FAULT DETECTION AND LOCATION IN DISTRIBUTION SYSTEMS

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FINAL EXAM
ALL TEXT ADDITIONS TO GIVEN TEMPLATE FILE IN GREEN COLOUR
(Style Normal_Valtari)

INPUT DATA

$\delta := 0.001$ **Voltage Phase Angle**

$es := 70 \cdot e^{j \cdot \delta \cdot deg}$ **Source S Voltage** $es = 70 + 1.222i \times 10^{-3}$

$er := 0$

Source R Voltage is zero in patent 5

$Z1S := 4.104 + j \cdot 11.276$ **Source S Positive Sequence Impedance**

$Z0S := 25.357 + j \cdot 54.378$ **Source S Zero Sequence Impedance**

$Z1R := 0.518 + j \cdot 1.932$ **Source R Positive Sequence Impedance**

$Z0R := 3 \cdot Z1R$ **Source S Zero Sequence Impedance**

$Z1L := 1.035 + j \cdot 3.864$ **Positive Sequence Line Impedance**

$Z0L := 3 \cdot Z1L$ **Zero Sequence Line Impedance**

$Z0L := 30 + 39i$ $Z1L := 10 + 13i$ **Corrected values lecture xmcd (creelman)**

$Z0S := 0$ $Z1S := 0$ **ZS was set to zero in the patent 5**

Fault Location

$m := 0.4$

Fault Impedances (for AG fault case) $INF := 10^{10}$

$ZFA := 0 + j \cdot 0$ $ZFB := INF + j \cdot 0$ $ZFC := INF + j \cdot 0$

Fault Resistance

$$ZFG := 0.85 + j \cdot 0$$

CONSTANTS

$rad := 1$ $\quad deg := \frac{\pi}{180} \cdot rad$

Operator

$a := e^{j \cdot 120 \cdot deg}$ $\quad a = -0.5 + 0.866i$

$$BAL := \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \quad one := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad zero := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Three phase voltages at S and R

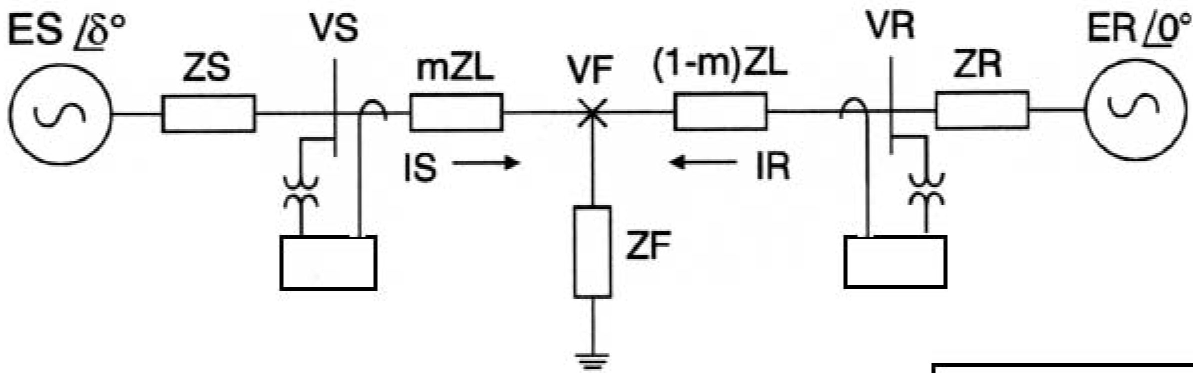
$ES := es \cdot BAL$

$$ES = \begin{pmatrix} 70 + 1.222i \times 10^{-3} \\ -34.999 - 60.622i \\ -35.001 + 60.621i \end{pmatrix}$$

$ER := er \cdot BAL$

$$ER = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

CIRCUIT EQUATION



$$
\begin{aligned}
ES &= [ZS + m \cdot ZL] \cdot IS + 0 + VF \\
ER &= + 0 + [ZR + (1\text{-}m) \cdot ZL] \cdot IR + VF \\
0 &= - ZF \cdot IS - ZF \cdot IR + VF
\end{aligned}
\quad \longrightarrow \quad
\begin{aligned}
ES &= ZSS \cdot IS + 0 + VF \\
ER &= + 0 + ZRR \cdot IR + VF \\
0 &= - ZF \cdot IS - ZF \cdot IR + VF
\end{aligned}
$$

$$
\left(
\begin{aligned}
ZSS &= [ZS + m \cdot ZL] \\
ZRR &= [ZR + (1\text{-}m) \cdot ZL]
\end{aligned}
\right)
$$

In 3-phase matrix form, the equation looks like this:

$$
\begin{bmatrix} ES \\ ER \\ null \end{bmatrix} = \begin{bmatrix} ZSS & zero & one \\ zero & ZRR & one \\ -ZF & -ZF & one \end{bmatrix} \begin{bmatrix} IS \\ IR \\ VF \end{bmatrix}
$$

How do we form the soure impedance ZS and ZR?

Let us consider the link between 3-phase circuit and symmetrical components

Coversion of positive sequence and zero sequence impedances to Self and Mutual impedances

$$zs(z0,z1) := \frac{2\cdot z1 + z0}{3} \qquad zm(z0,z1) := \frac{z0 - z1}{3}$$

Conversion Matrix Format

$$Z(z0,z1) := \begin{pmatrix} zs(z0,z1) & zm(z0,z1) & zm(z0,z1) \\ zm(z0,z1) & zs(z0,z1) & zm(z0,z1) \\ zm(z0,z1) & zm(z0,z1) & zs(z0,z1) \end{pmatrix}$$

Now Conversion

$$ZS := Z(Z0S, Z1S) \qquad ZL := Z(Z0L, Z1L) \qquad ZR := Z(Z0R, Z1R)$$

$$ZS = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad ZR = \blacksquare$$

$$ZL = \begin{pmatrix} 16.667 + 21.667i & 6.667 + 8.667i & 6.667 + 8.667i \\ 6.667 + 8.667i & 16.667 + 21.667i & 6.667 + 8.667i \\ 6.667 + 8.667i & 6.667 + 8.667i & 16.667 + 21.667i \end{pmatrix}$$

