find $V_1(F)$, $V_2(F)$, $V_3(F)$

$I_{12}(F)$, $I_{13}(F)$, $I_{32}(F)$



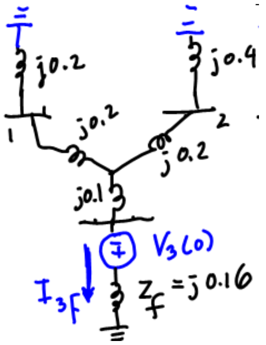


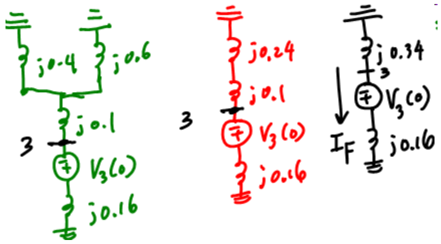
Continued



$$\frac{0.8\times0.4}{0.8+0.4+0.4} = 0.2$$

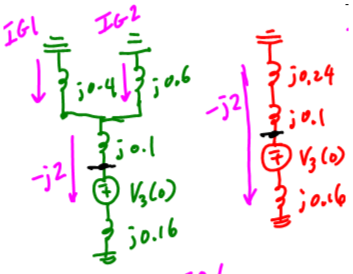
$$\frac{0.4\times0.4}{0.8+0.4+0.4} = 0.1$$





$$I_{3F} = \frac{V_3(0) \swarrow 1pu}{j0.34+j0.16} = -j2$$

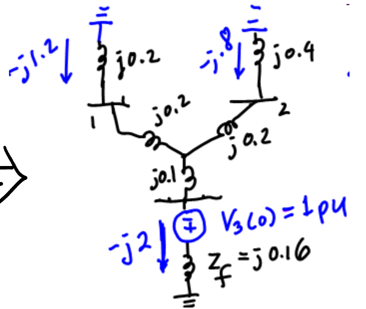
Continued with Calculation



$$I_{G1} = -j2\,\frac{j0.6}{j1.0} = -j1.2$$

$$I_{G2} = -j0.8$$

$\Rightarrow$



$$V_{1F} = -(j0.2)(-j1.2) = -0.24\text{pu}$$

$$V_{2F} = -(j0.4)(-j.8) = -0.32\text{pu}$$

$$V_{3F} = (j0.16)(-j2) - 1 = -0.68\text{pu}$$

$$V_1 = V_1(0) + V_{1F} = 0.76\text{pu}$$

$$V_2 = V_2(0) + V_{2F} = 0.68\text{pu}$$

$$V_3 = V_3(0) + V_{3F} = 0.32\text{pu}$$

$\Leftarrow$

$$I_{12} = \frac{0.76-0.68}{j0.8} = -j0.1\text{ pu}$$

$$I_{13} = \frac{0.76-0.32}{j0.4} = -j1.1\text{ pu}$$

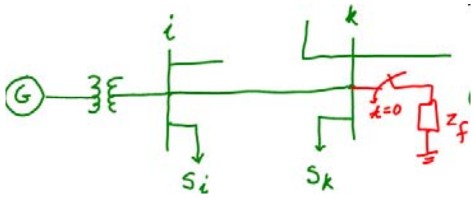
$$I_{23} = \frac{0.68-0.32}{j0.4} = -j0.9\text{pu}$$

System Fault Analysis with Bus Impedance Matrix

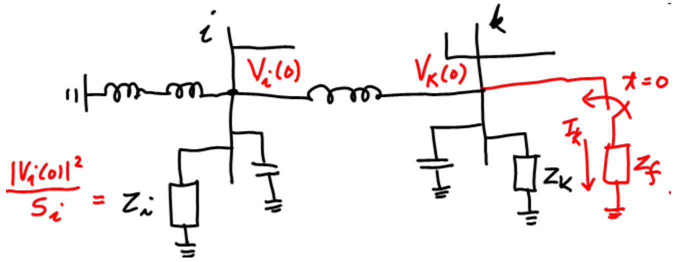
- What we employed in the previous example
  - Thevenin Theorem
    - Thevenin Voltage
    - Thevenin Impedance
    - Circuit Manipulation (reduction)
  - Size Problem
    - Small power circuits only
    - Impractical for real power system
- A systemic, computer-programmable method for real power system of any size
  - Bus Impedance Matrix Approach

N-bus power system

- A component: buses *i* and *k*.
- Fault at bus *k*



- Equivalent Circuit
  - Pi for transmission line
  - Load Impedance



Bus Equation

- Then, what are Ybus and Zbus?
- Why do we always use Ybus then inverse it to Zbus?
    - Ybus is better with Power Flow Analysis
    - Zbus approach is better in Fault Analysis

Pre-Fault Voltage

$$V_{bus}(0) = \begin{bmatrix} V_1(0) \\ \vdots \\ V_i(0) \\ \vdots \\ V_K(0) \\ \vdots \\ V_n(0) \end{bmatrix}$$

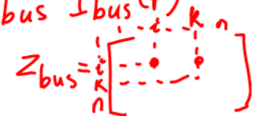
Change in Voltage due to fault

$$\Delta V_{bus} = \begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_i \\ \vdots \\ \Delta V_K \\ \vdots \\ \Delta V_n \end{bmatrix}$$

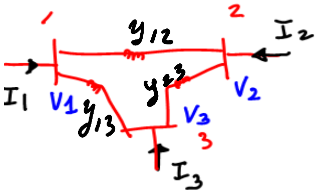
$$V_{bus}(F) = V_{bus}(0) + \Delta V_{bus}$$

From $V = ZI$

$$\Delta V_{bus} = Z_{bus} \cdot I_{bus}(F)$$



Ybus



$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

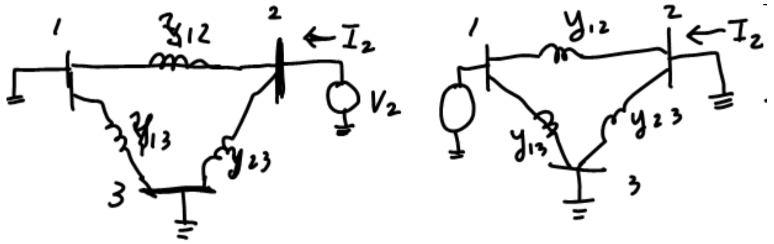
$$I_2 = Y_{21} V_1 + Y_{22} V_2 + Y_{23} V_3$$

$$\rightarrow \quad Y_{22} = \frac{I_2 - Y_{21} V_1 - Y_{23} V_3}{V_2} \longrightarrow \left. \frac{I_2}{V_2} \right|_{V_1 = V_3 = 0}$$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2 = V_3 = 0}$$

$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1 = V_3 = 0}$$

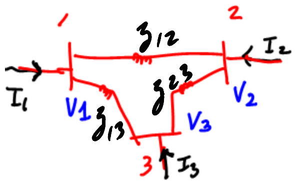
Ybus Example



$$Y_{22} = \frac{I_2}{V_2} = y_{12} + y_{23} \qquad Y_{21} = \frac{I_2}{V_1} = -y_{12}$$

- So, Ybus is:
  - Easy to formulate

Zbus



$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 + Z_{23} I_3$$

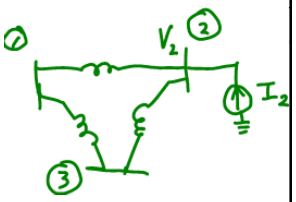
$$\rightarrow Z_{22} = \frac{V_2 - Z_{21} I_1 - Z_3 I_3}{I_2}$$ Driving Point Impedance

$$= \left. \frac{V_2}{I_2} \right|_{I_1 = I_3 = 0}$$

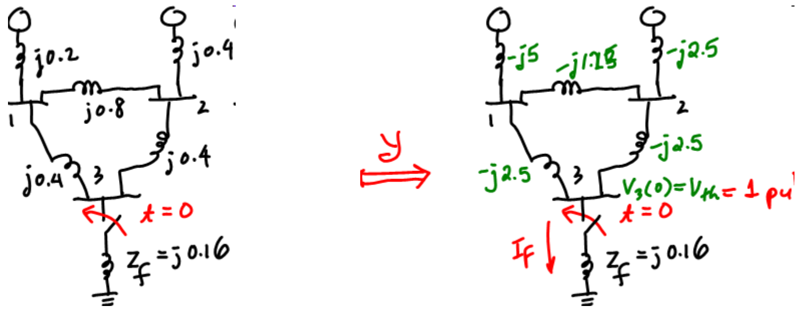
$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2 = I_3 = 0}, \qquad Z_{23} = \left. \frac{V_2}{I_3} \right|_{I_1 = I_2 = 0}$$

Zbus Problem and Promise

- ZBus is:
  - Driving Point Impedance: Thevenin Equivalent Impedance
  - Involves all circuit elements
  - Avoid if possible
  - Instead get
    - YBus = Inverse (Zbus)
  - Algorithms to directly build Zbus available
  - Best for fault calculation, though
    - In principle, drawing Zbus element at a bus is done by injection of current at the node, and no current injections at all other buses
    - Injection of the current : fault current
- Strategy in fault calculation in power system
  - Formation of Ybus
  - Inverse Ybus to get Zbus
  - Injection of Current at a faulted bus
  - Calculate voltages and currents for the entire network



Fault Calculation using Bus Matrix - Example



$$Y_{bus} = \begin{bmatrix} -j8.75 & j1.25 & j2.5 \\ j1.25 & -j6.25 & j2.5 \\ j2.5 & j2.5 & -j5 \end{bmatrix} \qquad Z_{bus} = \begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}$$

$$I_f = \frac{V_3(0)}{Z_{33} + Z_f} = \frac{1}{j0.5} = -j2$$

Adding a branch in Zbus

Voltage and Current Calculation

Since a fault is a short circuit, then $\Delta V_k = -V_f$.

Fault_Bus.xmcd

$$\begin{bmatrix} V_1(F) \\ V_2(F) \\ V_3(F) \end{bmatrix} = \begin{bmatrix} V_1(0) \\ V_2(0) \\ V_3(0) \end{bmatrix} - \overset{Z_{bus}}{\begin{bmatrix} j0.16 & j0.08 & j0.12 \\ j0.08 & j0.24 & j0.16 \\ j0.12 & j0.16 & j0.34 \end{bmatrix}} \overset{I}{\begin{bmatrix} 0 \\ 0 \\ -j2 \end{bmatrix}}$$

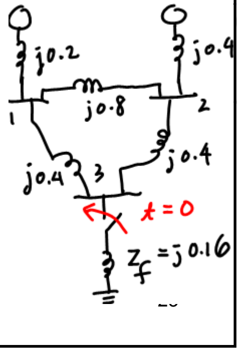
$$V_1(F) = 1 - (j0.12)(-j2) = 0.76$$
$$V_2(F) = 1 - (j0.16)(-j2) = 0.68$$
$$V_3(F) = 1 - (j0.34)(-j2) = 0.32$$

$$I_{12} = \frac{V_1(F) - V_2(F)}{z_{12}} = \frac{0.76-0.68}{j0.8} = -j0.1$$

From actual Impedance ($\neq Z_{12}$)

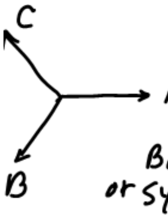
$$I_{13} = \frac{V_1(F) - V_3(F)}{z_{13}} = \frac{0.76-0.32}{j0.4} = -j1.1$$

$$I_{23} = \frac{V_2(F) - V_3(F)}{z_{23}} = \frac{0.68-0.32}{j0.4} = -j0.9$$



Asymmetrical Fault Analysis

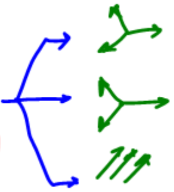
- Motive and Principle of Symmetrical Components



Balanced System or Symmetrical system

(1ϕ) per-phase $\xrightarrow{\text{analysis}}$ → Construct the other 2 phases

Unbalanced system or un-symmetrical system



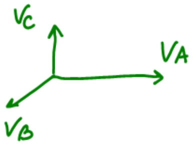
Balanced

Balanced → Super-position

Balanced

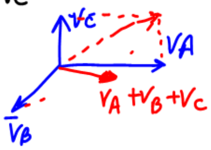
Decomposition

Unbalanced Example



$\bar{V}_A+\bar{V}_B+\bar{V}_C \neq 0$

Sum



$V_A+V_B+V_C=V_R$

Is it symmetrical?

