Fault Detection and Location in Distribution Systems
Post-Graduate Course (4 cr)
1.-3. June 2010 @ TKK

Take-Home Exam
Pat2_US20080150544_Premelani

Antti Koto
antti.koto@tut.fi

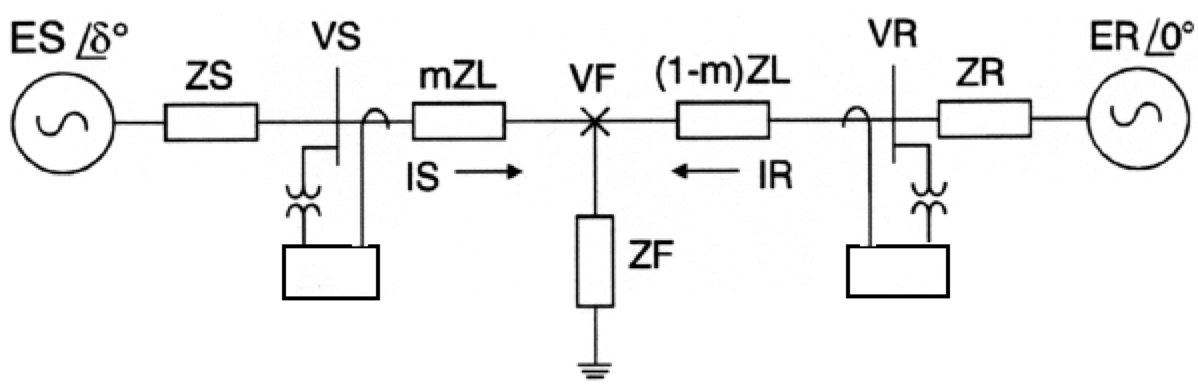
Tampere University of Technology
Student number

###
Generation of ideal 2-source data

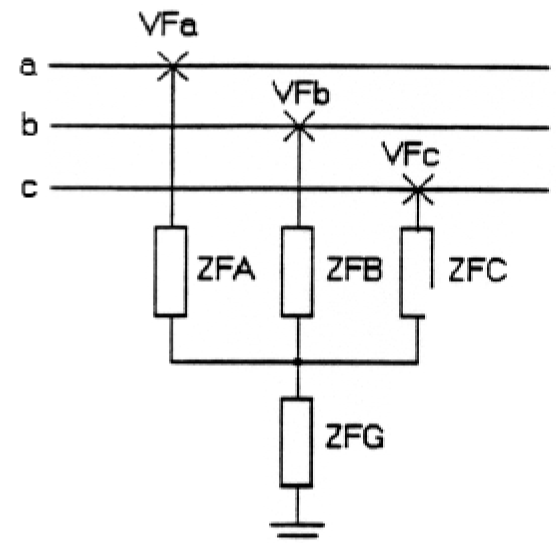
Fault Calculation Example NINEGEN_SEL_FI_ST.XMCD MAY 2010

NOTE:Some parameter values are changed

SIMPLE ONE LINE DIAGRAM



FAULT IMPEDANCE



INPUT DATA

Voltage Phase Angle $\delta := 1.1$

Source S Voltage $es := 70 \cdot e^{j \cdot \delta \cdot deg} = 69.987 + 1.344i$

Source R Voltage $er := 70$ $er := 0$

Source S Positive Sequence Impedance $Z1S := 4.104 + j \cdot 11.276$

Source S Zero Sequence Impedance $Z0S := 25.357 + j \cdot 54.378$

Source R Positive Sequence Impedance $Z1R := 0.518 + j \cdot 1.932$

Source R Zero Sequence Impedance $Z0R := 3 \cdot Z1R$ $Z0R = 1.554 + 5.796i$

Positive Sequence Line Impedance $Z1L := 1.035 + j \cdot 3.864$

Zero Sequence Line Impedance $Z0L := 3 \cdot Z1L$ $Z0L = 3.105 + 11.592i$

Fault Location

$m := 0.01$ $m := 0.10$ $m := 0.50$ $m := 0.75$ $m := 0.98$

Fault Impedances (for AG fault case) $INF := 10^{10}$

$ZFA := 0 + j \cdot 0$ $ZFB := INF + j \cdot 0$ $ZFC := INF + j \cdot 0$

Fault Resistance

$ZFG := 0$ $ZFG := 0.85 + j \cdot 0$ $ZFG := 10 + j \cdot 0$

CONSTANTS $rad := 1$ $deg := \frac{\pi}{180} \cdot rad$

Operator $a := e^{j \cdot 120 \cdot deg} = -0.5 + 0.866i$

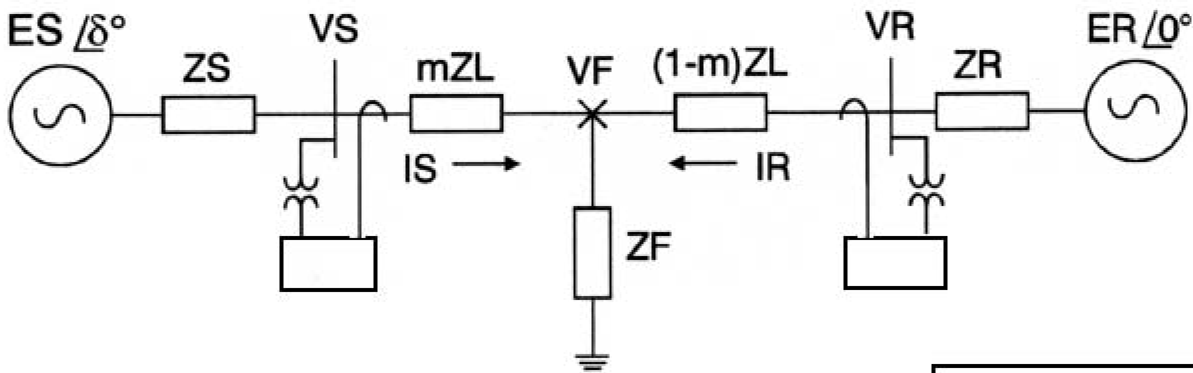
$$BAL := \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \quad one := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad zero := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Three phase voltages at S and R

$$ES := es \cdot BAL \qquad ES = \begin{pmatrix} 69.987 + 1.344i \\ -33.83 - 61.283i \\ -36.157 + 59.939i \end{pmatrix}$$

$ER := er \cdot BAL \qquad ER = \begin{pmatrix} 70 \\ -35 - 60.622i \\ -35 + 60.622i \end{pmatrix}$

CIRCUIT EQUATION



$$\begin{aligned} ES &= [ZS + m \cdot ZL] \cdot IS + 0 + VF \\ ER &= + 0 + [ZR + (1\text{-}m) \cdot ZL] \cdot IR + VF \\ 0 &= - ZF \cdot IS - ZF \cdot IR + VF \end{aligned}$$

$$\begin{aligned} ES &= ZSS \cdot IS + 0 + VF \\ ER &= + 0 + ZRR \cdot IR + VF \\ 0 &= - ZF \cdot IS - ZF \cdot IR + VF \end{aligned}$$

$$\begin{aligned} ZSS &= [ZS + m \cdot ZL] \\ ZRR &= [ZR + (1\text{-}m) \cdot ZL] \end{aligned}$$

In 3-phase matrix form, the equation looks like this:

$$\begin{bmatrix} ES \\ ER \\ null \end{bmatrix} = \begin{bmatrix} ZSS & zero & one \\ zero & ZRR & one \\ -ZF & -ZF & one \end{bmatrix} \begin{bmatrix} IS \\ IR \\ VF \end{bmatrix}$$

How do we form the soure impedance ZS and ZR?