Source and Line Impedances to the Fault

$$ZSS := ZS + m\cdot ZL \qquad ZSS = \begin{pmatrix} 6.667 + 8.667i & 2.667 + 3.467i & 2.667 + 3.467i \\ 2.667 + 3.467i & 6.667 + 8.667i & 2.667 + 3.467i \\ 2.667 + 3.467i & 2.667 + 3.467i & 6.667 + 8.667i \end{pmatrix}$$

$$ZRR := ZR + (1 - m)\cdot ZL \qquad ZRR = \begin{pmatrix} 10.863 + 16.22i & 4.345 + 6.488i & 4.345 + 6.488i \\ 4.345 + 6.488i & 10.863 + 16.22i & 4.345 + 6.488i \\ 4.345 + 6.488i & 4.345 + 6.488i & 10.863 + 16.22i \end{pmatrix}$$

Build System Part of the Impedance Matrix

$$\begin{bmatrix} ES \\ ER \\ null \end{bmatrix} = \begin{bmatrix} ZSS & zero & one \\ zero & ZRR & one \\ -ZF & -ZF & one \end{bmatrix} \begin{bmatrix} IS \\ IR \\ VF \end{bmatrix}$$

$$ES = \begin{bmatrix} ES_a \\ ES_b \\ ES_c \end{bmatrix} \quad ER = \begin{bmatrix} ER_a \\ ER_b \\ ER_c \end{bmatrix} \quad VF = \begin{bmatrix} VF_a \\ VF_b \\ VF_c \end{bmatrix} \quad IS = \begin{bmatrix} IS_a \\ IS_b \\ IS_c \end{bmatrix} \quad IR = \begin{bmatrix} IR_a \\ IR_b \\ IR_c \end{bmatrix}$$

$$ZTOP := augment(augment(ZSS, zero), one)$$

$$ZTOP = \begin{pmatrix} 6.667+8.667i & 2.667+3.467i & 2.667+3.467i & 0 & 0 & 0 & 1 & 0 & 0 \\ 2.667+3.467i & 6.667+8.667i & 2.667+3.467i & 0 & 0 & 0 & 0 & 1 & 0 \\ 2.667+3.467i & 2.667+3.467i & 6.667+8.667i & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$ZMID := augment(augment(zero, ZRR), one)$$

$$ZMID = \begin{pmatrix} 0 & 0 & 0 & 10.863+16.22i & 4.345+6.488i & 4.345+6.488i & 1 & 0 & 0 \\ 0 & 0 & 0 & 4.345+6.488i & 10.863+16.22i & 4.345+6.488i & 0 & 1 & 0 \\ 0 & 0 & 0 & 4.345+6.488i & 4.345+6.488i & 10.863+16.22i & 0 & 0 & 1 \end{pmatrix}$$

$$ZSYS := stack(ZTOP, ZMID)$$

$$ZSYS = \begin{pmatrix} 6.667+8.667i & 2.667+3.467i & 2.667+3.467i & 0 & 0 & 0 & 1 & 0 & 0 \\ 2.667+3.467i & 6.667+8.667i & 2.667+3.467i & 0 & 0 & 0 & 0 & 1 & 0 \\ 2.667+3.467i & 2.667+3.467i & 6.667+8.667i & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 10.863+16.22i & 4.345+6.488i & 4.345+6.488i & 1 & 0 & 0 \\ 0 & 0 & 0 & 4.345+6.488i & 10.863+16.22i & 4.345+6.488i & 0 & 1 & 0 \\ 0 & 0 & 0 & 4.345+6.488i & 4.345+6.488i & 10.863+16.22i & 0 & 0 & 1 \end{pmatrix}$$

Pre-fault conditions:

$$ZPRE := ZS + ZL + ZR$$

$$ZPRE = \begin{pmatrix} 17.53+24.887i & 7.012+9.955i & 7.012+9.955i \\ 7.012+9.955i & 17.53+24.887i & 7.012+9.955i \\ 7.012+9.955i & 7.012+9.955i & 17.53+24.887i \end{pmatrix}$$

$$IPRE := ZPRE^{-1} \cdot (ES - ER)$$

$$IPRE = \begin{pmatrix} 2.207-3.133i \\ -3.817-0.345i \\ 1.61+3.478i \end{pmatrix}$$

Pre_fault voltage at S end

$$VSP := ES - ZS \cdot IPRE$$

$$VSP = \begin{pmatrix} 70+1.222i\times 10^{-3} \\ -34.999-60.622i \\ -35.001+60.621i \end{pmatrix} \qquad ES = \begin{pmatrix} 70+1.222i\times 10^{-3} \\ -34.999-60.622i \\ -35.001+60.621i \end{pmatrix}$$

Build the voltage Vector

$$null := (0 \;\; 0 \;\; 0)$$

$$E := stack\left(stack(ES, ER), null^T\right)$$

$$TS := augment(augment(one, zero), zero)$$

$$TR := augment(augment(zero, one), zero)$$

$$E = \begin{pmatrix} 70 + 1.222i \times 10^{-3} \\ -34.999 - 60.622i \\ -35.001 + 60.621i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$TS = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$TR = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Building Fault Part of the Impedance Matrix:

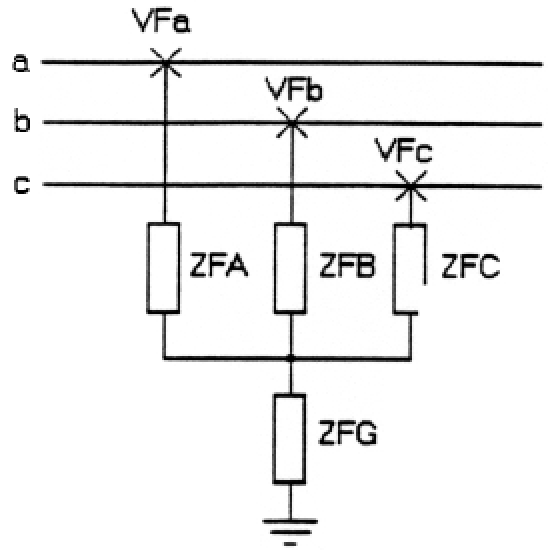
- the fault impedance ZF
- voltage drop due to each phase current flowing individually in the phase-fault impedances (ZFA, ZFB, or ZFC) and mutually in the ground-fault impedance ZFG.