Let $V_0=\frac{1}{3}V_R$,

$$V_A+V_B+V_C-V_R=0 \rightarrow V_A+V_B+V_C-3V_0=0$$

$$\rightarrow \underbrace{(V_A-V_0)}_{V_a}+\underbrace{(V_B-V_0)}_{V_b}+\underbrace{(V_C-V_0)}_{V_c}=0$$

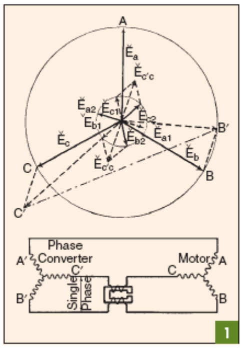
$V_a+V_b+V_c=0$, But Symmetrical? NO!

- We could make the sum zero, but we still have to decompose the **unbalanced phasor** into **symmetrical sets**

Symmetrical Sets

- Can we be assured that two such symmetrical sets exist?
- Can we decompose them into the two symmetrical sets?
- The answer is yes
  - Two such symmetrical sets exist.
  - Charles Fortescue's paper proved that.

> "Method of Symmetrical Co-Ordinates Applied to the Solution of Polyphase Networks"



This illustration from "Polyphase Networks" shows a vector diagram illustrating a method of using phase converters to supply a balanced three-phase electromotive force to a symmetrical load, such as an induction motor. The configuration shown achieves a low single-phase power factor (*Trans. AIEE*, vol. 37, pt. 2, 1918).

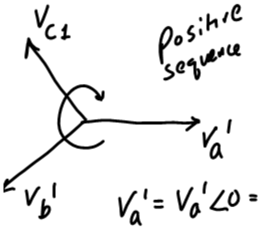
Symmetrical Components

- Theorem: We can represent ANY unsymmetrical set of 3 phasors as the sum of 3 constituent sets, each having 3 phasors:
  - A positive (a-b-c) sequence set and
  - A negative (a-c-b) sequence set and
  - An equal set
- The three sets
  - Positive (V_a^1, V_b^1, V_c^1)
  - Negative (V_a^2, V_b^2, V_c^2)
  - Zero (V_a^0, V_b^0, V_c^0)

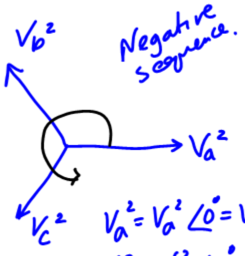
$$V_a = V_a^0 + V_a^1 + V_a^2$$
$$V_b = V_b^0 + V_b^1 + V_b^2$$
$$V_c = V_c^0 + V_c^1 + V_c^2$$

sequence components..

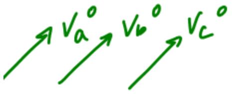
Pos, Neg, and Zero Sequences



$V_a^1 = V_a^1 \angle 0 = V_a^1$
$V_b^1 = V_a^1 \angle 240° = a^2 V_a^1$
$V_c^1 = V_a^1 \angle 120° = a V_a^1$



$V_a^2 = V_a^2 \angle 0° = V_a^2$
$V_b^2 = V_a^2 \angle 120° = aV_a^2$
$V_c^2 = V_a^2 \angle 240° = a^2 V_a^2$



Zero-Seq (0)

Equal, in-phase

$V_a^0 = V_b^0 = V_c^0$

Some Physical Sense out of Sequence Components?

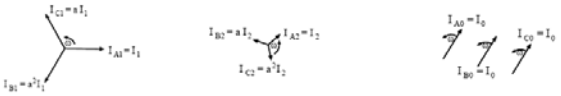
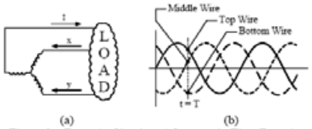
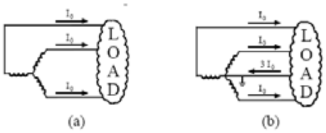


Figure 7 – (a) Positive, (b) Negative, and (c) Zero Sequence Components Expressed in Terms of Phase A Quantities



(a) (b)

Figure 9 – Example Circuit and Currents in Time Domain



(a) (b)

Figure 10 – Zero-Sequence Current and Return Path

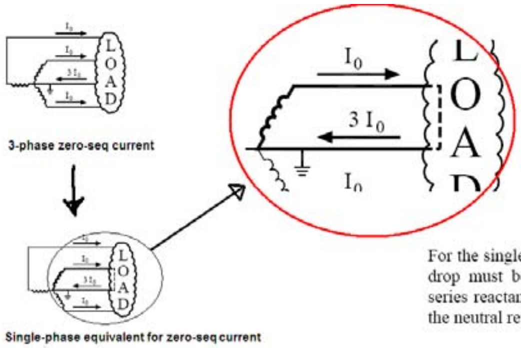
For the positive and negative sequence, *a current supplied by one phase conductor is returned to the source by the other two.* This relationship is always true in three-phase circuits.

There is no angular displacement between the phases, so whatever instantaneous current flows on the top wire also must flow on the middle and bottom wires.

Fig. 10(a) shows a total of 3 I_0 delivered from the source to the load. A return path to the source must exist. *The zero sequence current is supplied to the load on the phase conductors, but it cannot return to the source on the phase conductors.* A fourth conductor must be present to serve as the return path. The neutral conductor returns the zero-sequence current supplied by each phase conductor, or 3 I_0, as shown in Fig. 10(b). *If a fourth conductor (return path) does not exist, zero-sequence current will not flow.* This property is a characteristic of three-phase circuits.

- Source: R. E. Fehr, "A Novel Approach for Understanding Symmetrical Components and Sequence Networks of Three-Phase Power Systems," TE-2006-000213

More for the physical sense?



This situation causes a problem since the current supplied (I_0) and the current returned (3 I_0) are different.

For the single-phase equivalent to be valid, the correct voltage drop must be calculated for the neutral return path. If the series reactance in the return path is X_N, the voltage drop for the neutral return path is found using Ohm's Law.

$$V = (3\ I_0) \times X_N$$

Any impedance in the neutral return path is subjected to three times the zero-sequence current as is flowing in each of the phase conductors; therefore, to provide the proper voltage drop, any impedance in the neutral portion of the circuit must be *tripled* when modeling the circuit as sequence networks .

But the calculated voltage drop remains correct if the coefficient is simply grouped with the term (X_N)

$$V = I_0 \times (3\ X_N)$$

Matrix Form of Conversion

$$V_a = V_a^{\,0} + V_a^{\,1} + V_a^{\,2}$$
$$V_b = V_b^{\,0} + V_b^{\,1} + V_b^{\,2}$$
$$V_c = V_c^{\,0} + V_c^{\,1} + V_c^{\,2}$$

$$a = \angle 120^0 = e^{j\frac{2\pi}{3}}$$

$V_{012} \longrightarrow V_{abc}$

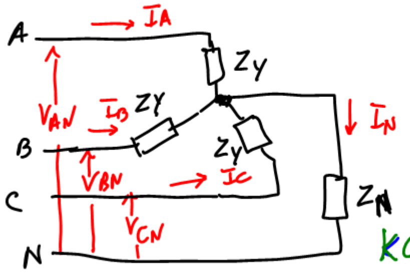
$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \qquad \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = A \begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix}$$

$V_{abc} \longrightarrow V_{012}$

$$\begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix} = A^{-1} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

Sequence Impedance

Y-connected Balanced Load :



KVL

$V_{AN} = I_A Z_Y + I_N Z_N$

$V_{BN} = I_B Z_Y + I_n Z_N$

$V_{CN} = I_C Z_Y + I_n Z_n$

KCL @ Load Junction

$I_A + I_B + I_C = I_N$

$V_{AN} = I_A Z_Y + (I_A + I_B + I_C) Z_N \rightarrow (Z_Y + Z_N) I_A + Z_N I_B + Z_N I_C$

$V_{BN} = I_B Z_Y + (I_A + I_B + I_C) Z_N \rightarrow Z_N I_A + (Z_Y + Z_N) I_B + Z_N I_C$

$V_{CN} = I_C Z_Y + (I_A + I_B + I_C) Z_N \rightarrow Z_N I_A + Z_N I_B + (Z_Y + Z_N) I_C$

Continued

$$\begin{bmatrix} V_{AN} \\ V_{BN} \\ V_{CN} \end{bmatrix} = \begin{bmatrix} Z_y+Z_N & Z_N & Z_N \\ Z_N & Z_y+Z_N & Z_N \\ Z_N & Z_N & Z_y+Z_N \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix} \Rightarrow V_{ABC} = Z_{ABC} I_{ABC}$$

$\Downarrow$ Target

$$V_{012} = Z_{012} \cdot I_{012}$$

From $V_{ABC} = A \cdot V_{012}$ & $I_{ABC} = A \cdot I_{012}$

$$A \cdot V_{012} = Z_{ABC} \cdot A \cdot I_{012} \quad ①$$

$$① \cdot A^{-1} : \quad A^{-1} \cdot A \cdot V_{012} = A^{-1} \cdot Z_{ABC} \cdot A \cdot I_{012}$$

$$V_{012} = \underbrace{A^{-1} \cdot Z_{ABC} \cdot A}_{Z_{012}} \cdot I_{012}$$

Z_{012}

$$Z_{012} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_y+Z_N & Z_N & Z_N \\ Z_N & Z_y+Z_N & Z_N \\ Z_N & Z_N & Z_y+Z_N \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$Z_{012} = \begin{bmatrix} Z_0 \\ Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} Z_y+3Z_N & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix}$$

$$V_{012} = Z_{012} \cdot I_{012}$$

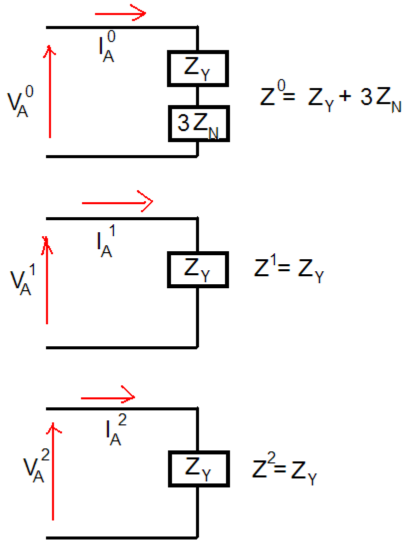
$$\begin{bmatrix} V_{AN}^0 \\ V_{AN}^1 \\ V_{AN}^2 \end{bmatrix} = \begin{bmatrix} Z_y+3Z_N & 0 & 0 \\ 0 & Z_y & 0 \\ 0 & 0 & Z_y \end{bmatrix} \begin{bmatrix} I_A^0 \\ I_A^1 \\ I_A^2 \end{bmatrix}$$

$$V_{AN}^0 = (Z_y+3Z_N) I_A^0, \quad V_{AN}^1 = Z_y \cdot I_A^1, \quad V_{AN}^2 = Z_y \cdot I_A^2$$