Let us consider the link between 3-phase circuit and symmetrical components

Conversion of positive sequence and zero sequence impedances to Self and Mutual impedances

$$zs(z0,z1) := \frac{2\cdot z1 + z0}{3} \qquad zm(z0,z1) := \frac{z0 - z1}{3}$$

Conversion Matrix Format

$$Z(z0,z1) := \begin{pmatrix} zs(z0,z1) & zm(z0,z1) & zm(z0,z1) \\ zm(z0,z1) & zs(z0,z1) & zm(z0,z1) \\ zm(z0,z1) & zm(z0,z1) & zs(z0,z1) \end{pmatrix}$$

Now Conversion

$$ZS := Z(Z0S, Z1S) \qquad ZL := Z(Z0L, Z1L) \qquad ZR := Z(Z0R, Z1R)$$

$$ZS = \begin{pmatrix} 11.188 + 25.643i & 7.084 + 14.367i & 7.084 + 14.367i \\ 7.084 + 14.367i & 11.188 + 25.643i & 7.084 + 14.367i \\ 7.084 + 14.367i & 7.084 + 14.367i & 11.188 + 25.643i \end{pmatrix}$$

$$ZR = \begin{pmatrix} 0.863 + 3.22i & 0.345 + 1.288i & 0.345 + 1.288i \\ 0.345 + 1.288i & 0.863 + 3.22i & 0.345 + 1.288i \\ 0.345 + 1.288i & 0.345 + 1.288i & 0.863 + 3.22i \end{pmatrix}$$

Source and Line Impedances to the Fault

$$ZSS := ZS + m\cdot ZL \qquad ZSS = \begin{pmatrix} 12.051 + 28.863i & 7.429 + 15.655i & 7.429 + 15.655i \\ 7.429 + 15.655i & 12.051 + 28.863i & 7.429 + 15.655i \\ 7.429 + 15.655i & 7.429 + 15.655i & 12.051 + 28.863i \end{pmatrix}$$

$$ZRR := ZR + (1 - m)\cdot ZL \qquad ZRR = \begin{pmatrix} 1.726 + 6.44i & 0.69 + 2.576i & 0.69 + 2.576i \\ 0.69 + 2.576i & 1.726 + 6.44i & 0.69 + 2.576i \\ 0.69 + 2.576i & 0.69 + 2.576i & 1.726 + 6.44i \end{pmatrix}$$

Build System Part of the Impedance Matrix

$$\begin{bmatrix} ES \\ ER \\ null \end{bmatrix} = \begin{bmatrix} ZSS & zero & one \\ zero & ZRR & one \\ -ZF & -ZF & one \end{bmatrix} \begin{bmatrix} IS \\ IR \\ VF \end{bmatrix} \quad ES = \begin{bmatrix} ES_a \\ ES_b \\ ES_c \end{bmatrix} \quad ER = \begin{bmatrix} ER_a \\ ER_b \\ ER_c \end{bmatrix} \quad VF = \begin{bmatrix} VF_a \\ VF_b \\ VF_c \end{bmatrix} \quad IS = \begin{bmatrix} IS_a \\ IS_b \\ IS_c \end{bmatrix} \quad IR = \begin{bmatrix} IR_a \\ IR_b \\ IR_c \end{bmatrix}$$

ZTOP := augment(augment(ZSS, zero), one)

$$ZTOP = \begin{pmatrix} 12.051 + 28.863i & 7.429 + 15.655i & 7.429 + 15.655i & 0 & 0 & 0 & 1 & 0 & 0 \\ 7.429 + 15.655i & 12.051 + 28.863i & 7.429 + 15.655i & 0 & 0 & 0 & 0 & 1 & 0 \\ 7.429 + 15.655i & 7.429 + 15.655i & 12.051 + 28.863i & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

ZMID := augment(augment(zero, ZRR), one)

$$ZMID = \begin{pmatrix} 0 & 0 & 0 & 1.726 + 6.44i & 0.69 + 2.576i & 0.69 + 2.576i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.69 + 2.576i & 1.726 + 6.44i & 0.69 + 2.576i & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.69 + 2.576i & 0.69 + 2.576i & 1.726 + 6.44i & 0 & 0 & 1 \end{pmatrix}$$

ZSYS := stack(ZTOP, ZMID)

$$ZSYS = \begin{pmatrix} 12.051 + 28.863i & 7.429 + 15.655i & 7.429 + 15.655i & 0 & 0 & 0 & 1 & 0 & 0 \\ 7.429 + 15.655i & 12.051 + 28.863i & 7.429 + 15.655i & 0 & 0 & 0 & 0 & 1 & 0 \\ 7.429 + 15.655i & 7.429 + 15.655i & 12.051 + 28.863i & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1.726 + 6.44i & 0.69 + 2.576i & 0.69 + 2.576i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.69 + 2.576i & 1.726 + 6.44i & 0.69 + 2.576i & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.69 + 2.576i & 0.69 + 2.576i & 1.726 + 6.44i & 0 & 0 & 1 \end{pmatrix}$$

Pre-fault conditions:

ZPRE := ZS + ZL + ZR

$$ZPRE = \begin{pmatrix} 13.777 + 35.303i & 8.12 + 18.231i & 8.12 + 18.231i \\ 8.12 + 18.231i & 13.777 + 35.303i & 8.12 + 18.231i \\ 8.12 + 18.231i & 8.12 + 18.231i & 13.777 + 35.303i \end{pmatrix}$$

$ISPRE := ZPRE^{-1} \cdot (ES - ER)$

$$ISPRE = \begin{pmatrix} 0.071 + 0.024i \\ -0.014 - 0.073i \\ -0.056 + 0.049i \end{pmatrix}$$

$$IRPRE := ZPRE^{-1} \cdot (ER - ES) \qquad IRPRE = \begin{pmatrix} -0.071 - 0.024i \\ 0.014 + 0.073i \\ 0.056 - 0.049i \end{pmatrix}$$

Pre_fault voltage at S end

$$VSP := ES - ZS \cdot ISPRE \qquad VSP = \begin{pmatrix} 69.97 + 0.447i \\ -34.597 - 60.819i \\ -35.372 + 60.372i \end{pmatrix} \qquad ES = \begin{pmatrix} 69.987 + 1.344i \\ -33.83 - 61.283i \\ -36.157 + 59.939i \end{pmatrix}$$

$$VRP := ZS \cdot IRPRE - ER \qquad VRP = \begin{pmatrix} -70.017 - 0.896i \\ 34.232 + 61.085i \\ 35.785 - 60.189i \end{pmatrix}$$

Build the voltage Vector

$$null := (0 \;\; 0 \;\; 0)$$

$$E := stack\left(stack(ES, ER), null^T\right)$$

$$TS := augment(augment(one, zero), zero)$$

$$TR := augment(augment(zero, one), zero)$$

$$E = \begin{pmatrix} 69.987 + 1.344i \\ -33.83 - 61.283i \\ -36.157 + 59.939i \\ 70 \\ -35 - 60.622i \\ -35 + 60.622i \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$TS = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$TR = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Building Fault Part of the Impedance Matrix:

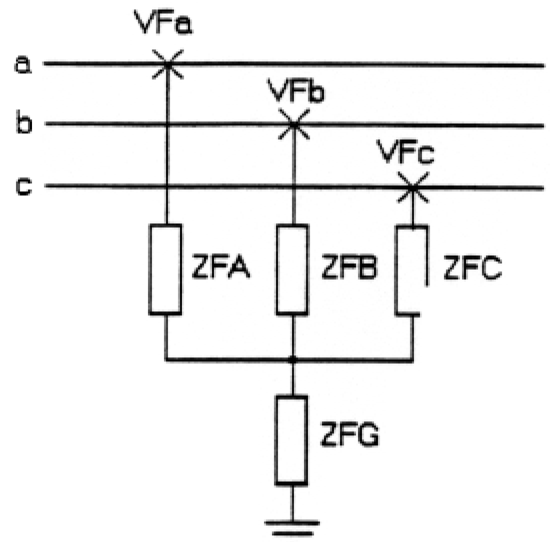
- the fault impedance ZF
- voltage drop due to each phase current flowing individually in the phase-fault impedances (ZFA, ZFB, or ZFC) and mutually in the ground-fault impedance ZFG.