$$ZF = \begin{bmatrix} ZFAG & ZFG & ZFG \\ ZFG & ZFBG & ZFG \\ ZFG & ZFG & ZFCG \end{bmatrix} \quad \text{where: } \begin{cases} ZFAG = ZFA + ZFG \\ ZFBG = ZFB + ZFG \\ ZFCG = ZFC + ZFG \end{cases}$$

$ZFAG := ZFA + ZFG$

$ZFBG := ZFB + ZFG$

$ZFCG := ZFC + ZFG$



$ZFAG = 0.85$

$ZFBG = 1 \times 10^{10}$

$$ZF := \begin{pmatrix} ZFAG & ZFG & ZFG \\ ZFG & ZFBG & ZFG \\ ZFG & ZFG & ZFCG \end{pmatrix}$$

$$ZF = \begin{pmatrix} 0.85 & 0.85 & 0.85 \\ 0.85 & 1 \times 10^{10} & 0.85 \\ 0.85 & 0.85 & 1 \times 10^{10} \end{pmatrix}$$

$$\begin{bmatrix} ES \\ ER \\ null \end{bmatrix} = \begin{bmatrix} ZSS & zero & one \\ zero & ZRR & one \\ -ZF & -ZF & one \end{bmatrix} \begin{bmatrix} IS \\ IR \\ VF \end{bmatrix}$$

$FABCG := \text{augment}(\text{augment}(-ZF, -ZF), one)$

$$FABCG = \begin{pmatrix} -0.85 & -0.85 & -0.85 & -0.85 & -0.85 & -0.85 & 1 & 0 & 0 \\ -0.85 & -1 \times 10^{10} & -0.85 & -0.85 & -1 \times 10^{10} & -0.85 & 0 & 1 & 0 \\ -0.85 & -0.85 & -1 \times 10^{10} & -0.85 & -0.85 & -1 \times 10^{10} & 0 & 0 & 1 \end{pmatrix}$$

FINAL Z MATRIX

$$ZABCG := stack(ZSYS, FABCG)$$

$$ZABCG = \begin{pmatrix} 6.667+8.667i & 2.667+3.467i & 2.667+3.467i & 0 & 0 & 0 & 1 & 0 & 0 \\ 2.667+3.467i & 6.667+8.667i & 2.667+3.467i & 0 & 0 & 0 & 0 & 1 & 0 \\ 2.667+3.467i & 2.667+3.467i & 6.667+8.667i & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 10.863+16.22i & 4.345+6.488i & 4.345+6.488i & 1 & 0 & 0 \\ 0 & 0 & 0 & 4.345+6.488i & 10.863+16.22i & 4.345+6.488i & 0 & 1 & 0 \\ 0 & 0 & 0 & 4.345+6.488i & 4.345+6.488i & 10.863+16.22i & 0 & 0 & 1 \\ -0.85 & -0.85 & -0.85 & -0.85 & -0.85 & -0.85 & 1 & 0 & 0 \\ -0.85 & -1\times10^{10} & -0.85 & -0.85 & -1\times10^{10} & -0.85 & 0 & 1 & 0 \\ -0.85 & -0.85 & -1\times10^{10} & -0.85 & -0.85 & -1\times10^{10} & 0 & 0 & 1 \end{pmatrix}$$

$$YABCG := ZABCG^{-1}$$

Fault Currents:

$$IABCG := YABCG \cdot E$$

$$E = \begin{pmatrix} 70+1.222i\times10^{-3} \\ -34.999-60.622i \\ -35.001+60.621i \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad IABCG = \begin{pmatrix} 4.864-5.872i \\ -3.817-0.345i \\ 1.61+3.478i \\ -0.823+1.505i \\ 3.817+0.345i \\ -1.61-3.478i \\ 3.435-3.711i \\ -38.101-41.302i \\ -39.932+36.43i \end{pmatrix}$$

S - End Fault Currents:

$IS := TS \cdot IABCG$

$$IS = \begin{pmatrix} 4.864 - 5.872i \\ -3.817 - 0.345i \\ 1.61 + 3.478i \end{pmatrix}$$

R - End Fault Currents:

$IR := TR \cdot IABCG$

$$IR = \begin{pmatrix} -0.823 + 1.505i \\ 3.817 + 0.345i \\ -1.61 - 3.478i \end{pmatrix}$$

S - End Voltages

$VS := ES - ZS \cdot IS$

$$VS = \begin{pmatrix} 70 + 1.222i \times 10^{-3} \\ -34.999 - 60.622i \\ -35.001 + 60.621i \end{pmatrix}$$

Line Prefault Load Currents from S Bus

$Ia := IPRE_0$ $\quad |Ia| = 3.833$ $\quad \frac{arg(Ia)}{deg} = -54.838$ $\quad 0.32 \cdot \frac{180}{3.14} = 18.344$

$Ib := IPRE_1$ $\quad |Ib| = 3.833$ $\quad \frac{arg(Ib)}{deg} = -174.838$

$$IPRE = \begin{pmatrix} 2.207 - 3.133i \\ -3.817 - 0.345i \\ 1.61 + 3.478i \end{pmatrix}$$

$Ic := IPRE_2$ $\quad |Ic| = 3.833$ $\quad \frac{arg(Ic)}{deg} = 65.162$

Line Prefault Voltages at S Bus

$Va := VSP_0$ $\quad |Va| = 70$ $\quad \frac{arg(Va)}{deg} = 1 \times 10^{-3}$

$$VSP = \begin{pmatrix} 70 + 1.222i \times 10^{-3} \\ -34.999 - 60.622i \\ -35.001 + 60.621i \end{pmatrix}$$

$Vb := VSP_1$ $\quad |Vb| = 70$ $\quad \frac{arg(Vb)}{deg} = -119.999$

$Vc := VSP_2$ $\quad |Vc| = 70$ $\quad \frac{arg(Vc)}{deg} = 120.001$

Line Fault Currents from S Bus

$Iasf := IS_0$ $\quad |Iasf| = 7.624$ $\quad \frac{arg(Iasf)}{deg} = -50.363$

$$IS = \begin{pmatrix} 4.864 - 5.872i \\ -3.817 - 0.345i \\ 1.61 + 3.478i \end{pmatrix}$$