- Off-diagonal terms =0 (**Uncoupled)**
  - the only current that determines the zero sequence voltage is the zero sequence current.
  - the only current that determines the positive sequence voltage is the positive sequence current.
  - the only current that determines the negative sequence voltage is the negative sequence current.
  - 3 separate, distinct single phase equations

Kolme's Company

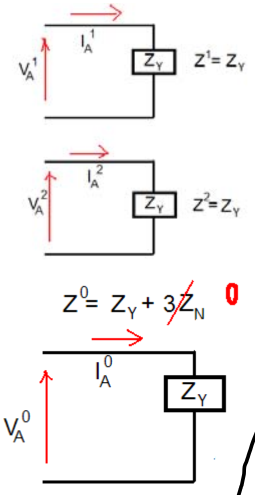
3 single phase circuits



- Some questions:
- Why doesn't the neutral impedance appear in the positive & negative sequence networks?
  - *Because the positive and negative sequence networks contain balanced currents only, and balanced currents sum to 0 and therefore do not contribute to flow in the neutral.*
- Why do we have 3Zn in the zero sequence network instead of just Zn?
  - *Voltage drop is $Z_N*I_N=Z_N*3*I^0=I^0*3Z_N$. Therefore, to obtain the correct voltage drop seen in the neutral conductor with a flow of only ,model the zero-sequence impedance as 3Zn.*

$I_A^0 = \frac{1}{3}(I_A+I_B+I_C) = \frac{1}{3}I_n$

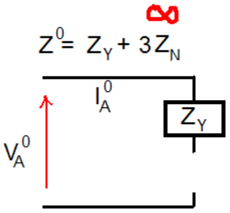
Some Questions - Continued



- What do these three networks look like if the neutral is solidly grounded (no neutral impedance)?
  - *Positive and negative sequence networks are the same. Zero sequence is the same except Zn=0.*

•What if the neutral is ungrounded (floating)?

•Positive and negative sequence networks are the same. Zero sequence has an open circuit, which means $I_N=I^0=0$.



Asymmetric Load/Line?

- What if the load (or line, or load and line) is not symmetric?
- General Impedance matrix
  - Self-Impedance: Zaa, Zbb, Zcc
  - Non-Zero Mutual Impedance: Zab, Zbc,Zacc

$$Z_{012} = A^{-1} Z_{abc} A = \begin{bmatrix} Z_{012}^{00} & Z_{012}^{01} & Z_{012}^{02} \\ Z_{012}^{10} & Z_{012}^{11} & Z_{012}^{12} \\ Z_{012}^{20} & Z_{012}^{21} & Z_{012}^{22} \end{bmatrix} \qquad Z_{abc} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

Requirement for Decoupling

$$Z_{012} = \begin{bmatrix} Z_{012}^{00} & Z_{012}^{01} & Z_{012}^{02} \\ Z_{012}^{10} & Z_{012}^{11} & Z_{012}^{12} \\ Z_{012}^{20} & Z_{012}^{21} & Z_{012}^{22} \end{bmatrix}$$

$$Z_{012}^{00} = \frac{1}{3}\left(Z_{aa} + Z_{bb} + Z_{cc} + 2Z_{ab} + 2Z_{ac} + 2Z_{bc}\right)$$

$$Z_{012}^{11} = Z_{012}^{22} = \frac{1}{3}\left(Z_{aa} + Z_{bb} + Z_{cc} - Z_{ab} - Z_{ac} - Z_{bc}\right)$$

$$Z_{012}^{01} = Z_{012}^{20} = \frac{1}{3}\left(Z_{aa} + a^2 Z_{bb} + aZ_{cc} - aZ_{ab} - a^2 Z_{ac} - Z_{bc}\right) ✓$$

$$Z_{012}^{02} = Z_{012}^{10} = \frac{1}{3}\left(Z_{aa} + aZ_{bb} + a^2 Z_{cc} - a^2 Z_{ab} - aZ_{ac} - Z_{bc}\right) ✓$$

$$Z_{012}^{12} = \frac{1}{3}\left(Z_{aa} + a^2 Z_{bb} + aZ_{cc} + 2aZ_{ab} + 2a^2 Z_{ac} + 2Z_{bc}\right) ✓$$

$$Z_{012}^{21} = \frac{1}{3}\left(Z_{aa} + aZ_{bb} + a^2 Z_{cc} + 2a^2 Z_{ab} + 2aZ_{ac} + 2Z_{bc}\right) ✓$$

- For our sequence circuits to be decoupled (and thus obtain the advantage of symmetrical component decomposition) the off-diagonal elements of Z_{012} must be 0.
- Conditions for the off-diagonal elements of Z_{012} to be 0:

$$Z_{aa} = Z_{bb} = Z_{cc}$$

$$Z_{ab} = Z_{ac} = Z_{bc}$$

Observations

- When
  - Diagonal phase impedances are equal
  - Off-diagonal phase impedances are equal

$$Z_{012}^{00} = Z_{aa} + 2Z_{ab}$$

$$Z_{012}^{11} = Z_{012}^{22} = Z_{aa} - Z_{ab}$$

$$Z_{012}^{01} = Z_{012}^{20} = Z_{012}^{02} = Z_{012}^{10} = Z_{012}^{12} = Z_{012}^{21} = 0$$

$$Z_{012} = \begin{bmatrix} Z_{aa} + 2Z_{ab} & 0 & 0 \\ 0 & Z_{aa} - Z_{ab} & 0 \\ 0 & 0 & Z_{aa} - Z_{ab} \end{bmatrix}$$

- Observations:
  - the positive and negative sequence impedances are always equal, independent of whether the load is symmetric or not. This is true for transmission lines, cables, and transformers. (* It is not true for rotating machines because positive sequence currents, rotating in the same direction as the rotor, produce fluxes in the rotor iron differently than the negative sequence currents which rotate in the opposite direction as the rotor.)
  - Zero Sequence impedance is not the same even in the symmetric load, line, unless mutual phase impedances are zero; Zab=Zbc=Zac=0

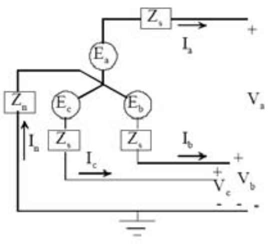
Symmetric Component Approach in Fault Analysis

- Symmetric Component provides 3 decoupled systems for analysis of unbalanced sources applied to a symmetrical system.
- Faulted systems (except for 3-phase faults) are not symmetrical systems, so it would appear that symmetric component is not much good for asymmetrical faults.
- Practical way
  - Replace the fault with an unbalanced source, then the network becomes symmetric.
  - Then get the sequence components of the unbalanced source at the fault point,
  - Perform per-phase analysis on each sequence circuit.

Developing Sequence Networks for Fault Analysis

- (1) Develop the sequence network for the system under analysis.
  - Express **abc** voltages as a function of **abc** currents and **abc** impedances.
  - Substitute symmetric components for **abc** voltages and currents (e.g., $\boldsymbol{V_{abc}}$**=A**$\boldsymbol{V_{012}}$ and $\boldsymbol{I_{abc}}$**=A**$\boldsymbol{I_{012}}$).
  - Manipulate to obtain the form $\boldsymbol{V_{012}}$**=**$\mathbf{Z_{012}}$*****$\boldsymbol{I_{012}}$.
  - Obtain the Thevenin equivalents looking into the network from the fault point.
- (2) Connect the Sequence Networks to capture the influence of the particular fault type.
- (3)Compute the Sequence fault current from the circuit resulting from step 2.
- (4) Compute the phase currents I_a, I_b, and I_c from $\boldsymbol{I_{abc}}$**=A***$\boldsymbol{I_{012}}$.

Sequence Network



$$E^{abc} = \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} E_a$$

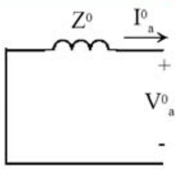
$$V_a = E_a - Z_s I_a - Z_n I_n$$
$$V_b = E_b - Z_s I_b - Z_n I_n$$
$$V_c = E_c - Z_s I_c - Z_n I_n$$

$$\begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = \begin{pmatrix} E_a \\ E_b \\ E_c \end{pmatrix} - \begin{pmatrix} Z_s + Z_n & Z_n & Z_n \\ Z_n & Z_s + Z_n & Z_n \\ Z_n & Z_n & Z_s + Z_n \end{pmatrix} \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix}$$

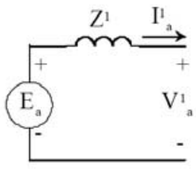
$$E_a^{012} = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{pmatrix} \begin{pmatrix} E_a \\ a^2 E_a \\ a E_a \end{pmatrix} = \begin{pmatrix} 0 \\ E_a \\ 0 \end{pmatrix}$$

$$V_a^{012} = E_a^{012} - Z^{012} I_a^{012}$$

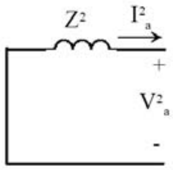
$$\boxed{V_a^0 = 0 - Z^0 I_a^0}$$



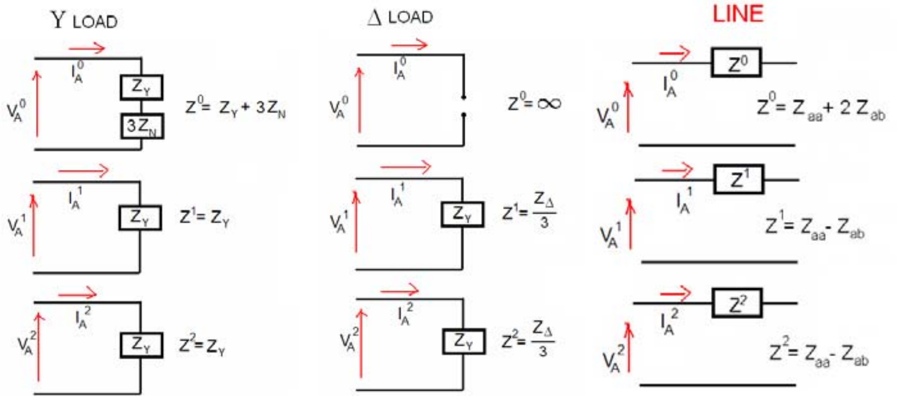
$$\boxed{V_a^1 = E_a - Z^1 I_a^1}$$



$$\boxed{V_a^2 = 0 - Z^2 I_a^2}$$



Sequence Networks – Load and Line

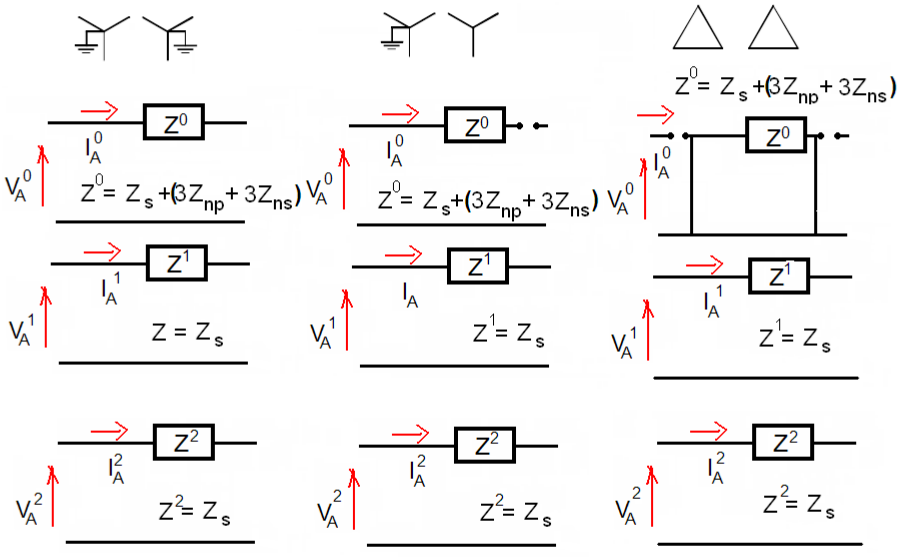


- Typical Overhead Line
  - Zab = 0.4*Zaa
  - Therefore, Z^0=1.8Zaa Z^1=0.6Zaa Z^2=0.6Zaa
    →Z^0=3Z^1