$$ZF = \begin{bmatrix} ZFAG & ZFG & ZFG \\ ZFG & ZFBG & ZFG \\ ZFG & ZFG & ZFCG \end{bmatrix} \quad \text{where: } \begin{cases} ZFAG = ZFA + ZFG \\ ZFBG = ZFB + ZFG \\ ZFCG = ZFC + ZFG \end{cases}$$

$ZFAG := ZFA + ZFG$

$ZFBG := ZFB + ZFG$

$ZFCG := ZFC + ZFG$



$ZFAG = 0.85$

$ZFBG = 1 \times 10^{10}$

$$ZF := \begin{pmatrix} ZFAG & ZFG & ZFG \\ ZFG & ZFBG & ZFG \\ ZFG & ZFG & ZFCG \end{pmatrix} \qquad ZF = \begin{pmatrix} 0.85 & 0.85 & 0.85 \\ 0.85 & 1 \times 10^{10} & 0.85 \\ 0.85 & 0.85 & 1 \times 10^{10} \end{pmatrix}$$

$$\begin{bmatrix} ES \\ ER \\ null \end{bmatrix} = \begin{bmatrix} ZSS & zero & one \\ zero & ZRR & one \\ -ZF & -ZF & one \end{bmatrix} \begin{bmatrix} IS \\ IR \\ VF \end{bmatrix}$$

FABCG := augment(augment(−ZF, −ZF), one)

$$FABCG = \begin{pmatrix} -0.85 & -0.85 & -0.85 & -0.85 & -0.85 & -0.85 & 1 & 0 & 0 \\ -0.85 & -1 \times 10^{10} & -0.85 & -0.85 & -1 \times 10^{10} & -0.85 & 0 & 1 & 0 \\ -0.85 & -0.85 & -1 \times 10^{10} & -0.85 & -0.85 & -1 \times 10^{10} & 0 & 0 & 1 \end{pmatrix}$$

FINAL Z MATRIX

ZABCG := stack(ZSYS, FABCG)

$$ZABCG = \begin{pmatrix} 12.051 + 28.863i & 7.429 + 15.655i & 7.429 + 15.655i & 0 & 0 & 0 & 1 & 0 & 0 \\ 7.429 + 15.655i & 12.051 + 28.863i & 7.429 + 15.655i & 0 & 0 & 0 & 0 & 1 & 0 \\ 7.429 + 15.655i & 7.429 + 15.655i & 12.051 + 28.863i & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1.726 + 6.44i & 0.69 + 2.576i & 0.69 + 2.576i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.69 + 2.576i & 1.726 + 6.44i & 0.69 + 2.576i & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.69 + 2.576i & 0.69 + 2.576i & 1.726 + 6.44i & 0 & 0 & 1 \\ -0.85 & -0.85 & -0.85 & -0.85 & -0.85 & -0.85 & 1 & 0 & 0 \\ -0.85 & -1 \times 10^{10} & -0.85 & -0.85 & -1 \times 10^{10} & -0.85 & 0 & 1 & 0 \\ -0.85 & -0.85 & -1 \times 10^{10} & -0.85 & -0.85 & -1 \times 10^{10} & 0 & 0 & 1 \end{pmatrix}$$

$YABCG := ZABCG^{-1}$

$$YABCG = \begin{pmatrix} 0.017 - 0.045i & -4.483 \times 10^{-3} + 0.014i & -4.483 \times 10^{-3} + 0.014i & -3.999 \times 10^{-4} + 0.015i & 4.483 \times 10^{-3} - 0.014i & 4.483 \times 10^{-} \\ -4.483 \times 10^{-3} + 0.014i & 0.013 - 0.039i & -4.158 \times 10^{-3} + 0.014i & 3.239 \times 10^{-3} - 0.011i & -0.013 + 0.039i & 4.158 \times 10^{-} \\ -4.483 \times 10^{-3} + 0.014i & -4.158 \times 10^{-3} + 0.014i & 0.013 - 0.039i & 3.239 \times 10^{-3} - 0.011i & 4.158 \times 10^{-3} - 0.014i & -0.013 + \\ -3.999 \times 10^{-4} + 0.015i & 3.239 \times 10^{-3} - 0.011i & 3.239 \times 10^{-3} - 0.011i & 0.055 - 0.142i & -3.239 \times 10^{-3} + 0.011i & -3.239 \times 10^{-} \\ 4.483 \times 10^{-3} - 0.014i & -0.013 + 0.039i & 4.158 \times 10^{-3} - 0.014i & -3.239 \times 10^{-3} + 0.011i & 0.013 - 0.039i & -4.158 \times 10^{-} \\ 4.483 \times 10^{-3} - 0.014i & 4.158 \times 10^{-3} - 0.014i & -0.013 + 0.039i & -3.239 \times 10^{-3} + 0.011i & -4.158 \times 10^{-3} + 0.014i & 0.013 - \\ 0.014 - 0.026i & -1.058 \times 10^{-3} + 3.256i \times 10^{-3} & -1.058 \times 10^{-3} + 3.256i \times 10^{-3} & 0.047 - 0.109i & 1.058 \times 10^{-3} - 3.256i \times 10^{-3} & 1.058 \times 10^{-3} - \\ -0.103 - 0.015i & 0.208 + 0.016i & -0.014 + 2.809i \times 10^{-3} & -0.302 - 0.041i & 0.792 - 0.016i & 0.014 - 2.8(\\ -0.103 - 0.015i & -0.014 + 2.809i \times 10^{-3} & 0.208 + 0.016i & -0.302 - 0.041i & 0.014 - 2.809i \times 10^{-3} & 0.792 - \end{pmatrix}$$

Fault Currents:

$$IABCG := YABCG \cdot E \qquad E = \begin{pmatrix} 69.987 + 1.344i \\ -33.83 - 61.283i \\ -36.157 + 59.939i \\ 70 \\ -35 - 60.622i \\ -35 + 60.622i \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad IABCG = \begin{pmatrix} 1.249 - 2.095i \\ -0.103 + 0.194i \\ -0.145 + 0.317i \\ 3.812 - 8.949i \\ 0.103 - 0.194i \\ 0.145 - 0.317i \\ 4.302 - 9.388i \\ -63.029 - 64.742i \\ -63.546 + 56.466i \end{pmatrix}$$

S - End Fault Currents:

$$IS := TS \cdot IABCG \qquad IS = \begin{pmatrix} 1.249 - 2.095i \\ -0.103 + 0.194i \\ -0.145 + 0.317i \end{pmatrix}$$

R - End Fault Currents:

$$IR := TR \cdot IABCG \qquad IR = \begin{pmatrix} 3.812 - 8.949i \\ 0.103 - 0.194i \\ 0.145 - 0.317i \end{pmatrix}$$

S - End Voltages

$$VS := ES - ZS \cdot IS \qquad VS = \begin{pmatrix} 11.382 - 7.315i \\ -61.072 - 64.096i \\ -61.847 + 57.095i \end{pmatrix} \qquad VSP = \begin{pmatrix} 69.97 + 0.447i \\ -34.597 - 60.819i \\ -35.372 + 60.372i \end{pmatrix}$$

R - End Voltages

$$VR := (ZR \cdot IR - ER) \qquad VR = \begin{pmatrix} -37.149 + 4.69i \\ 49.015 + 62.68i \\ 49.273 - 58.546i \end{pmatrix} \qquad VRP = \begin{pmatrix} -70.017 - 0.896i \\ 34.232 + 61.085i \\ 35.785 - 60.189i \end{pmatrix}$$