$Ibsf := IS_1$ $\quad |Ibsf| = 3.833$ $\quad \frac{arg(Ibsf)}{deg} = -174.838$

$Icsf := IS_2$ $\quad |Icsf| = 3.833$ $\quad \frac{arg(Icsf)}{deg} = 65.162$

Line Fault Currents from R Bus

$$IR = \begin{pmatrix} -0.823 + 1.505i \\ 3.817 + 0.345i \\ -1.61 - 3.478i \end{pmatrix}$$

$Iarf := IR_0$ $\quad |Iarf| = 1.715$ $\quad \frac{arg(Iarf)}{deg} = 118.661$

$Ibrf := IR_1$ $\quad |Ibrf| = 3.833$ $\quad \frac{arg(Ibrf)}{deg} = 5.162$

$Icrf := IR_2$ $\quad |Icrf| = 3.833$ $\quad \frac{arg(Icrf)}{deg} = -114.838$

$$VSP = \begin{pmatrix} 70 + 1.222i \times 10^{-3} \\ -34.999 - 60.622i \\ -35.001 + 60.621i \end{pmatrix}$$

Line Fault Voltages at S Bus

$Vaf := VS_0$ $\quad |Vaf| = 70$ $\quad \frac{arg(Vaf)}{deg} = 1 \times 10^{-3}$

$Vbf := VS_1$ $\quad |Vbf| = 70$ $\quad \frac{arg(Vbf)}{deg} = -119.999$

$$VS = \begin{pmatrix} 70 + 1.222i \times 10^{-3} \\ -34.999 - 60.622i \\ -35.001 + 60.621i \end{pmatrix}$$

$Vcf := VS_2$ $\quad |Vcf| = 70$ $\quad \frac{arg(Vcf)}{deg} = 120.001$

Residual Current Ir

$$Ir := \sum_{j=0}^{2} IS_j \qquad Ir = 2.657 - 2.738i$$

$$IS = \begin{pmatrix} 4.864 - 5.872i \\ -3.817 - 0.345i \\ 1.61 + 3.478i \end{pmatrix}$$

$$IrPRE := \sum_{j=0}^{2} IPRE_j = 1.776i \times 10^{-15}$$

Phase A SLG Fault

$$Va = m \cdot Z1L \cdot (Ia + k_0 \cdot Ir) + R_F \cdot I_F$$

$$m = \frac{Va - R_F \cdot I_F}{Z1L \cdot (Ia + k_0 \cdot Ir)}$$

$Vaf = 70 + 1.222i \times 10^{-3}$ $\quad Z1L = 10 + 13i$

$Iasf = 4.864 - 5.872i$

$Rf := Re(ZFAG)$

$$k0 := \frac{Z0L - Z1L}{3 \cdot Z1L}$$

$Rf = 0.85$

$Ir = 2.657 - 2.738i$

$If := Iasf + Iarf = 4.041 - 4.366i$ **Fault Current**

Impedance to the Fault

$$mCalc := \frac{(Vaf - Rf \cdot If)}{(Iasf + k0 \cdot Ir) \cdot Z1L} = 0.4$$

Fault Distance

$|mCalc| = 0.4$

$m = 0.4$

$|Z1L| = 16.401$

Line Length

$$Ds := \frac{Iasf}{If}$$ **Distrbution Factor**

$$Ns := \frac{Iasf}{Iasf - IPRE_0} = 1.992 - 0.157i$$

Loading Factor

$m = 0.4$

$$mCalc2 := \frac{1}{Z1L} \cdot \left(\frac{Vaf}{Iasf + k0 \cdot Ir} - \frac{Rf}{Ds \cdot Ns} \right) = 0.405 - 9.125i \times 10^{-3}$$

Fault Distance

$|mCalc2| = 0.405$

Zero Sequence Current Distribution Factor, Dn

$$d_s = \frac{Z_H + (1-m)Z_L}{Z_G + Z_H + Z_L}$$

$arg(Ds) = -0.055$

$$Ds := \frac{Z0R + (1 - m) \cdot Z0L}{Z0S + Z0R + Z0L} = 0.641 + 0.015i$$

$beta := arg(Ds) = 0.024$

$theta := -beta = -0.024$

distance L (which is supposed to be same as m) by [modified] Takagi Method

$$L := \frac{\mathrm{Im}\left[Vaf\,\overline{\left(Ir\cdot e^{j\cdot theta}\right)}\right]}{\mathrm{Im}\left[Z1L\cdot(Iasf + k0\cdot Ir)\cdot\overline{\left(Ir\cdot e^{j\cdot theta}\right)}\right]}$$

$L = 0.4$

So How do we generate digital signals of Voltage and Current of the Simulation 4 Cycles with 7680 samples per second (128 samples per cycle in 60HZ system)?

$k := 0..511$

$delT := 0.0001302$

$\frac{1}{delT} = 7.68\times 10^{3}$

$\frac{7680}{60} = 128$

$Van_k := \left|VSP_0\right|\cdot\sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(VSP_0\right)\right)$

$T1_k := k\cdot delT$

$T2_k := 512\cdot delT + k\cdot delT$

$T3_k := 1024\cdot delT + k\cdot delT$

$Vbn_k := \left|VSP_1\right|\cdot\sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(VSP_1\right)\right)$

$Vcn_k := \left|VSP_2\right|\cdot\sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(VSP_2\right)\right)$

$Vaf_k := \left|VS_0\right|\cdot\sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(VS_0\right)\right)$

$Vbf_k := \left|VS_1\right|\cdot\sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(VS_1\right)\right)$

$Vcf_k := \left|VS_2\right|\cdot\sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(VS_2\right)\right)$

$Ian_k := \left|IPRE_0\right|\cdot\sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(IPRE_0\right)\right)$

$Ibn_k := \left|IPRE_1\right|\cdot\sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(IPRE_1\right)\right)$

$Icn_k := \left|IPRE_2\right|\cdot\sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(IPRE_2\right)\right)$