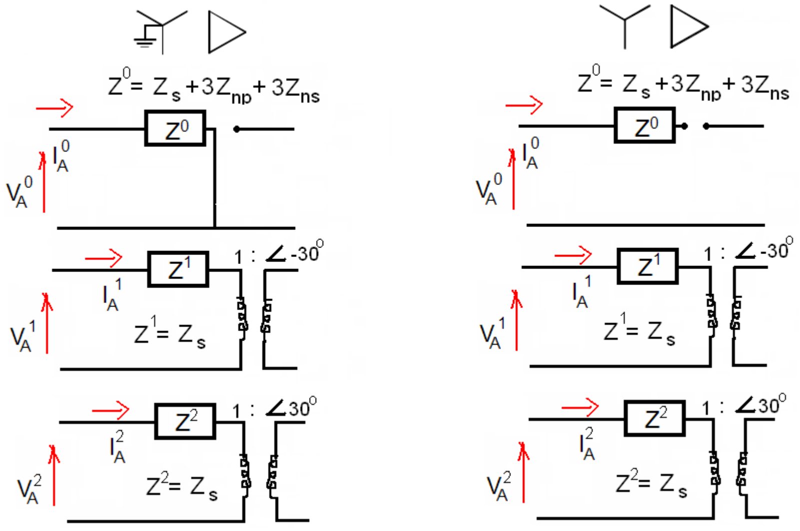
Sequence Networks for Transformer

- Many different types of transformers
- General Guidelines
  - Exciting current is negligible so shunt path is infinite impedance and we only have the series *Z* (*winding resistance and leakage reactance*) in *abc* model.
    - **Z^1 =Z^2 =Zs**, where Zs is the transformer winding resistance and leakage reactance.
    - **Z^0 =Zs** +(3*Znp + 3*Zns), where, **Znp is neutral impedance on primary**, and **Zns is neutral impedance on secondary**
  - Transformers in Δ-Y or Y-Δ configuration are always connected so that positive sequence voltages on the high side lead positive sequence voltages on the low side by 30º (industry convention).

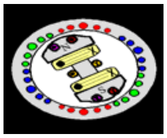
Transformer



Transformer



Rotating Machine



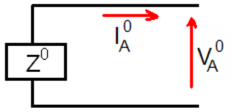
- **Reactance**
  - Positive sequence reactance
    - Use Xd, X’d, or X’’d, depending on time frame of interest.
  - Negative sequence reactance
    - the negative sequence reactance is generally assumed equal to X’’d.
  - Zero-sequence
    - The zero sequence reactance is typically quite small.
    - The reason - the zero sequence currents in the a, b, and c windings are in-phase. Their individual fluxes in the air gap sum to zero and therefore induce no voltage, so the only effect to be modeled is due to leakage reactance, Zg^0.
    - As with loads, if the neutral is grounded through an impedance Zn, model 3Zn in the zero sequence network

$$Z^0 = Z_g^0 + 3Z_n$$

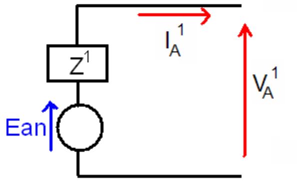
- **Voltage source**
  - **Generators produce balanced positive sequence voltages, no negative or zero sequence voltages.**
  - **Model a voltage source only in the positive sequence circuit.**

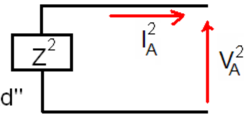
Rotating Machines

$Z^0 = Z_g^0 + 3\ Z_n$



Z^1= Xd, Xd', or Xd''

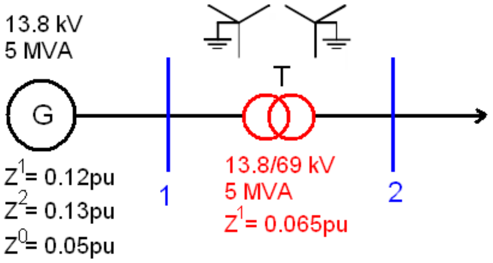




Z^2= Xd''

		Smooth Rotor	Salient Pole	Synchronous Condensers	Motor
X^1	X_d	1.1	1.15	1.8	1.2
	X'_d	0.23	0.37	0.4	0.35
	X''_d	0.12	0.24	0.25	0.30
X^2		0.13	0.29	0.27	0.35
X^0		0.05	0.11	0.09	0.16

Example for Sequence Network (with Fault Cases)

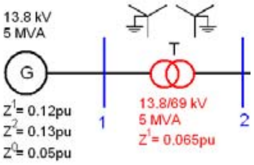


Seq_Comp.xmcd

- Q: Determine the positive, negative, and zero-sequence Thevenin equivalents as seen from Bus 2, on a 1 MVA base.

Process Steps

- (1) Impedance Unification under 1MVA



$X_g^0 = 0.05 \times \frac{1}{5} = 0.01$ pu

$X_g^1 = 0.12 \times \frac{1}{5} = 0.024$ pu

$X_g^2 = 0.13 \times \frac{1}{5} = 0.026$ pu

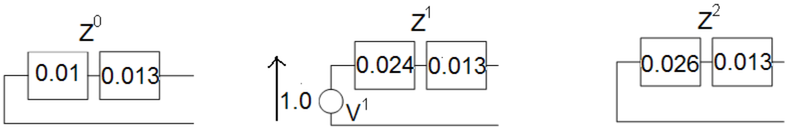
$X_T^0 = 0.065 \times \frac{1}{5} = 0.013$ pu

$= X_T^1 = X_T^2$

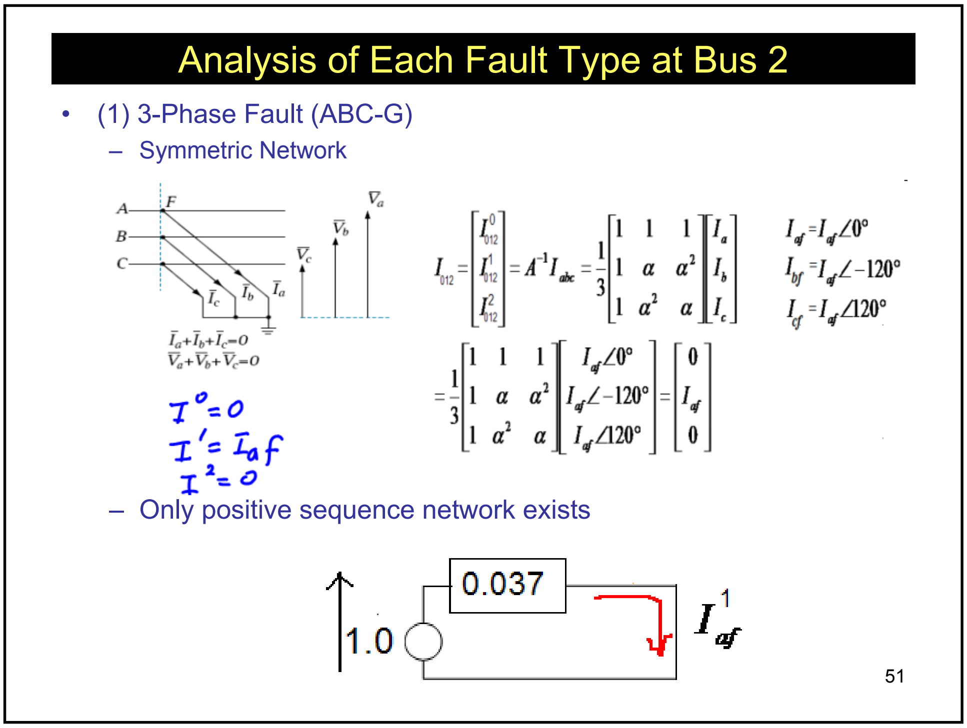
$Z_{b2} = Z_{b1} \cdot \frac{MVA_{b2}}{MVA_{b1}}$

$Z_{b2} = Z_{b1} \cdot \left(\frac{kV_{b1}}{kV_{b2}}\right)^2$

- (2) Sequence Networks (These are Thevenin equivalents seen at Bus 2)



$Z^0 = 0.023$ $Z^1 = 0.037$ $Z^2 = 0.039$ $V^1 = 1.0$



Analysis of Each Fault Type at Bus 2

- (1) 3-Phase Fault (ABC-G)
  - Symmetric Network

$\bar{I}_a+\bar{I}_b+\bar{I}_c=0$

$\bar{V}_a+\bar{V}_b+\bar{V}_c=0$

$$I_{012}=\begin{bmatrix} I_{012}^{0} \\ I_{012}^{1} \\ I_{012}^{2} \end{bmatrix}=A^{-1}I_{abc}=\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$I_{af}=I_{af}\angle 0°$

$I_{bf}=I_{af}\angle -120°$

$I_{cf}=I_{af}\angle 120°$

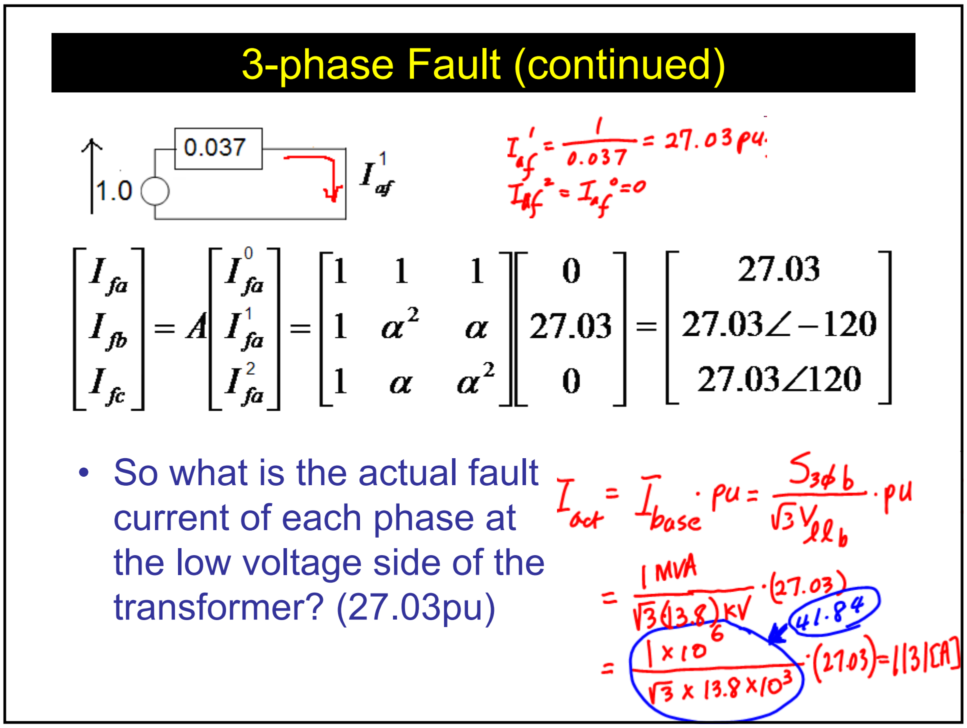
$$=\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_{af}\angle 0° \\ I_{af}\angle -120° \\ I_{af}\angle 120° \end{bmatrix}=\begin{bmatrix} 0 \\ I_{af} \\ 0 \end{bmatrix}$$