Line Prefault Load Currents from S Bus

$Ia := ISPRE_0$ $|Ia| = 0.075$ $\frac{arg(Ia)}{deg} = 18.883$ $0.32 \cdot \frac{180}{3.14} = 18.344$ $ISPRE = \begin{pmatrix} 0.071 + 0.024i \\ -0.014 - 0.073i \\ -0.056 + 0.049i \end{pmatrix}$

$Ib := ISPRE_1$ $|Ib| = 0.075$ $\frac{arg(Ib)}{deg} = -101.117$

$Ic := ISPRE_2$ $|Ic| = 0.075$ $\frac{arg(Ic)}{deg} = 138.883$ $IRPRE = \begin{pmatrix} -0.071 - 0.024i \\ 0.014 + 0.073i \\ 0.056 - 0.049i \end{pmatrix}$

Line Prefault Voltages at S Bus

$Va := VSP_0$ $|Va| = 69.971$ $\frac{arg(Va)}{deg} = 0.366$

$Vb := VSP_1$ $|Vb| = 69.971$ $\frac{arg(Vb)}{deg} = -119.634$ $VSP = \begin{pmatrix} 69.97 + 0.447i \\ -34.597 - 60.819i \\ -35.372 + 60.372i \end{pmatrix}$

$Vc := VSP_2$ $|Vc| = 69.971$ $\frac{arg(Vc)}{deg} = 120.366$

Line Fault Currents from S Bus

$Iasf := IS_0$ $|Iasf| = 2.439$ $\frac{arg(Iasf)}{deg} = -59.199$

$Ibsf := IS_1$ $|Ibsf| = 0.22$ $\frac{arg(Ibsf)}{deg} = 117.835$ $IS = \begin{pmatrix} 1.249 - 2.095i \\ -0.103 + 0.194i \\ -0.145 + 0.317i \end{pmatrix}$

$Icsf := IS_2$ $|Icsf| = 0.348$ $\frac{arg(Icsf)}{deg} = 114.519$

Line Fault Currents from R Bus

$Iarf := IR_0$ $|Iarf| = 9.727$ $\frac{arg(Iarf)}{deg} = -66.928$

$Ibrf := IR_1 \quad |Ibrf| = 0.22 \quad \frac{arg(Ibrf)}{deg} = -62.165$

$Icrf := IR_2 \quad |Icrf| = 0.348 \quad \frac{arg(Icrf)}{deg} = -65.481$

$$IR = \begin{pmatrix} 3.812 - 8.949i \\ 0.103 - 0.194i \\ 0.145 - 0.317i \end{pmatrix}$$

Line Fault Voltages at S Bus

$$VSP = \begin{pmatrix} 69.97 + 0.447i \\ -34.597 - 60.819i \\ -35.372 + 60.372i \end{pmatrix}$$

$Vasf := VS_0 \quad |Vasf| = 13.53 \quad \frac{arg(Vasf)}{deg} = -32.728$

$Vbsf := VS_1 \quad |Vbsf| = 88.533 \quad \frac{arg(Vbsf)}{deg} = -133.616$

$Vcsf := VS_2 \quad |Vcsf| = 84.171 \quad \frac{arg(Vcsf)}{deg} = 137.288$

$$VS = \begin{pmatrix} 11.382 - 7.315i \\ -61.072 - 64.096i \\ -61.847 + 57.095i \end{pmatrix}$$

Line Fault Voltage at R Bus

$$VRP = \begin{pmatrix} -70.017 - 0.896i \\ 34.232 + 61.085i \\ 35.785 - 60.189i \end{pmatrix}$$

$Varf := VR_0 \quad |Varf| = 37.444 \quad \frac{arg(Varf)}{deg} = 172.805$

$Vbrf := VR_1 \quad |Vbrf| = 79.57 \quad \frac{arg(Vbrf)}{deg} = 51.975$

$Vcrf := VR_2 \quad |Vcrf| = 76.521 \quad \frac{arg(Vcrf)}{deg} = -49.915$

$$VR = \begin{pmatrix} -37.149 + 4.69i \\ 49.015 + 62.68i \\ 49.273 - 58.546i \end{pmatrix}$$

Residual Current and Voltage Vsr, Vrr, Isr, Irr

$$Isrf := \sum_{j=0}^{2} IS_j = 1.002 - 1.584i \qquad arg(Isrf) = -1.007$$

$$Irrf := \sum_{j=0}^{2} IR_j = 4.059 - 9.46i \qquad arg(Irrf) = -1.165$$

$$Vsrf := \sum_{j=0}^{2} VS_j = -111.537 - 14.316i \qquad arg(Vsrf) = -3.014$$

$$Vrrf := \sum_{j=0}^{2} VR_j = 61.139 + 8.825i \qquad arg(Vrrf) = 0.143$$

$$ISPREr := \sum_{j=0}^{2} ISPRE_j = 0$$

$$IRPREr := \sum_{j=0}^{2} IRPRE_j = 0$$

$$VSPr := \sum_{j=0}^{2} VSP_j = -1.421 \times 10^{-14} + 2.132i \times 10^{-14}$$

$$VRPr := \sum_{j=0}^{2} VRP_j = 7.105 \times 10^{-15} - 2.842i \times 10^{-14}$$

$$Z0s := \frac{Vsrf}{Isrf} = -25.357 - 54.378i$$

$$Z0r := \frac{Vrrf}{Irrf} = 1.554 + 5.796i$$

$$IS = \begin{pmatrix} 1.249 - 2.095i \\ -0.103 + 0.194i \\ -0.145 + 0.317i \end{pmatrix}$$

$$IR = \begin{pmatrix} 3.812 - 8.949i \\ 0.103 - 0.194i \\ 0.145 - 0.317i \end{pmatrix}$$

$$VS = \begin{pmatrix} 11.382 - 7.315i \\ -61.072 - 64.096i \\ -61.847 + 57.095i \end{pmatrix}$$

$$VR = \begin{pmatrix} -37.149 + 4.69i \\ 49.015 + 62.68i \\ 49.273 - 58.546i \end{pmatrix}$$

So How do we generate digital signals of Voltage and Current of the Simulation 4 Cycles with 7680 samples per second (128 samples per cycle in 60HZ system)?

For S side

$k := 0 .. 511$

$delT := 0.0001302$ $\qquad \frac{1}{7680} = 1.302 \times 10^{-4}$ $\qquad \frac{1}{delT} = 7.68 \times 10^{3}$ $\qquad \frac{7680}{60} = 128$

$T1_k := k \cdot delT$

$T2_k := 512 \cdot delT + k \cdot delT$

$T3_k := 1024 \cdot delT + k \cdot delT$

$Van_k := \left|VSP_0\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VSP_0\right)\right)$

$Vbn_k := \left|VSP_1\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VSP_1\right)\right)$

$Vcn_k := \left|VSP_2\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VSP_2\right)\right)$

$Vaf_k := \left|VS_0\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VS_0\right)\right)$

$Vbf_k := \left|VS_1\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VS_1\right)\right)$

$Vcf_k := \left|VS_2\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VS_2\right)\right)$

$Ian_k := \left|ISPRE_0\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(ISPRE_0\right)\right)$

$Ibn_k := \left|ISPRE_1\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(ISPRE_1\right)\right)$

$Icn_k := \left|ISPRE_2\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(ISPRE_2\right)\right)$