$T1_k =$

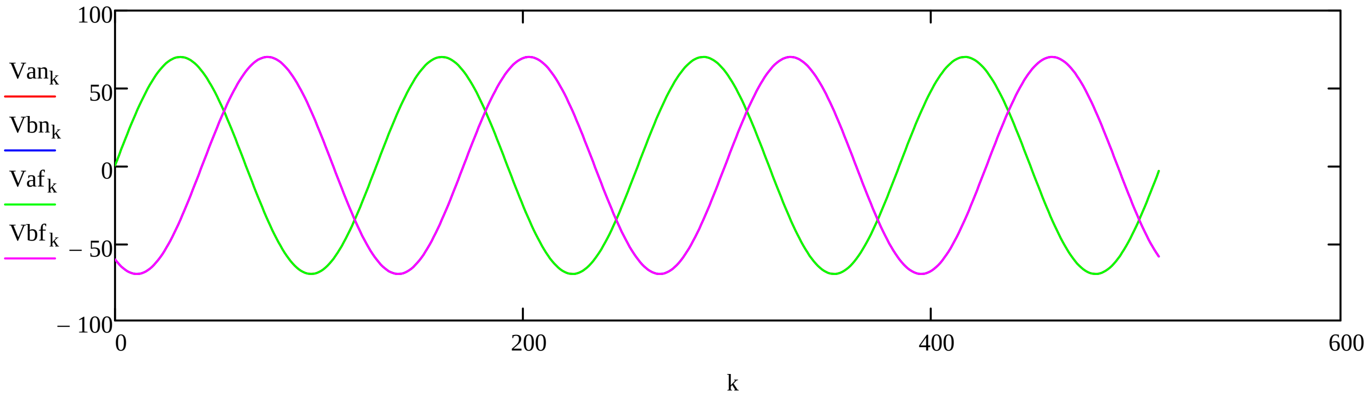
0
$1.302\cdot 10^{-4}$
$2.604\cdot 10^{-4}$
$3.906\cdot 10^{-4}$
$5.208\cdot 10^{-4}$
$6.51\cdot 10^{-4}$
$7.812\cdot 10^{-4}$
$9.114\cdot 10^{-4}$
$1.042\cdot 10^{-3}$
$1.172\cdot 10^{-3}$
$1.302\cdot 10^{-3}$

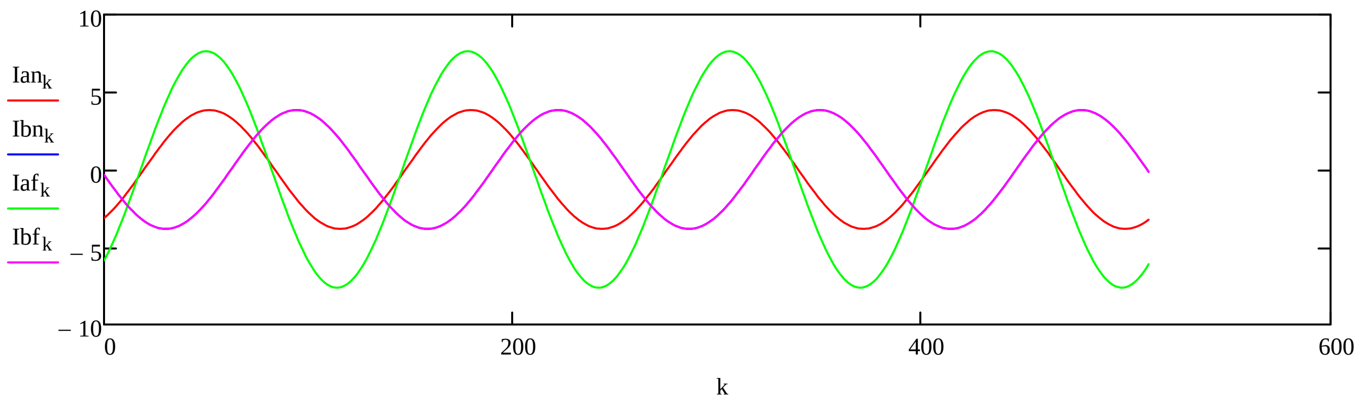
$$Iaf_k := \left|IS_0\right| \cdot \sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(IS_0\right)\right)$$

$$Ibf_k := \left|IS_1\right| \cdot \sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(IS_1\right)\right)$$

$$Icf_k := \left|IS_2\right| \cdot \sin\left(2\cdot\pi\cdot 60\cdot k\cdot delT + \arg\left(IS_2\right)\right)$$

$1.432\cdot 10^{-3}$
$1.562\cdot 10^{-3}$
$1.693\cdot 10^{-3}$
$1.823\cdot 10^{-3}$
...





Let us make Normal (4 cycle)+ Fault (4 cycle) +Normal (4 cycle)