$I^{0}=0$

$I^{1}=I_{af}$

$I^{2}=0$

  - Only positive sequence network exists



3-phase Fault (continued)

$I_{af}^{1}=\frac{1}{0.037}=27.03pu.$

$I_{af}^{2}=I_{af}^{0}=0$

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix}=A\begin{bmatrix} I_{fa}^{0} \\ I_{fa}^{1} \\ I_{fa}^{2} \end{bmatrix}=\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}\begin{bmatrix} 0 \\ 27.03 \\ 0 \end{bmatrix}=\begin{bmatrix} 27.03 \\ 27.03\angle -120 \\ 27.03\angle 120 \end{bmatrix}$$

- So what is the actual fault current of each phase at the low voltage side of the transformer? (27.03pu)

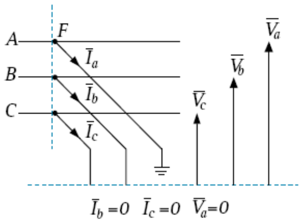
$$I_{act}=\bar{I}_{base}\cdot pu=\frac{S_{3\phi b}}{\sqrt{3}V_{ll_b}}\cdot pu$$

$$=\frac{1\,MVA}{\sqrt{3}(13.8)KV}\cdot(27.03)$$

$$=\frac{1\times 10^{6}}{\sqrt{3}\times 13.8\times 10^{3}}\cdot(27.03)=1131[A]$$

41.84

(2) Single Phase Ground Fault (A-G)



$$I_{012} = \begin{bmatrix} I_{012}^0 \\ I_{012}^1 \\ I_{012}^2 \end{bmatrix} = A^{-1} I_{abc} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_{af} \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} I_{af} \\ I_{af} \\ I_{af} \end{bmatrix}$$

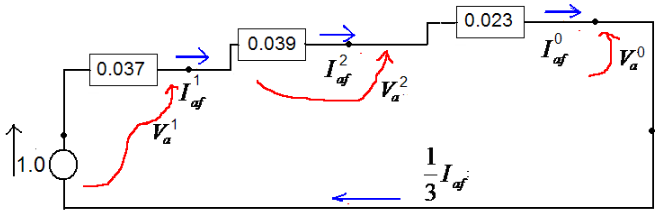
From $V_a = \boxed{V_a^0 + V_a^1 + V_a^2 = 0}$ $\boxed{\frac{1}{3} I_{af} = I_a^0 = I_a^1 = I_a^2}$

- What would be the sequence network which satisfies above two equations?

53

(2) A-G continued

From $V_a = \boxed{V_a^0 + V_a^1 + V_a^2 = 0}$ $\boxed{\frac{1}{3} I_{af} = I_a^0 = I_a^1 = I_a^2}$



$$I_{fa}^1 = I_{fa}^2 = I_{fa}^0 = \frac{1.0}{0.037 + 0.039 + 0.023} = 10.10\,pu$$

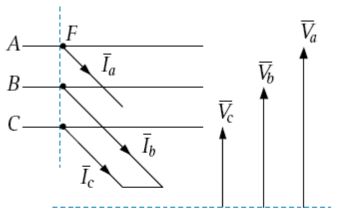
$$I_{fa} = 3 \cdot I^0$$

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = A \begin{bmatrix} I_{fa}^0 \\ I_{fa}^1 \\ I_{fa}^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 10.10 \\ 10.10 \\ 10.10 \end{bmatrix} = \begin{bmatrix} 30.30 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{I_{f_{1\phi}} > I_{F_{3\phi}}}$$

$$\underbrace{(41.84)}_{I_{base}} \underbrace{(30.30)}_{pu} = \underbrace{1268}_{I_{Actual}} [A]$$

(3) Line-to-Line Fault (BC)



$\bar{I}_a = 0 \quad \bar{I}_b = -\bar{I}_c \quad \bar{V}_b = \bar{V}_c$

$$I_{012} = \begin{bmatrix} I_{012}^0 \\ I_{012}^1 \\ I_{012}^2 \end{bmatrix} = A^{-1} I_{abc} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_{bf} \\ -I_{bf} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ \sqrt{3} I_{bf}\angle 90° \\ \sqrt{3} I_{bf}\angle -90° \end{bmatrix}$$

$$\begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix}$$

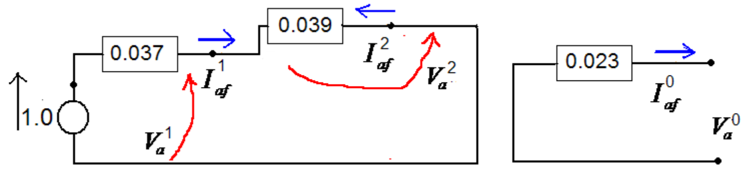
$I^0 = 0 \qquad I^1 = -I^2$

$V^1 = V^2$

Sequence Network Formation Condition

(3) BC continued

- Sequence Network Formation



- Fault Current Calculation

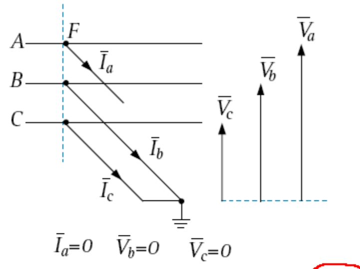
$$I_{fa}^1 = -I_{fa}^2 = \frac{1.0}{0.037+0.039} = 13.16\,pu$$

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = A\begin{bmatrix} I_{fa}^0 \\ I_{fa}^1 \\ I_{fa}^2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}\begin{bmatrix} 0 \\ 13.16 \\ -13.16 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \sqrt{3}(13.16)\angle 90° \\ \sqrt{3}(13.16)\angle -90° \end{bmatrix} = \begin{bmatrix} 0 \\ 22.79\angle 90° \\ 22.79\angle -90° \end{bmatrix}$$

$I_{fb_{Actual}}$ = (41.84)(22.79)=954 [A]

(4) Two-Line-to-Ground Fault (BC-G)



$$I_{012} = \begin{bmatrix} I_{012}^{0} \\ I_{012}^{1} \\ I_{012}^{2} \end{bmatrix} = A^{-1} I_{abc} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_{bf} \\ I_{cf} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} I_{bf} + I_{cf} \\ \alpha I_{bf} + \alpha^2 I_{cf} \\ \alpha^2 I_{bf} + \alpha I_{cf} \end{bmatrix}$$

$$\begin{aligned} I^0 + I^2 &= I_{bf} + I_{cf} + \alpha^2 I_{bf} + \alpha I_{cf} \\ &= I_{bf} + \alpha^2 I_{bf} + I_{cf} + \alpha I_{cf} \\ &= (1+\alpha^2) I_{bf} + (1+\alpha) I_{cf} \\ &= (-\alpha) I_{bf} + (-\alpha^2) I_{cf} \\ &= -(\alpha I_{bf} + \alpha^2 I_{cf}) \\ &= -I^1 \end{aligned}$$

$$\boxed{I^1 = -(I^0 + I^2)}$$

$$\begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} V_a \\ V_a \\ V_a \end{bmatrix}$$

$$\boxed{V^0 = V^1 = V^2}$$

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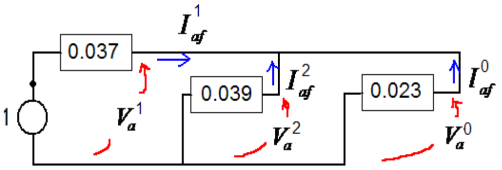
(4) BC-G continued

- Sequence Network Formation – Parallel



$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}\begin{bmatrix} -12.22 \\ 19.42 \\ -7.2 \end{bmatrix}$$

$$I_{fa}^1 = \frac{1.0}{0.037 + 0.039 // 0.023} = 19.42\,pu$$

$$I_{fa}^2 = -19.42 \frac{0.023}{0.039 + 0.023} = -7.2\,pu$$

$$I_{fa}^0 = -19.42 \frac{0.039}{0.039 + 0.023} = -12.22\,pu$$

$$= \begin{bmatrix} 0 \\ 29.45\angle 129° \\ 29.45\angle -129° \end{bmatrix}$$

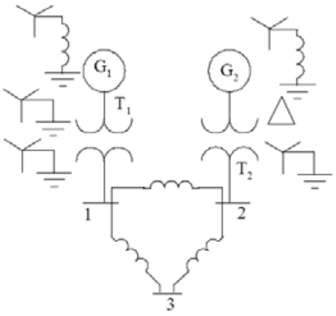
- Actual Fault Currents

(41.84)(29.45)=1232 [A]

Seq_Comp.xmcd

Fault with Fault Impedance, Zf

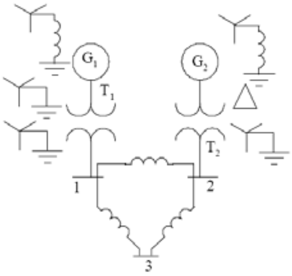
- From the one-line diagram of a simple power system, the neutral of each generator is grounded through a current-limiting reactor of 0.25/3 per unit in a 100-MVA base.
- The generators are running on no-load at their rated voltage and rated frequency with their emfs in phase.
- (Q) Determine the fault current for the following faults:
  - (a) A balanced three-phase fault at bus 3 through fault impedance Z_f=j0.1 pu.
  - (b) A single line-to-ground fault at bus 3 through fault impedance Z_f=j0.1 pu.
  - (c) A line-to-line fault at bus 3 through fault impedance Z_f=j0.1 pu.
  - (d) A double line-to-ground fault at bus 3 through fault impedance Z_f=j0.1 pu.



System Data

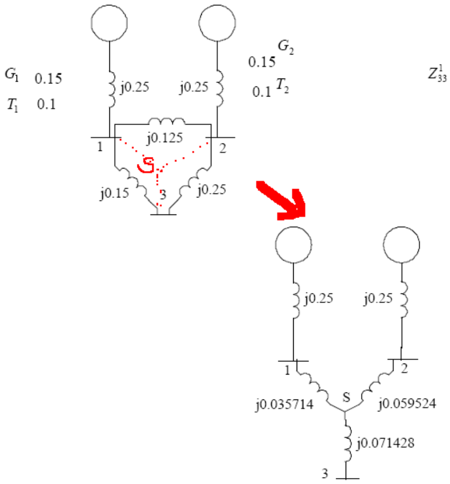
- The system data expressed in per unit on a common 100-MVA base:

Item	Base MVA	Voltage rating	X^1	X^2	X^0
G_1	100	20 kV	0.15	0.15	0.05
G_2	100	20 kV	0.15	0.15	0.05
T_1	100	20/220 kV	0.1	0.1	0.1
T_2	100	20/220 kV	0.1	0.1	0.1
L_{12}	100	220 kV	0.125	0.125	0.3
L_{13}	100	220 kV	0.15	0.15	0.35
L_{23}	100	220 kV	0.25	0.25	0.7125



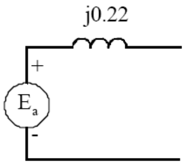
Pre-Processing

- Thevenin equivalent circuit seen from bus 3.