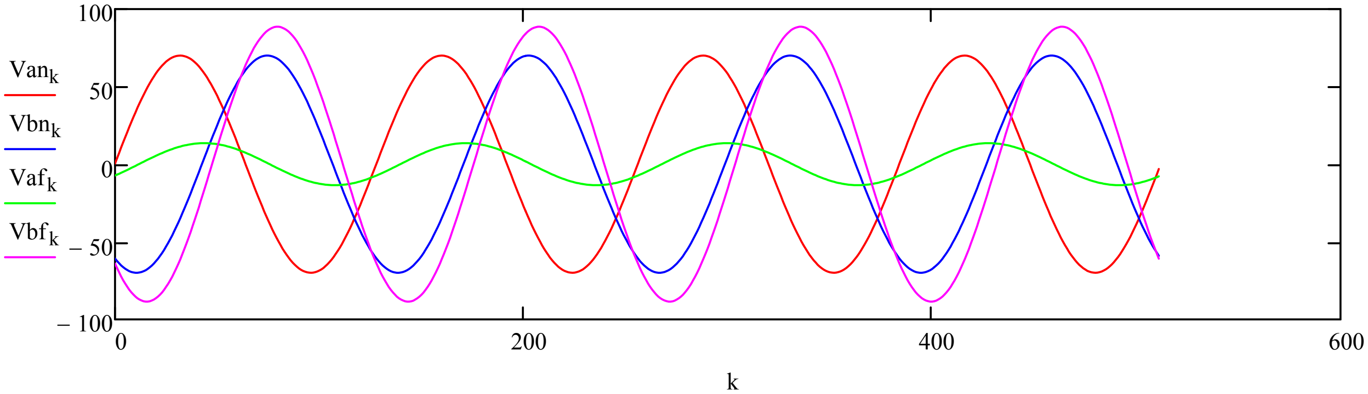
$Iaf_k := \left|IS_0\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(IS_0\right)\right)$

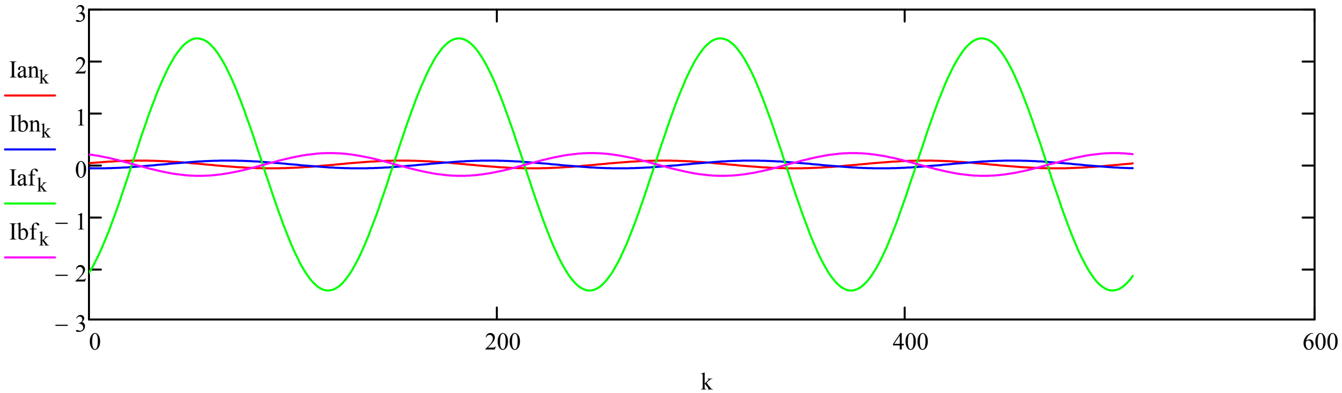
$Ibf_k := \left|IS_1\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(IS_1\right)\right)$

$Icf_k := \left|IS_2\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(IS_2\right)\right)$

$T1_k =$

0
$1.302 \cdot 10^{-4}$
$2.604 \cdot 10^{-4}$
$3.906 \cdot 10^{-4}$
$5.208 \cdot 10^{-4}$
$6.51 \cdot 10^{-4}$
$7.812 \cdot 10^{-4}$
$9.114 \cdot 10^{-4}$
$1.042 \cdot 10^{-3}$
$1.172 \cdot 10^{-3}$
$1.302 \cdot 10^{-3}$
$1.432 \cdot 10^{-3}$
$1.562 \cdot 10^{-3}$
$1.693 \cdot 10^{-3}$
$1.823 \cdot 10^{-3}$
...





Let us make Normal (4 cycle)+ Fault (4 cycle) +Normal (4 cycle)