$$Seg1 := augment(T1, Ian, Ibn, Icn, Van, Vbn, Vcn)$$

$$Seg2 := augment(T2, Iaf, Ibf, Icf, Vaf, Vbf, Vcf)$$

$Seg3 := augment(T3, Ian, Ibn, Icn, Van, Vbn, Vcn)$

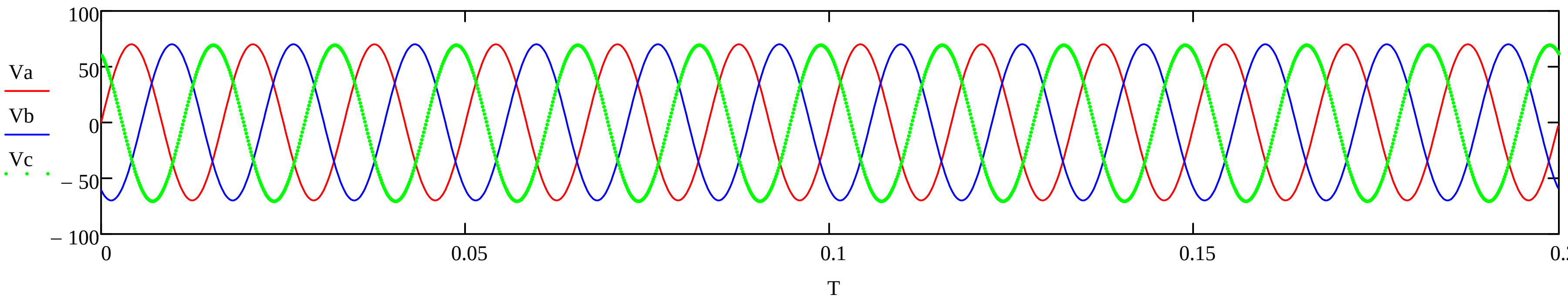
$Final := stack(Seg1, Seg2, Seg3)$

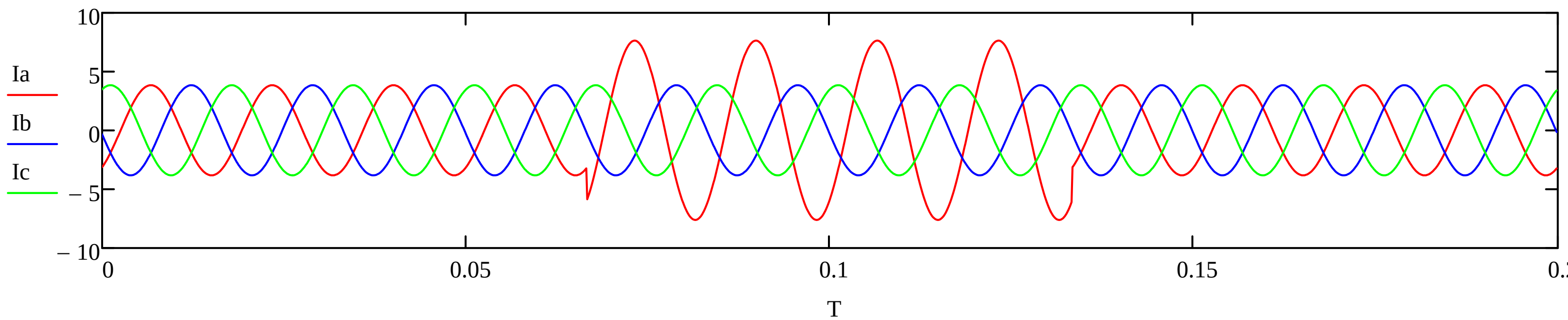
$T := Final^{\langle 0 \rangle}$ $Ia := Final^{\langle 1 \rangle}$ $Ib := Final^{\langle 2 \rangle}$ $Ic := Final^{\langle 3 \rangle}$ $Va := Final^{\langle 4 \rangle}$ $Vb := Final^{\langle 5 \rangle}$ $Vc := Final^{\langle 6 \rangle}$

$Ir := Ia + Ib + Ic$

$Vr := Va + Vb + Vc$

Acquired currents and voltages.





FFT calculation:

$m := \mathrm{length}(Va)$

$m = 1.536 \times 10^3$

$window := 128$

$wind := window - 1$

$Isa := Ia + k0 \cdot Ir$ $\quad Isb := Ib + k0 \cdot Ir$ $\quad Isc := Ic + k0 \cdot Ir$ $\quad k0 = 0.667$

$dd := 0 .. \frac{m}{window} - 1$ $\quad \frac{(m - window)}{8} = 176$

$kk := 0 .. m - window$

$k := 0 .. \frac{m - window}{8}$

$Ua_k := \mathrm{submatrix}(Va, k \cdot 8, k \cdot 8 + wind, 0, 0)$ $\quad Ub_k := \mathrm{submatrix}(Vb, k \cdot 8, k \cdot 8 + wind, 0, 0)$ $\quad Uc_k := \mathrm{submatrix}(Vc, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$Ao_k := \mathrm{submatrix}(Ir, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$Aa_k := \mathrm{submatrix}(Isa, k \cdot 8, k \cdot 8 + wind, 0, 0)$ $\quad Ab_k := \mathrm{submatrix}(Isb, k \cdot 8, k \cdot 8 + wind, 0, 0)$ $\quad Ac_k := \mathrm{submatrix}(Isc, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$Pa_k := \mathrm{FFT}(Ua_k)$ $\quad Pb_k := \mathrm{FFT}(Ub_k)$ $\quad Pc_k := \mathrm{FFT}(Uc_k)$

$Fa_k := \mathrm{FFT}(Aa_k)$ $\quad Fb_k := \mathrm{FFT}(Ab_k)$ $\quad Fc_k := \mathrm{FFT}(Ac_k)$

$Pa_k := \mathrm{FFT}(Ua_k)$ $\quad Pb_k := \mathrm{FFT}(Ub_k)$ $\quad Pc_k := \mathrm{FFT}(Uc_k)$ $\quad Fo_k := \mathrm{FFT}(Ao_k)$

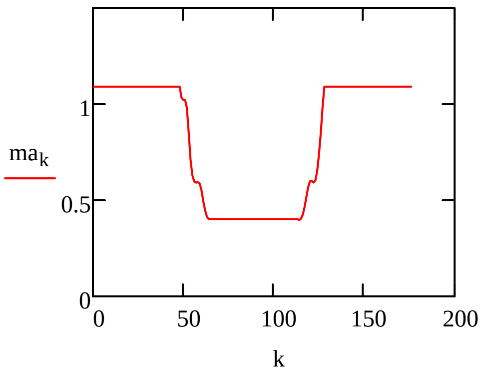
Novosel Algorithm
(Left to exam only for comparison, so that results can be compared with patent 5)