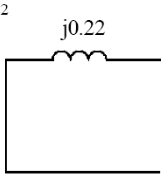
Pos-Seq Impedance at Bus 3

$$Z_{33}^{1} = \frac{(j0.2857143)(j0.3095238)}{j0.5952381} + j0.0714286 = j0.22$$

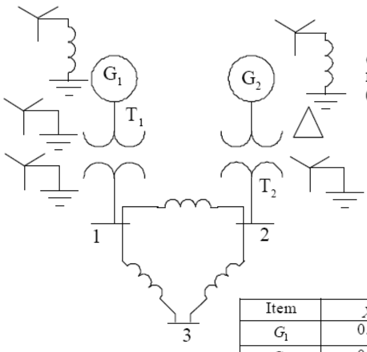


(Neg-Seq = Pos. Seq)

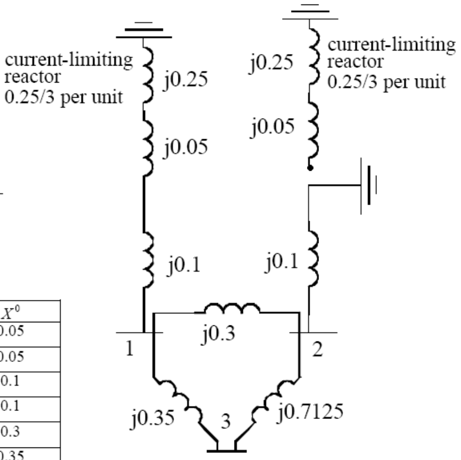
$$Z_{33}^{2} = Z_{33}^{1} = j0.22$$



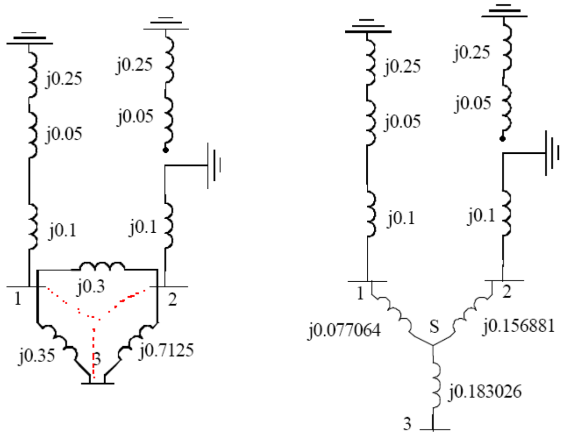
Zero-Seq Circuit



Item	X^0
G_1	0.05
G_2	0.05
T_1	0.1
T_2	0.1
L_{12}	0.3
L_{13}	0.35
L_{23}	0.7125



Final Zero-Seq Circuit

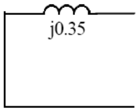


xA := 0.25 + 0.05 + 0.1 + 0.077064

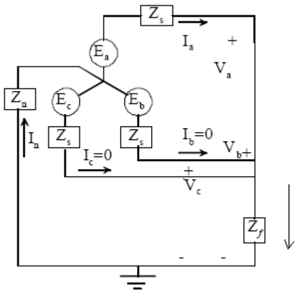
xB := 0.1 + 0.156881

xC := 0.183026

$$Z := xC + \frac{xA \cdot xB}{xA + xB} = 0.35$$



(a) Balanced 3-Phase Fault



$V_a + V_b + V_c = 0$

$I_a + I_b + I_c = 0$

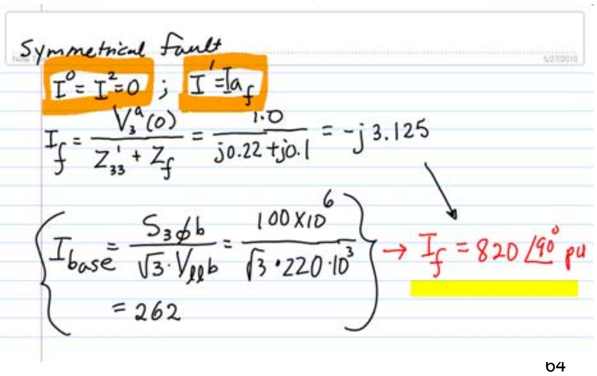
$I_b = a^2 \cdot I_a$

$I_c = a \cdot I_a$

$I_{012} = A^{-1} \cdot I_{abc} = \begin{bmatrix} 0 \\ I_a \\ 0 \end{bmatrix}$

$I^0 = 0$

$I^2 = 0$



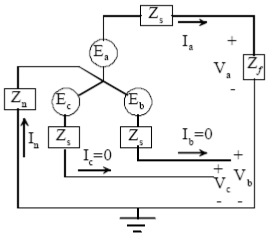
Symmetrical fault

$\boxed{I^0 = I^2 = 0}$; $\boxed{I^1 = I_{a_f}}$

$$I_f = \frac{V_3^a(0)}{Z'_{33} + Z_f} = \frac{1.0}{j0.22 + j0.1} = -j\,3.125$$

$$\left\{ I_{base} = \frac{S_{3\phi b}}{\sqrt{3} \cdot V_{llb}} = \frac{100 \times 10^6}{\sqrt{3} \cdot 220 \cdot 10^3} = 262 \right\} \rightarrow I_f = 820 \angle 90^\circ \text{ pu}$$

(b) Single Line to Ground Fault



$V_a = Z_f \cdot I_a \qquad I_b = I_c = 0$

$I^{012} = A^{-1} \cdot I_{abc} = \frac{1}{3}\begin{bmatrix} I_a \\ I_a \\ I_a \end{bmatrix} \qquad \underline{I^0 = I^1 = I^2 = \left(\frac{1}{3} I_a\right)}$

$V_a = Z_f \cdot 3I^0 \qquad V_a = V^0 + V^1 + V^2$

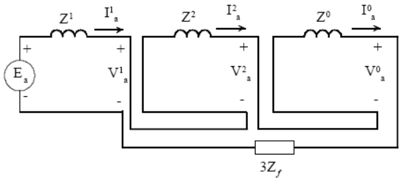
$V^1 = E - Z^1 I^1$

$V^2 = 0 - Z^2 I^2$

$V^0 = 0 - Z^0 I^0$

$Z_f \cdot 3 I_0 = 0 - Z^2 I^2 + E - Z^1 I^1 + 0 - Z^0 I^0$ ($I^2 \leftarrow I^0$, $I^1 \leftarrow I^0$)

$\rightarrow I^0 = \frac{E}{Z^1 + Z^2 + Z^0 + 3 \cdot Z_f}$



$I^0 = I^1 = I^2 \quad \& \quad Z^{012} = Z^1 + Z^2 + (Z^0 + 3 \cdot Z_f)$

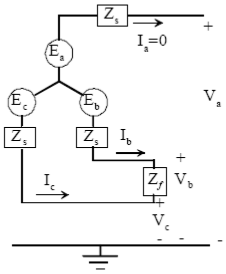
$\Rightarrow I^0 = I^1 = I^2 = \frac{V^1(0)}{Z^{012}} = \frac{1}{j0.22 + j0.22 + j0.35 + (3 \cdot j0.1)}$

$= -j0.9174 \text{pu}$

$I^{abc} = A \cdot I^{012}$

$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I^0 \\ I^1 \\ I^2 \end{pmatrix} = \begin{pmatrix} -j2.7523 \\ 0 \\ 0 \end{pmatrix} \Rightarrow I_f = (2.7523)(262) = \underline{722}$ I_{base}

(c) Line to Line Fault



$I_a = 0, \quad I_b = -I_c$

$V_b - V_c = Z_f \cdot I_b$

$a - a^2 = 1.732i$

$a^2 - a = -1.732i$

$I_{012} = A^{-1} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} \quad I^0 = 0$

$\rightarrow I^1 = \frac{1}{3}(a - a^2) I_b = -I^2$

$V_a = V_0 + V_1 + V_2$

$V_b = V_0 + a^2 V_1 + a V^2$

$V_c = V_0 + a V^1 + a^2 V^2$

$V_b - V_c = (a^2 - a)(V^1 - V^2)$

$V^1 = E - Z^1 I^1$

$V^2 = 0 - Z^2 I^2$

$Z_f \cdot I_b = (a^2 - a)(E - Z^1 I^1 + Z^2 I^2)$

$I_b \leftarrow \frac{3I^1}{a - a^2}$

$3 Z_f \cdot I^1 = \underline{(a^2 - a)(a - a^2)}(E - Z^1 I^1 - Z^2 I^1)$

$a := \exp\left(j \cdot \frac{2 \cdot \pi}{3}\right) \quad a^2 = -0.5 - 0.866i$

$(a^2 - a) \cdot (a - a^2) = 3$

$\therefore I^1 = \frac{E}{Z^1 + Z^2 + Z_f}$

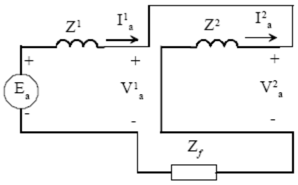
From $I^0 = 0$ & $I^1 = -I^2$

$I^1 = -I^2 = \frac{V^1(0)}{Z^1 + Z^2 + Z_f} = \frac{1}{j0.22 + j0.22 + j0.1}$

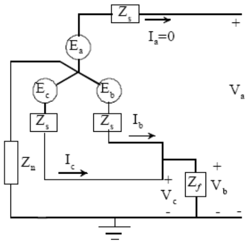
$= -j1.8519$

$I^{abc} = A \cdot I^{012} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} 0 \\ -j1.8519 \\ j1.8519 \end{pmatrix} = \begin{pmatrix} 0 \\ -3.2075 \\ 3.2075 \end{pmatrix}$

$I_b = -I_c = (3.2075)(262) = 842\,(A)$



(d) Double Line to Ground Fault



$I_a = 0 = I^0 + I^1 + I^2 = 0$

$V_b = V_c = Z_f(I_b + I_c)$

$= Z_f(I^0 + a^2 I^1 + a I^2 + I^0 + a I^1 + a^2 I^2)$

$= Z_f(2I^0 \underbrace{- I^1 - I^2}_{I^0}) = 3 \cdot Z_f \cdot I^0$

from

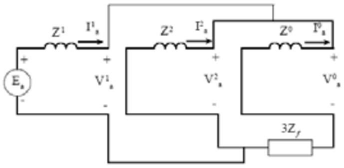
$V_b = V_c$ $\quad$ $\left.\begin{array}{l} V_b = V^0 + a^2 V^1 + a V^2 \\ V_c = V^0 + a V^1 + a^2 V^2 \end{array}\right) V^1 = V^2$

$\rightarrow V_b = V^0 + \underbrace{(a^2 + a)}_{-1} V^1 = 3 \cdot Z_f \cdot I^0$

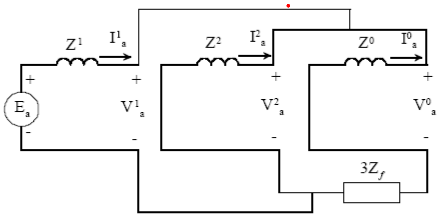
$\rightarrow \quad 0 - I^0 Z^0 - (E - I^1 Z^1) = 3 Z_f I^0$