$Seg1 := augment(T1, Ian, Ibn, Icn, Van, Vbn, Vcn)$

$Seg2 := augment(T2, Iaf, Ibf, Icf, Vaf, Vbf, Vcf)$

$Seg3 := augment(T3, Ian, Ibn, Icn, Van, Vbn, Vcn)$

$Final := stack(Seg1, Seg2, Seg3)$

$T := Final^{\langle 0 \rangle}$

$IaS := Final^{\langle 1 \rangle}$

$IbS := Final^{\langle 2 \rangle}$

$IcS := Final^{\langle 3 \rangle}$

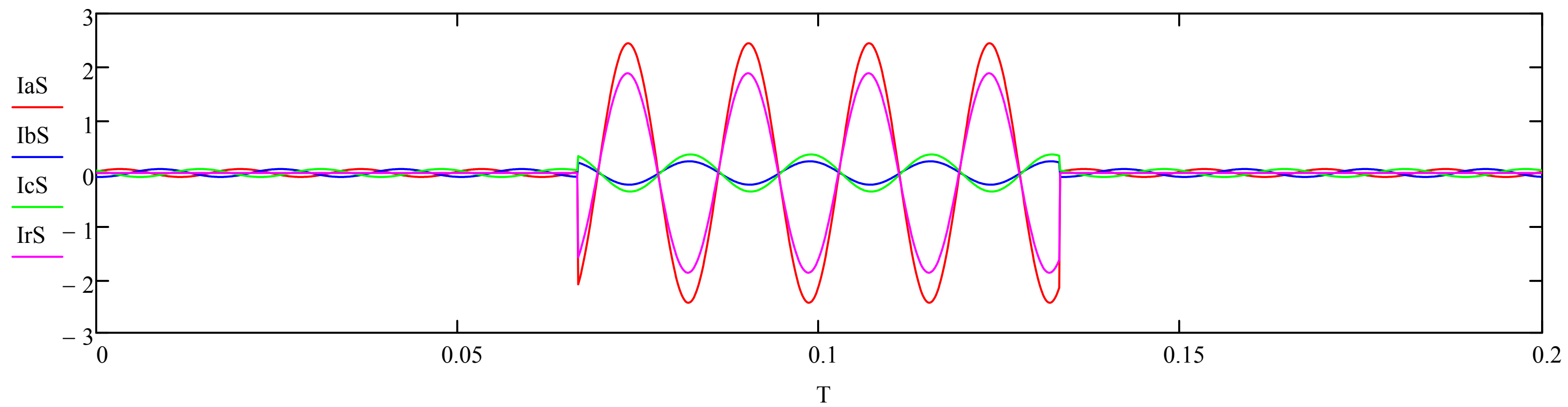
$VaS := Final^{\langle 4 \rangle}$

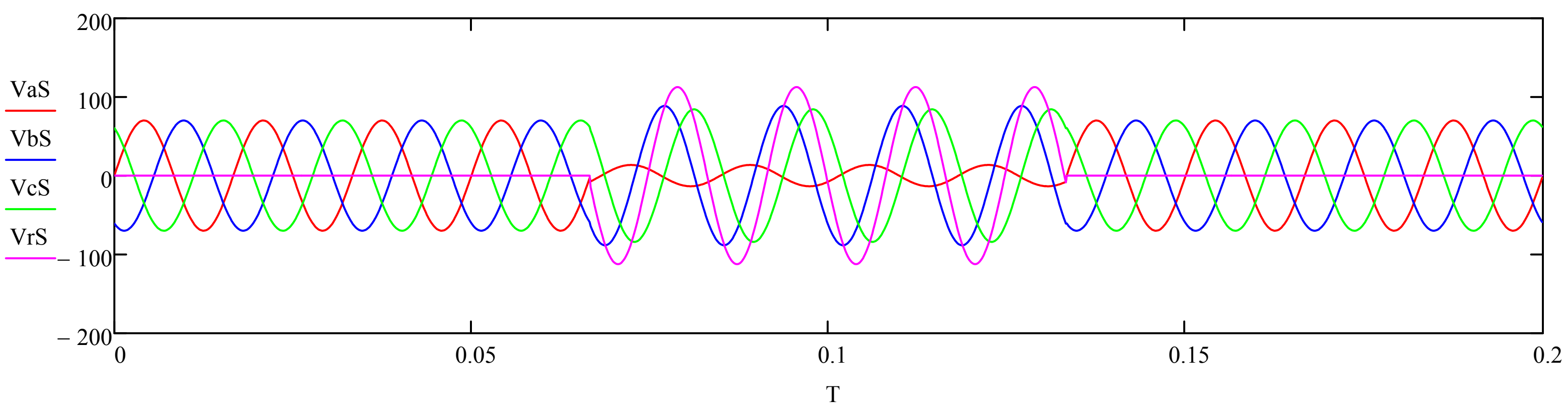
$VbS := Final^{\langle 5 \rangle}$

$VcS := Final^{\langle 6 \rangle}$

$IrS := IaS + IbS + IcS$

$VrS := VaS + VbS + VcS$





For R-Side

$k := 0 .. 511$

$delT := 0.0001302$ $\qquad \frac{1}{delT} = 7.68 \times 10^3 \qquad \frac{7680}{60} = 128$

$T1_k := k \cdot delT$

$T2_k := 512 \cdot delT + k \cdot delT$

$T3_k := 1024 \cdot delT + k \cdot delT$

$Van_k := \left|VRP_0\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VRP_0\right)\right)$

$Vbn_k := \left|VRP_1\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VRP_1\right)\right)$

$Vcn_k := \left|VRP_2\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VRP_2\right)\right)$

$Vaf_k := \left|VR_0\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VR_0\right)\right)$

$Vbf_k := \left|VR_1\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VR_1\right)\right)$

$Vcf_k := \left|VR_2\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(VR_2\right)\right)$

$Ian_k := \left|IRPRE_0\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(IRPRE_0\right)\right)$

$Ibn_k := \left|IRPRE_1\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(IRPRE_1\right)\right)$

$Icn_k := \left|IRPRE_2\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(IRPRE_2\right)\right)$

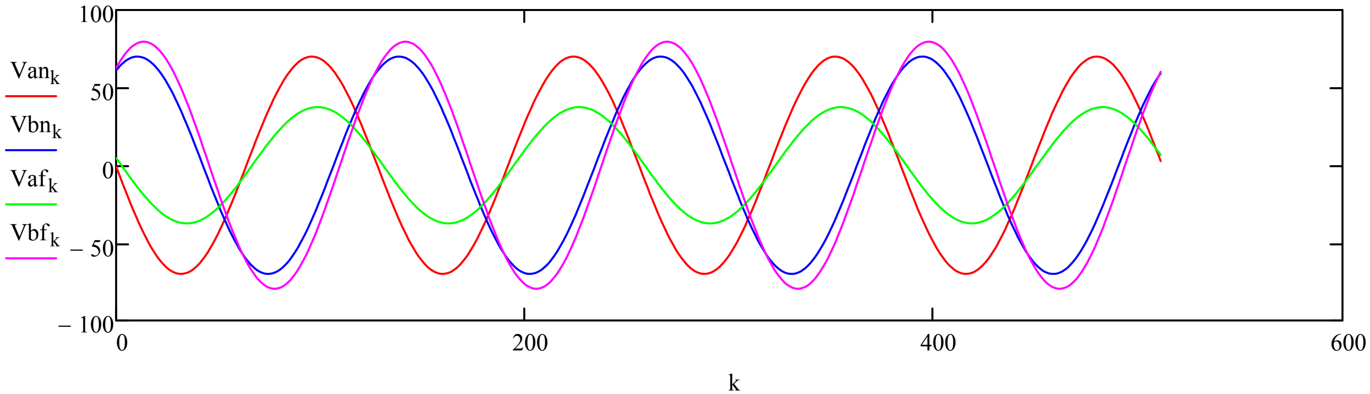
$Iaf_k := \left|IR_0\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(IR_0\right)\right)$

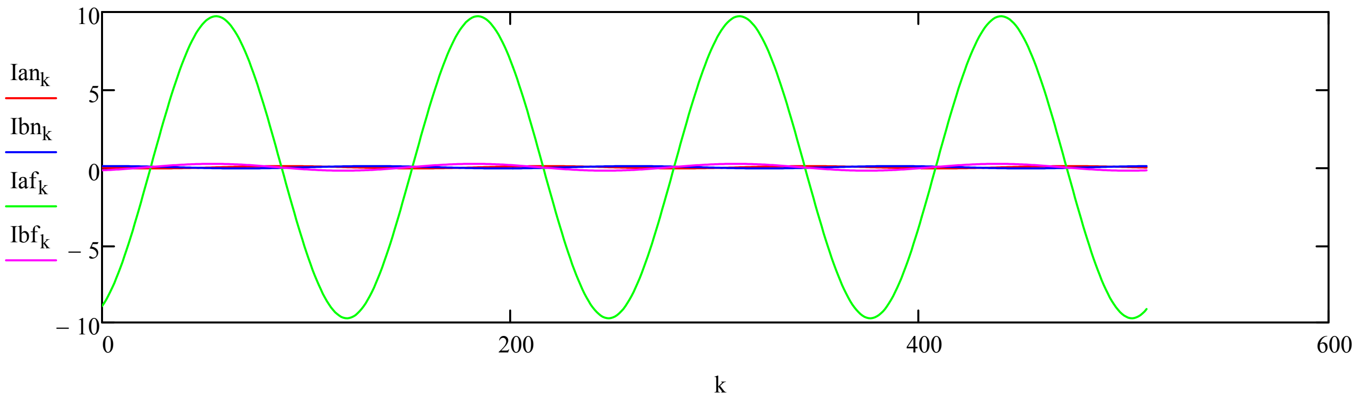
$Ibf_k := \left|IR_1\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(IR_1\right)\right)$

$Icf_k := \left|IR_2\right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot delT + \arg\left(IR_2\right)\right)$

$T1_k =$

0
$1.302 \cdot 10^{-4}$
$2.604 \cdot 10^{-4}$
$3.906 \cdot 10^{-4}$
$5.208 \cdot 10^{-4}$
$6.51 \cdot 10^{-4}$
$7.812 \cdot 10^{-4}$
$9.114 \cdot 10^{-4}$
$1.042 \cdot 10^{-3}$
$1.172 \cdot 10^{-3}$
$1.302 \cdot 10^{-3}$
$1.432 \cdot 10^{-3}$
$1.562 \cdot 10^{-3}$
$1.693 \cdot 10^{-3}$
$1.823 \cdot 10^{-3}$
...





Let us make Normal (4 cycle)+ Fault (4 cycle) +Normal (4 cycle)