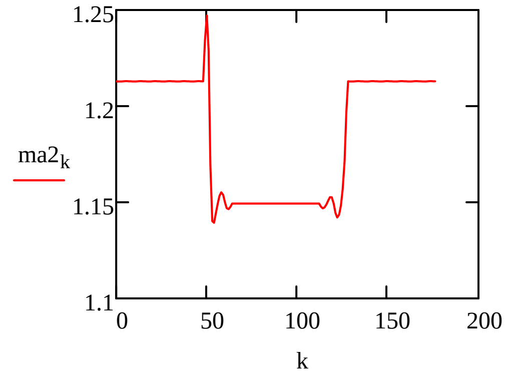
$IaN := 0$

$$k1a_k := \frac{\left(Pa_k\right)_{1,0}}{\left(Fa_k\right)_{1,0} \cdot Z1L} + \frac{Z1R}{Z1L} + 1 \qquad k2a_k := \frac{\left(Pa_k\right)_{1,0}}{\left(Fa_k\right)_{1,0} \cdot Z1L} \cdot \left(\frac{Z1R}{Z1L} + 1\right) \qquad k3a_k := \frac{\left[\left(Fa_k\right)_{1,0} - IaN\right]}{\left(Fa_k\right)_{1,0} \cdot Z1L} \cdot \left(\frac{Z1S + Z1R}{Z1L} + 1\right)$$

$$a1_k := 1 \qquad ab1_k := -\left(Re\left(k1a_k\right) - \frac{Im\left(k1a_k\right) \cdot Re\left(k3a_k\right)}{Im\left(k3a_k\right)}\right) \qquad ac1_k := Re\left(k2a_k\right) - \frac{Im\left(k2a_k\right) \cdot Re\left(k3a_k\right)}{Im\left(k3a_k\right)}$$

$$ma_k := \frac{-ab1_k - \sqrt{\left(ab1_k\right)^2 - 4 \cdot ac1_k \cdot a1_k}}{2 \cdot a1_k} \qquad ma2_k := \frac{-ab1_k + \sqrt{\left(ab1_k\right)^2 - 4 \cdot ac1_k \cdot a1_k}}{2 \cdot a1_k}$$





Result from Novosel algorithm: $ma_{100} = 0.402$

Patent 5 - actual EXAM algorithm:

C1 and C2 parameters

$$C1_k := ZL_{(0,0)}\left(Fa_k\right)_{1,0} + ZL_{(0,1)}\cdot\left(Fb_k\right)_{1,0} + ZL_{(0,2)}\cdot\left(Fc_k\right)_{1,0}$$

$$C2_k := ZR_{(0,0)}\left(Fa_k\right)_{1,0} + ZR_{(0,1)}\cdot\left(Fb_k\right)_{1,0} + ZR_{(0,2)}\cdot\left(Fc_k\right)_{1,0}$$

Parts of the polonomial

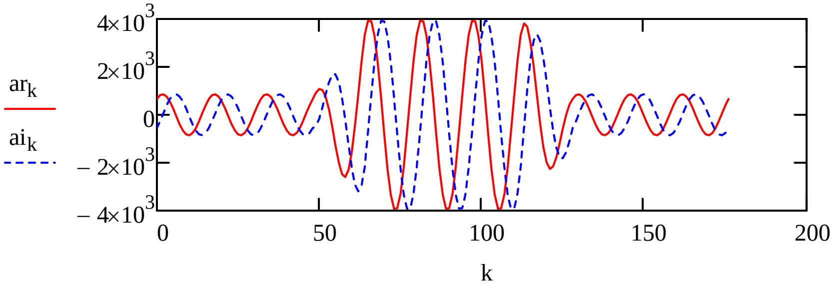
$$ar_k := \mathrm{Re}\left[C1_k\cdot ZL_{(0,0)}\right] \qquad ai_k := \mathrm{Im}\left[C1_k\cdot ZL_{(0,0)}\right]$$

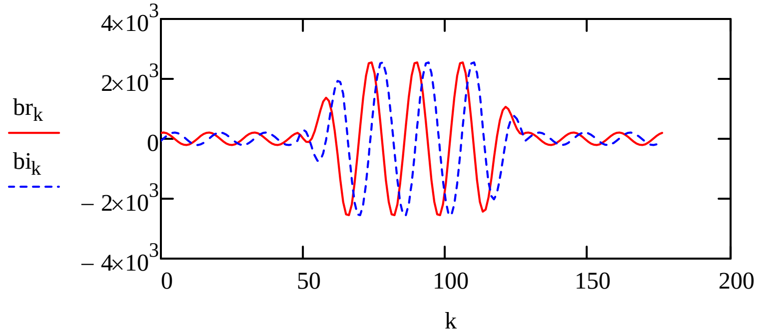
$$br_k := \mathrm{Re}\left[\left(Pa_k\right)_{1,0}\cdot ZL_{(0,0)} - C1_k\cdot ZL_{(0,0)} + C1_k\cdot ZR_{(0,0)}\right] \qquad bi_k := \mathrm{Im}\left[\left(Pa_k\right)_{1,0}\cdot ZL_{(0,0)} - C1_k\cdot ZL_{(0,0)} + C1_k\cdot ZR_{(0,0)}\right]$$

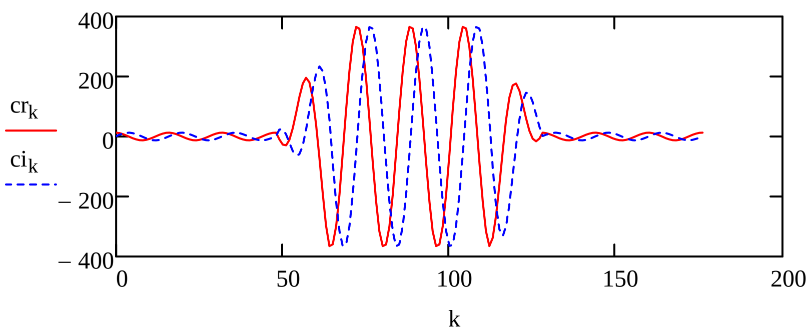
$$cr_k := \mathrm{Re}\left[ZR_{(0,0)}\cdot\left[\left(Pa_k\right)_{1,0} - C1_k\right]\right] \qquad ci_k := \mathrm{Im}\left[ZR_{(0,0)}\cdot\left[\left(Pa_k\right)_{1,0} - C1_k\right]\right]$$