$I^0 = -\frac{E - Z^1 I^1}{Z^0 + 3Z_f}$

Using V_c : $I^2 = -\frac{E - Z^1 I^1}{Z^2}$



(d) Double line to ground fault - continued



Seq_Comp_Zf.xmcd
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$I_a^1 = \frac{V(0)}{Z^1 + \frac{Z^2(Z^0 + 3Z_f)}{Z^2 + Z^0 + 3Z_f}}$ $\quad$ $I_a^0 = -I_a^1 \frac{Z^2}{Z^2 + Z^0 + 3Z_f}$ $\quad$ $I_a^2 = -I_a^1 \frac{Z^0 + 3Z_f}{Z^2 + Z^0 + 3Z_f}$

$I012 = \begin{pmatrix} 0.658i \\ -2.602i \\ 1.944i \end{pmatrix}$ $\quad$ $Iabc := A \cdot I012 = \begin{pmatrix} 0 \\ -3.936 + 0.987i \\ 3.936 + 0.987i \end{pmatrix}$

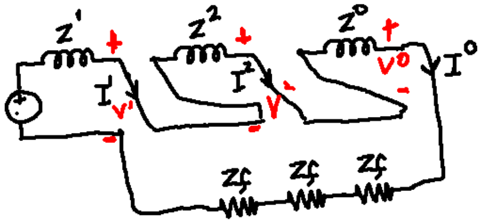
$IabcACT := Ibase \cdot Iabc = \begin{pmatrix} 0 \\ -1.033 \times 10^3 + 258.979i \\ 1.033 \times 10^3 + 258.979i \end{pmatrix}$

Actual Fault Current is the sum of Ib + Ic = $IabcACT_1 + IabcACT_2 = 517.958i$

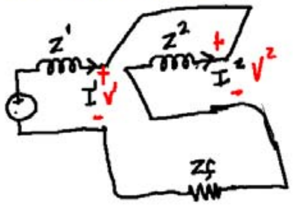
Fault Impedance Placement in Sequence Analysis

3-Phase Symmetrical Fault

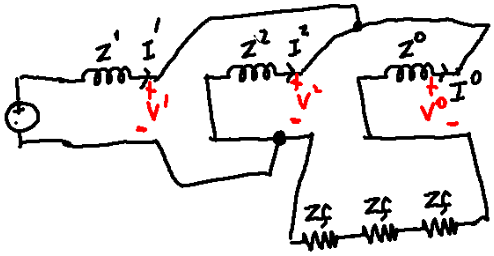
Single Line to Ground Fault



Line to Line Fault

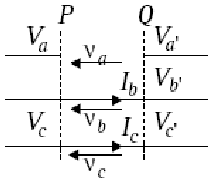


Double Line to Ground Fault

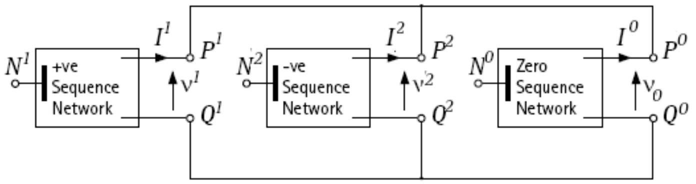


Sequence Circuit in Other Situations

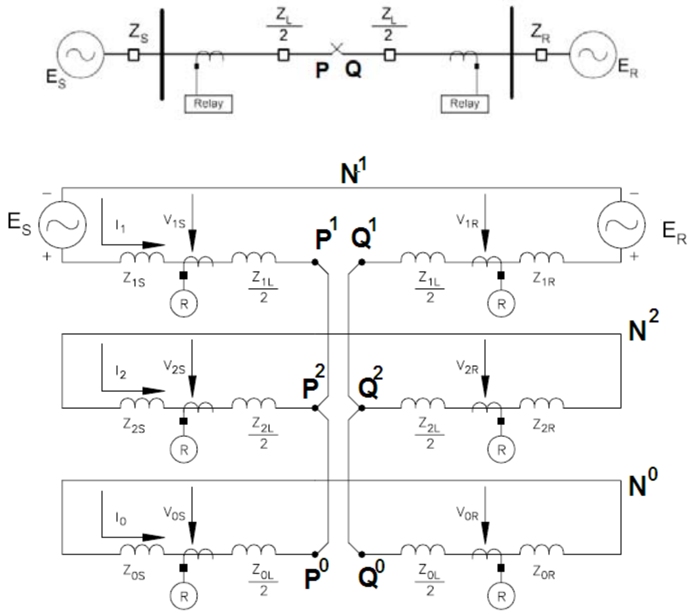
(1. Single Phase Open Circuit)



- Boundary conditions at the fault point
  - Ia =0
    - » $Ia=I^0 + I^1 + I^2 =0$
  - Vb=Vc=0
    - » $V^0=V^1=V^2=Va/3$

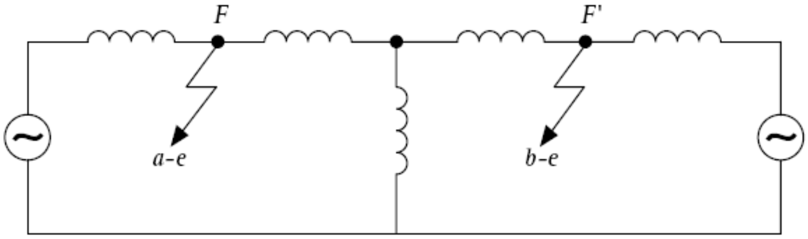


Single-Phase Open Example



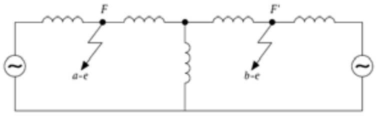
Sequence Circuit of cross-country Fault

- Cross-country fault
  - A situation where there are two faults affecting the same circuit, but in different locations and possibly involving different phases.
  - Example: A-G @F and B-G @F’



Sequence Network

- Boundary Conditions at F
  - Ib=Ic=0
    - » $Ia^0 = Ia^1 = Ia^2$
  - Va=0
    - » $Va^0+Va^1+Va^2=0$
- Boundary Conditions at F’
  - I’a=I’c=0
    - » $I'b^0=I'b^1=I'b^2$
  - V’b=0
    - » $V'b^0+V'b^1+V'b^2=0$
- Conversion of the currents and voltages at point *F* ’ to the sequence currents in the same phase as those at point *F*.

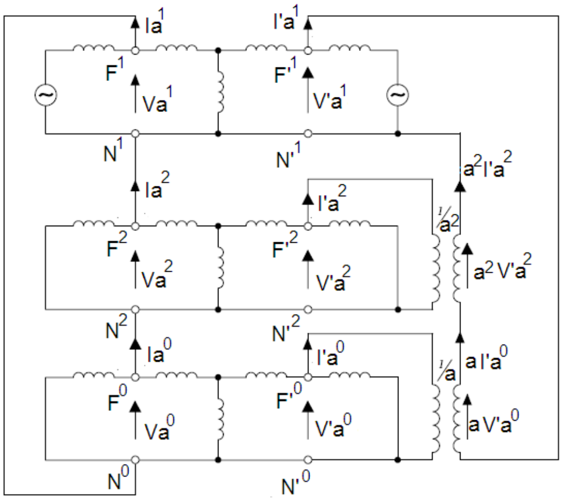


$$I'^{\,1}_{b} = I'^{\,2}_{b} = I'^{\,0}_{b} \rightarrow a^2 I'^{\,1}_{a} = aI'^{\,2}_{a} = I'^{\,0}_{a} \rightarrow I'^{\,1}_{a} = a^2 I'^{\,2}_{a} = aI'^{\,0}_{a}$$

$$V'^{\,1}_{b} + V'^{\,2}_{b} + V'^{\,0}_{b} = 0 \rightarrow a^2 V'^{\,1}_{a} + aV'^{\,2}_{a} + V'^{\,0}_{a} = 0 \rightarrow$$

$$V'^{\,1}_{a} + a^2 V'^{\,2}_{a} + aV'^{\,0}_{a} = 0$$

Final Equivalent Circuit



@F

$Ia^1 = Ia^2 = Ia^0$

$Va^1 + Va^2 + Va^0 = 0$

@F'

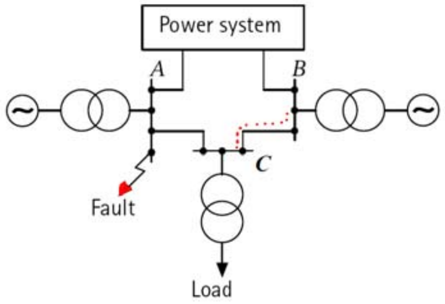
$I'a^1 = a^2 I'a^2 = a I'a^0$

$V'a^1 + a^2 V'a^2 + a V'a^0 = 0$

Phase-Shifting
Transformers

Fault Current Distribution

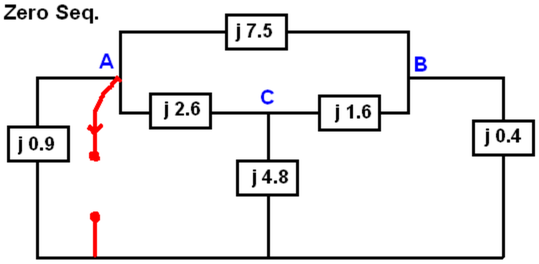
- So far, we focused on the fault current in the faulted branch.
- Practical Importance in the investigation on the effect of a fault in the branches other than the faulted branch → "fault current distribution"
- **Case**: A fault at A and want to find the currents in the line B-C due to the fault.



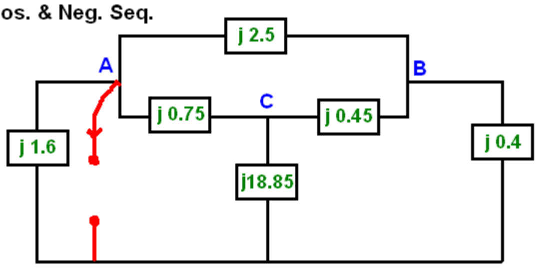
- Assumption: Positive and Negative Sequence Impedances are equal

Fault Current Distribution – Equivalent Sequence Networks (with typical values of impedance)

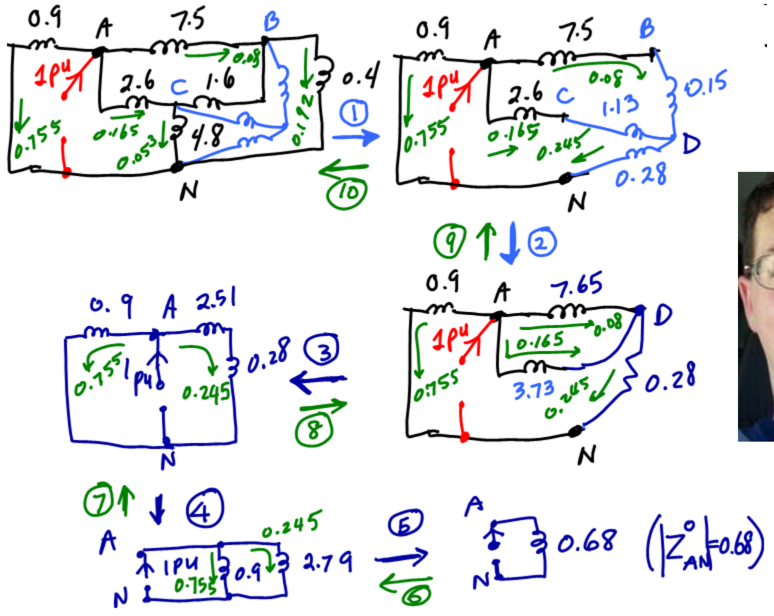
Zero Seq.