$Seg1 := augment(T1, Ian, Ibn, Icn, Van, Vbn, Vcn)$

$Seg2 := augment(T2, Iaf, Ibf, Icf, Vaf, Vbf, Vcf)$

$Seg3 := augment(T3, Ian, Ibn, Icn, Van, Vbn, Vcn)$

$Final := stack(Seg1, Seg2, Seg3)$

$T := Final^{\langle 0 \rangle}$

$IaR := Final^{\langle 1 \rangle}$

$IbR := Final^{\langle 2 \rangle}$

$IcR := Final^{\langle 3 \rangle}$

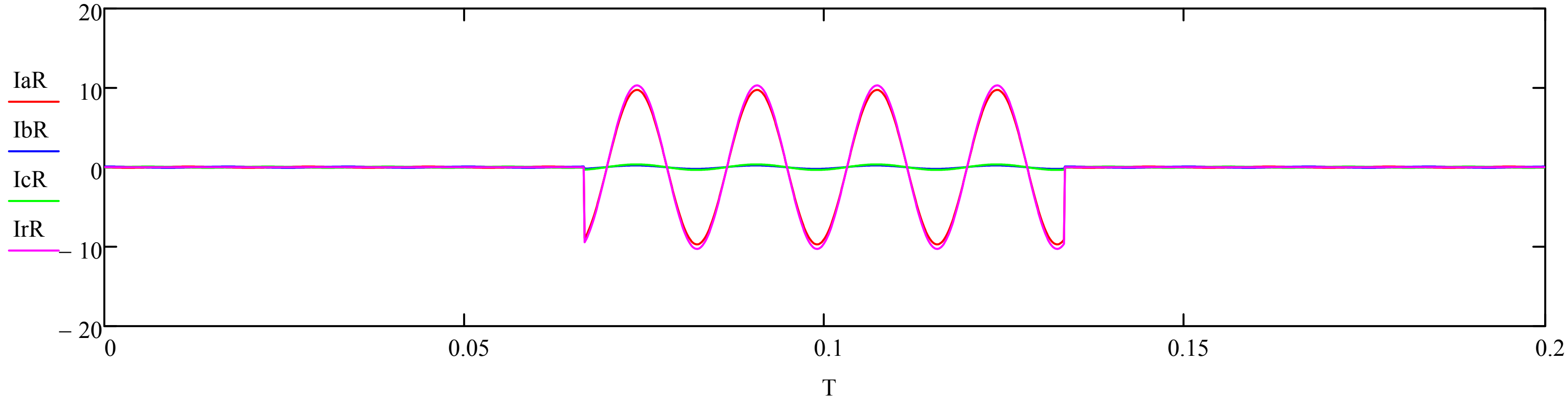
$VaR := Final^{\langle 4 \rangle}$

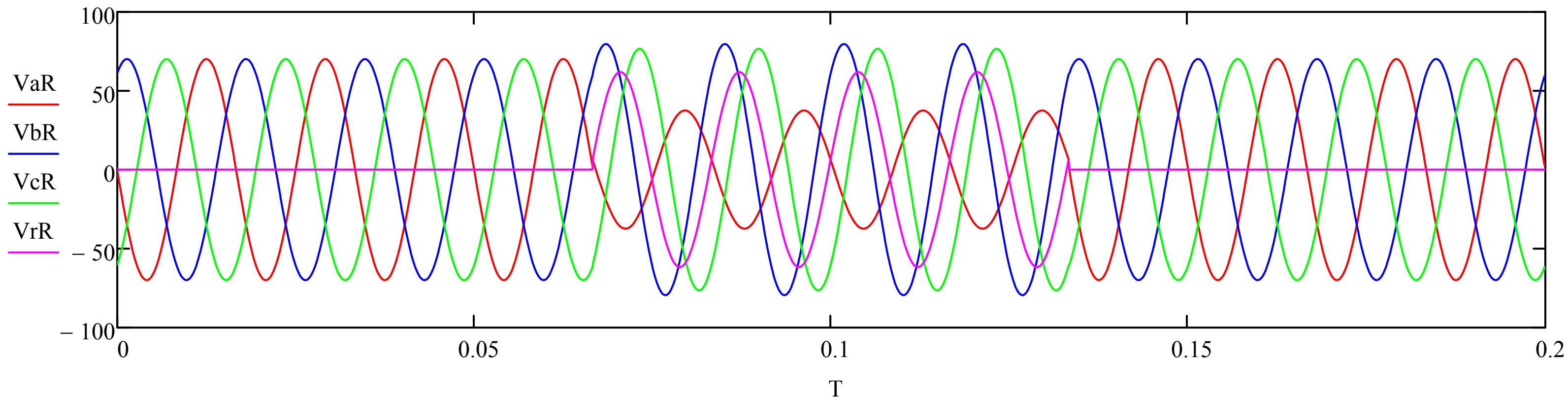
$VbR := Final^{\langle 5 \rangle}$

$VcR := Final^{\langle 6 \rangle}$

$IrR := IaR + IbR + IcR$

$VrR := VaR + VbR + VcR$





Now for all the calculations

$mm := 1536$

$window := 128$

$wind := window - 1$

$dd := 0 .. \frac{mm}{window} - 1$

$kk := 0 .. mm - window$

$k := 0 .. \frac{mm - window}{8}$

$UrS_k := submatrix(VrS, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$UrR_k := submatrix(VrR, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$ArS_k := submatrix(IrS, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$ArR_k := submatrix(IrR, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$UaS_k := submatrix(VaS, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$UaR_k := submatrix(VaR, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$AaS_k := submatrix(IaS, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$AaR_k := submatrix(IaR, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$UbS_k := submatrix(VbS, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$UbR_k := submatrix(VbR, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$AbS_k := submatrix(IbS, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$AbR_k := submatrix(IbR, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$UcS_k := submatrix(VcS, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$UcR_k := submatrix(VcR, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$AcS_k := submatrix(IcS, k \cdot 8, k \cdot 8 + wind, 0, 0)$

$VaS =$

	0
0	0.447
1	3.88
2	7.303
3	10.709
4	14.088
5	17.434
6	20.738
7	23.992
8	27.188
9	30.318
10	[illegible]

$UaS_k =$

	0
0	[128, 1]
1	[128, 1]
2	[128, 1]
3	[128, 1]
4	[128, 1]
5	[128, 1]
6	[128, 1]
7	[128, 1]
8	[128, 1]
9	[128, 1]

$AcR_k := \text{submatrix}(IcR, k\cdot 8, k\cdot 8 + wind, 0, 0)$

10	33.376
11	36.353
12	39.243
13	42.038
14	44.732
15	...

9	[128, 1]
10	[128, 1]
11	[128, 1]
12	[128, 1]
13	[128, 1]
14	[128, 1]
15	...

$PrS_k := FFT(UrS_k)$

$PrR_k := FFT(UrR_k)$

$FrS_k := FFT(ArS_k)$

$FrR_k := FFT(ArR_k)$

$PaS_k := FFT(UaS_k)$

$PaR_k := FFT(UaR_k)$

$FaS_k := FFT(AaS_k)$

$FaR_k := FFT(AaR_k)$

$PbS_k := FFT(UbS_k)$

$PbR_k := FFT(UbR_k)$

$FbS_k := FFT(AbS_k)$

$FbR_k := FFT(AbR_k)$

$PcS_k := FFT(UcS_k)$

$PcR_k := FFT(UcR_k)$

$FcS_k := FFT(AcS_k)$

$FcR_k := FFT(AcR_k)$

$PaS_k =$

	0
0	[65, 1]
1	[65, 1]
2	[65, 1]
3	[65, 1]
4	[65, 1]
5	[65, 1]
6	[65, 1]
7	[65, 1]
8	[65, 1]
9	[65, 1]
10	[65, 1]
11	[65, 1]
12	[65, 1]
13	[65, 1]
14	[65, 1]
15	...

Fourier transformed

$(PaS_k)_{1,0} =$

	0
0	0.217-34.986i
1	13.587-32.24i
2	24.889-24.587i
3	32.403-13.191i
4	34.984+0.213i
5	32.24+13.585i
6	24.588+24.888i
7	13.193+32.403i
8	-0.21+34.986i
9	-13.581+32.243i
10	-24.884+24.592i
11	-32.4+13.197i
12	-34.984-0.206i
13	-32.242-13.578i
14	-24.593-24.883i
15	...