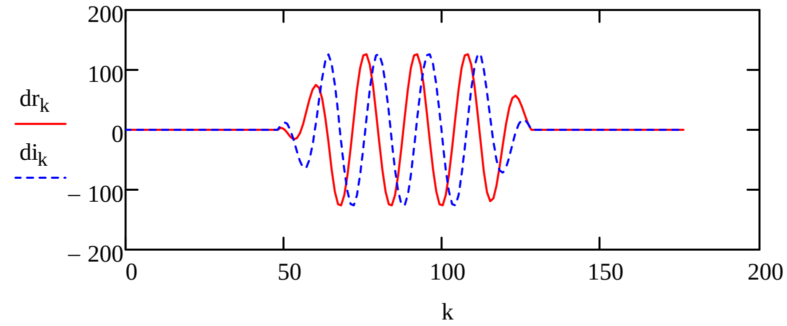
$$dr_k := \mathrm{Re}\left[\left(Pa_k\right)_{1,0} - C1_k - C2_k\right] \qquad di_k := \mathrm{Im}\left[\left(Pa_k\right)_{1,0} - C1_k - C2_k\right]$$

$$polya_k := ar_k - \frac{dr_k\cdot ai_k}{di_k} \qquad polyb_k := br_k - \frac{dr_k\cdot bi_k}{di_k} \qquad polyc_k := cr_k - \frac{dr_k\cdot ci_k}{di_k}$$





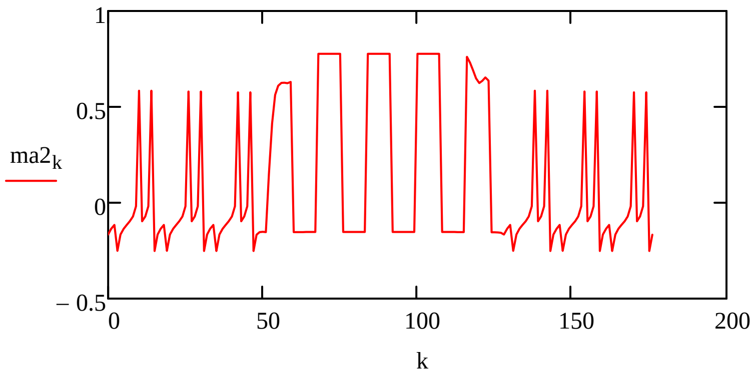


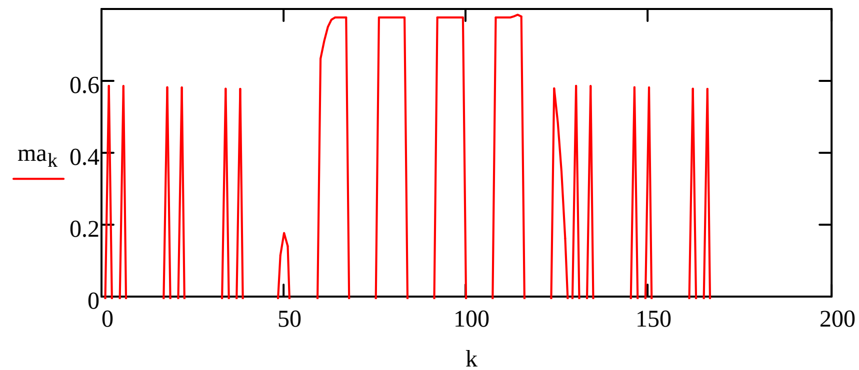


Solving the polynomial

$$ma_k := \frac{-polyb_k + \sqrt{(polyb_k)^2 - 4 \cdot polya_k \cdot polyc_k}}{2 \cdot polya_k}$$

$$ma2_k := \frac{-polyb_k - \sqrt{(polyb_k)^2 - 4 \cdot polya_k \cdot polyc_k}}{2 \cdot polya_k}$$





$ma_{110} = 0.776$

$ma2_{100} = 0.776$

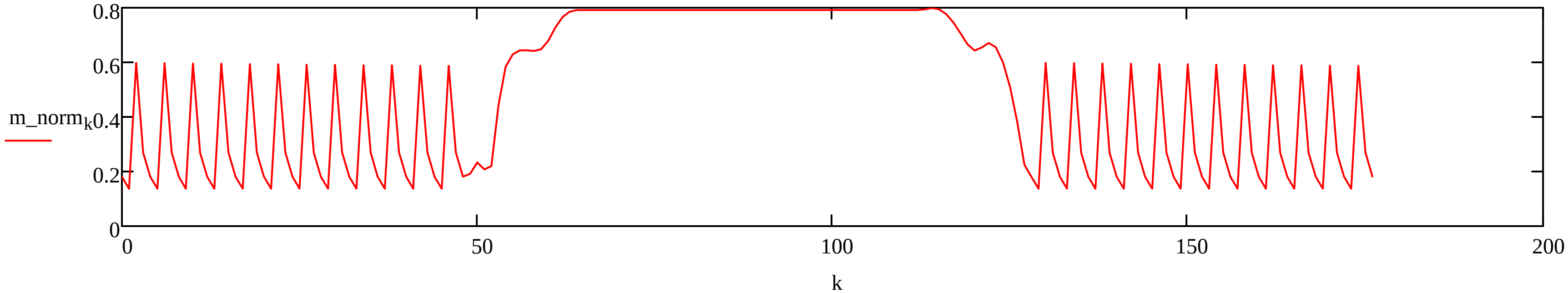
$result := 2 \cdot (1 - ma_{110})$

Result from Patent 5:

$result = 0.447$

Issue in Patent 5 is that there is oscillation in the result, because of changing real and imagynary parts.
Improvement to the algorithm is to use norm instead:

$$m\_norm_k := \sqrt{\left(ma_k \cdot ma_k + ma2_k \cdot ma2_k\right)}$$



$$m\_norm_{100} = 0.791$$

Result from Patent 5:

$$result := 2 \cdot \left(1 - m\_norm_{110}\right)$$

$$result = 0.417$$

The improvement makes the signal more stable, and it is easier to determine the fault distance.
But the algorithm is not very accurate, with different fault distances following results were acquired:

Distance	Patent 5	Improvement
0.1	0.09	0.07
0.2	0.2	0.17
0.3	0.32	0.29
0.4	0.45	0.42
0.5	0.6	0.56
0.6	0.76	0.72
0.7	0.94	0.9
0.8	1.2	1.1
0.9	1.4	1.3
1.0	1.6	1.45

Initial algorithm is better when fault distance is small and improvement is better when fault distance is long.

More testing with fault data was not done, since I did not find from files what was the actual correct fault distance