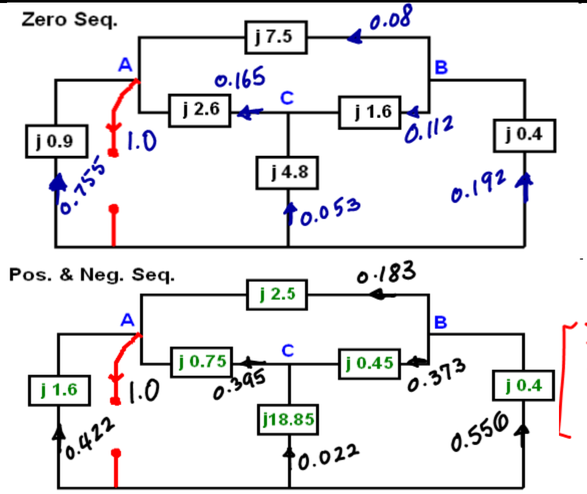
Pos. & Neg. Seq.



Zero-Sequence Fault Current Distribution-works



Zero and Pos & Neg-Sequence Networks Current Distributions



- *I*a’: Phase-a current flowing in the line B-C
- *I*0’ :Zero Seq Current on line B-C
- *I*0: Zero Seq Current at the Faulted Point A.

$$\begin{bmatrix} I_a' \\ I_b' \\ I_c' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I^{0'} \\ I^{1'} \\ I^{2'} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.112\, I^0 \\ 0.373\, I^1 \\ 0.373\, I^2 \end{bmatrix}$$

Fault Current Contribution (A-G Example Case)

$$\begin{bmatrix} I_a' \\ I_b' \\ I_c' \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I^{0'} \\ I^{1'} \\ I^{2'} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0.112\, I^0 \\ 0.373\, I^1 \\ 0.373\, I^2 \end{bmatrix}$$

$$I_a' = \underbrace{0.112}_{C_0} I^0 + \underbrace{0.373}_{C_1} I' + \underbrace{0.373}_{C_1} I^2$$

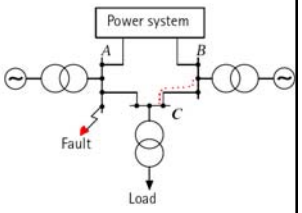
C_0 : Zero Seq. contribution

C_1 : pos. seq. contribution

For A-G fault, $I^0 = I' = I^2$

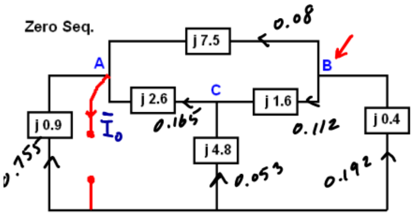
$$I_a' = (2C_1 + C_0) I^0 = 0.858 I^0$$

$$I_b' = I_c' = -C(C_1 - C_0) I^0 = -0.261 I^0$$



Fault Voltage Distribution (A-G Case)

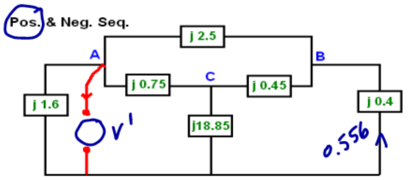
- Voltage at bus B due to the fault at A



$I^0 = I' = I^2$

$V_B^0 = (j0.4)(0.192 I_0)$

$= j\, 0.077 I_0$



$V_B^2 = (0.556)(j0.4) I_0$

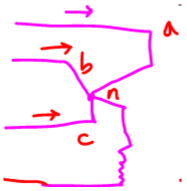
$= j\, 0.222 I_0$

$V_B' = V' - (j2.5)(0.183) I_0$

$\downarrow \; \to V_{Ba}'$, V_{Bb}', & V_{Bc}'

Effect of System Earth on Zero Sequence Quantities

- Observation
  - Zero Sequence Network is connected only in ground fault
  - I^0 flows in the earth path during earth fault.
  - I^0 flow is influenced by the method of earthing



$$I_R = I_a + I_b + I_c$$

$$V_R = V_a + V_b + V_c$$

$$I_{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$$I^0 = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3} I_R$$

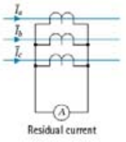
$$V^0 = \frac{1}{3}(V_a + V_b + V_c) = \frac{1}{3} V_R$$

$$I_R = 3I^0$$

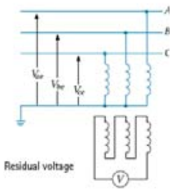
$$V_R = 3V^0$$

Residual Voltage and Residual Current

- Residual Current and Residual Voltage
  - Normal System Operation (Balanced)
    - All symmetrical (even capacitance)
    - No current flow between any two earth points
  - Unbalanced Earth Fault Case
    - Potential Difference between the earth points, resulting in a current flow in the earthed path
    - Vector sum of phase currents (phase voltages)



Residual current



Residual voltage

$$\left.\begin{aligned} \bar{I}_R &= \bar{I}_a + \bar{I}_b + \bar{I}_c \\ \overline{V}_R &= \overline{V}_{ae} + \overline{V}_{be} + \overline{V}_{ce} \end{aligned}\right\}$$

$$\left.\begin{aligned} \underline{\bar{I}_R = 3\bar{I}_0} \\ \underline{\overline{V}_R = 3\overline{V}_0} \end{aligned}\right\}$$

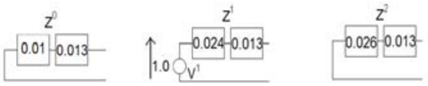
$$\left.\begin{aligned} \overline{V}_{ae} &= \overline{V}_{an} + \overline{V}_{ne} \\ \overline{V}_{be} &= \overline{V}_{bn} + \overline{V}_{ne} \\ \overline{V}_{ce} &= \overline{V}_{cn} + \overline{V}_{ne} \end{aligned}\right\}$$

since $\overline{V}_{bn} = a^2\,\overline{V}_{an}$, $\overline{V}_{cn} = a\overline{V}_{an}$ then:

$$\underline{\overline{V}_R = 3\overline{V}_{ne}}$$

The ratio of residual and phase values

- Fault Point as Reference – Point of Injection of Unbalanced Voltages (Currents) Into the Balanced System
- Practical Benefits (in distribution of residual quantities through the system) of
  - Residual Voltage in relation to the Normal System Voltage
  - Residual Current in relation to the 3-Phase Fault Current
  - The characteristics can be expressed in terms of the system impedance ratio (Z^0/Z^1)



$$I_p = \frac{V'^1}{Z^1} \qquad I^0 = \frac{V'^0}{Z^0}$$

$$\frac{I_R}{I_p} = \frac{3I^0}{\frac{V'^1}{Z^1}} = \frac{3\frac{V'^0}{Z^0}}{\frac{V'^1}{Z^1}} = \frac{3\, {}^{V'^0}\!/\!_{V'^1}}{\boxed{{}^{Z^0}\!/\!_{Z^1}}}$$

System Impedance Ratio (Z^0/Z^1 = K)

- Sequence Impedance Ratio Viewed from Fault.
- Dependent upon method of grounding, fault position, and system operating arrangement
- Characterization of the relationship between Residual and Phase values
- **Z^1**
  - **is mostly Reactance only** while
- **Z^0**
  - contains both earth reactance and resistance
- Approximation of K (= Z^0/Z^1)

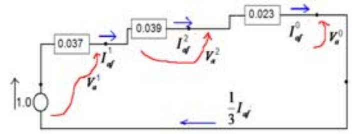
$$K = \frac{Z^0}{Z^1} = \frac{R^0 + jX^0}{R^1 + jX^1} \approx \frac{X^0}{X^1} + \frac{R^0}{jX^1} = \frac{X^0}{X^1} - j\frac{R^0}{X^1}$$

I_R/I_p and V_R/V_P for A-G Case

$$I_R = 3I^0 = 3\left(\frac{V^1}{Z^1+Z^2+Z^0}\right) = 3\left(\frac{V^1}{2Z^1+Z^0}\right)$$

$$= \frac{3\cdot V^1/Z^1}{2+Z^0/Z^1} = \frac{3}{2+K}\cdot\frac{V^1}{Z^1} = \frac{3}{2+K}\cdot I_P$$

$$\boxed{\therefore \frac{I_R}{I_P} = \frac{3}{2+K}}$$



$$V_R = 3V^0 = 3I^0Z^0 = \left(\frac{3V^1}{2Z^1+Z^0}\right)\cdot Z^0$$

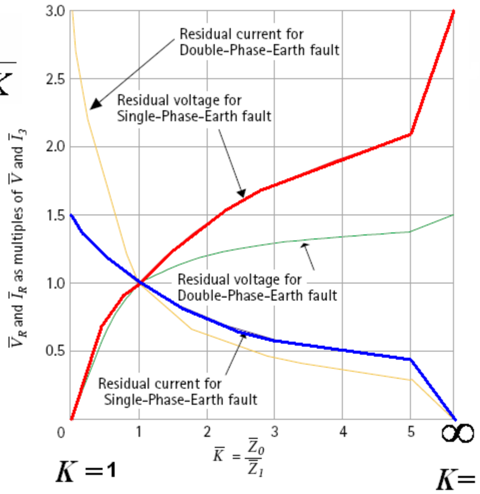
$$= \frac{3\cdot Z^0/Z^1}{2+Z^0/Z^1}\cdot V^1 = \frac{3K}{2+K}\cdot V^1$$

$$\boxed{\therefore \frac{V_R}{V^1} = \frac{3K}{2+K}}$$

$$\boxed{\frac{\dfrac{V_R}{V^1} = \dfrac{3K}{2+K}}{\dfrac{I_R}{I_P} = \dfrac{3}{2+K}} = \frac{\dfrac{V_R}{I_R}}{\dfrac{V^1}{I_P}} = K}$$

Variation of Residual Voltage and Current

$$\frac{V_R}{V^1} = \frac{3K}{2+K}$$



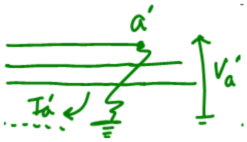
$$\frac{I_R}{I_P} = \frac{3}{2+K}$$

$K = 1$ **Solid Neutral** ← **Impedance Neutral** → $K = \infty$ **Isolated Neutral**

Residual Compensation and *k* Factor

$$\underline{V_a'} = I^{1'}Z^1 + I^{2'}Z^2 + I^{0'}Z^0 \rightarrow \underline{(I^{1'} + I^{2'})Z^1 + I^{0'}Z^0}$$

$$I_a' = I^{1'} + I^{2'} + I^{0'} \rightarrow \underline{I^{1'} + I^{2'} = I_a' - I^{0'}}$$



$$3I^{0'} = I_a' + I_b' + I_c' = I_r'$$

$$V_a' = (I_a' - I^{0'})Z^1 + I^{0'}Z^0 = Z^1 I_a' + I^{0'}(Z^0 - Z^1)$$

$$= Z^1 I_a' + \frac{I_r'(Z^0 - Z^1)}{3}$$

$$= Z^1 I_a' + Z^1 \cdot \frac{I_r'(Z^0 - Z^1)}{3 \cdot Z^1}$$

$$= Z^1\left(I_a' + kI_r'\right)$$

$$k = \frac{Z^0 - Z^1}{3Z^1} = \frac{\frac{Z^0}{Z^1} - 1}{3}$$

Summary

- Time Waveform and Vector/Phasor Expression
- Thevenin Approach in Fault Current Calculation
- Ybus – Zbus Approach in Systematic System Analysis
- Asymmetric Fault Cases
  - Symmetrical Component Approach
  - Fault Distribution
  - System Impedance Ratio ($K = Z^0/Z^1$)
  - Residual Compensation Factor (k)