50 Hz component

##

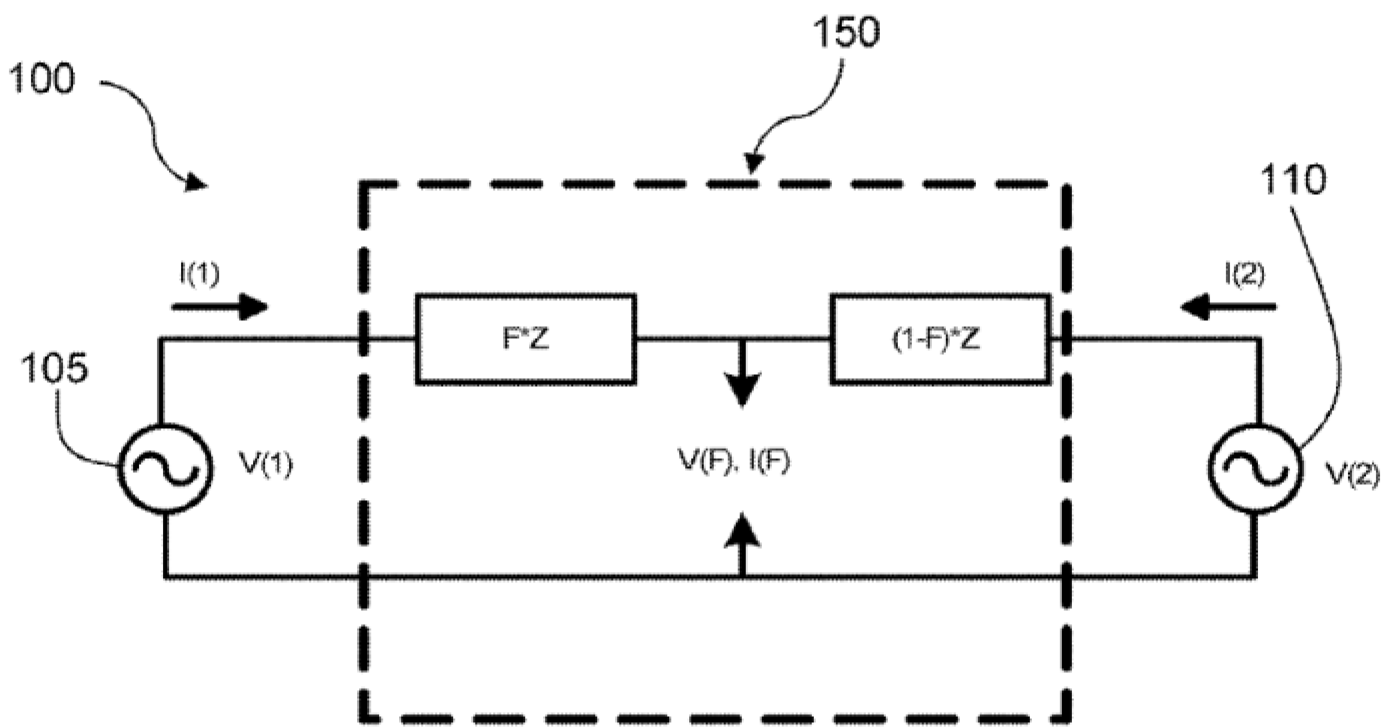
Implementation of the Premelani Method (Pat2_US20080150544_Premelani)

The patent publication includes:
1) fault location algorithm for two-ended system
2) fault location algorithm for three-terminal system
3) fault resistance calculations
4) charging current compensation to enhance the accuracy of fault locating system

In this take-home exam only 1) fault location algorithm for two-ended system part of the patent is used. In the patent it is claimed that the fault location algorithm is independent of faulted phase, fault type, fault resistance and zero-sequence (ground current) coupling to an adjacent transmission line, if any.

The algorithm uses data from two terminals in the two-ended system. These two terminals are bus number 1 (= S-bus) and bus number 2 (= R-bus) and the data includes phase voltages and currents from both terminals. In this take-home exam only the data generated in Mathcad can be used to test the functionality of the algorithm. Real data from Creelman substation has measurements only from one terminal, so it cannot be used.

TWO-ENDED SYSTEM



$$b^* == 1 - j^* \tan(alpha)$$

Clarke transforms of voltages and currents

Clarke transforms of voltages and currents

$$V=(1/3)*(2*VA-b*VB-(b*)*VC)$$ **where** $$b=1+j*\tan(\text{alpha})$$ **and**

$\text{alpha} := \frac{\pi}{4}$ $\text{alpha} := \frac{\pi}{3.4}$ $\text{alpha} = 0.924$

Value of alpha can be freely chosen. Value π/4 was suggested in the patent publication, but in this single-phase-to-ground fault case value π/3.4 was found out to give more reasonable and accurate resul

$b := 1 + j\cdot\tan(\text{alpha})$ $b = 1 + 1.324i$

$b\_conj := 1 - j\cdot\tan(\text{alpha})$ $b\_conj = 1 - 1.324i$

Data from S bus

$$Clarke\_V1_k := \left(\frac{1}{3}\right)\cdot\left[2\cdot\left(PaS_k\right)_{1,0} - b\cdot\left(PbS_k\right)_{1,0} - b\_conj\cdot\left(PcS_k\right)_{1,0}\right]$$

$$Clarke\_I1_k := \left(\frac{1}{3}\right)\cdot\left[2\cdot\left(FaS_k\right)_{1,0} - b\cdot\left(FbS_k\right)_{1,0} - b\_conj\cdot\left(FcS_k\right)_{1,0}\right]$$

Data from R bus

$$Clarke\_V2_k := \left(\frac{1}{3}\right)\cdot\left[2\cdot\left(PaR_k\right)_{1,0} - b\cdot\left(PbR_k\right)_{1,0} - b\_conj\cdot\left(PcR_k\right)_{1,0}\right]$$

$$Clarke\_I2_k := \left(\frac{1}{3}\right)\cdot\left[2\cdot\left(FaR_k\right)_{1,0} - b\cdot\left(FbR_k\right)_{1,0} - b\_conj\cdot\left(FcR_k\right)_{1,0}\right]$$

Equation for the fractional fault location F

$$F = \text{Real}\left[\frac{\frac{V(1)-V(2)}{Z} + I(2)}{I(1)+I(2)}\right]$$

Taken from patent publication, at [0036]:

Variable Z in the equation above is the total line impedance of the transmission line. This value is a complex ratio of the composite voltage and composite current measured at one end of the line with the other end under fault. Practically this impedance is equal to the negative or positive sequence impedance of the line.

So basically this says that the Z in the equation is equal to Z1L in this Mathcad implementation (Z1L = positive sequence impedance of the line). However, with the generated data used in this implementation, the value of Z has to include also positive sequence source impedances Z1S and Z1R as they are not modeled in the circuit described in the patent:

$Z1all := Z1S + Z1R + Z1L$ $\qquad Z1all = 5.657 + 17.072i$

When the two-ended system is in normal state (i.e. no fault has occurred) the denominator of the fault location equation i.e. (I(1) + I(2)) becomes zero.

Variable k runs from 0 to 176 and the denominator is non-zero when k = 49 ... 127 (see the table on right).
Fault location calculations and graph are presented below:

$f := 49 .. 127$

$$F_f := \mathrm{Re}\left[\frac{\dfrac{\left(Clarke\_V1_f - Clarke\_V2_f\right)}{Z1all} + Clarke\_I2_f}{Clarke\_I1_f + Clarke\_I2_f}\right]$$

$r := 128 .. 176$

$F_r := 0$

When the real fault location is m, the effective fault location in this implementation is mEffective which is calculated below. The difference between m and mEffective is that mEffetice takes also the source impedance values into account.

$Zeff := Z1S + m \cdot Z1L$ $\qquad Zeff = 4.622 + 13.208i$

$$mEffective := \frac{\mathrm{Im}(Zeff)}{\mathrm{Im}(Z1all)}$$

With the used network parameters the values of mEffective run approximately from 0.661 to 0.887 while the real fault location values (m) run from 0 to 1.

$$calculated\_m_k := \frac{F_k \cdot \mathrm{Im}(Z1all) - \mathrm{Im}(Z1S)}{\mathrm{Im}(Z1L)}$$

This fault location equation is based on results given by Premelani fault location equation and the equations of Zeff and mEffective presented above. Basically it transforms fault location F to the scale of 0 to 1 so that calculated_m should correspond to the real fault location m.

Fault location by Premelani algorithm $F_{100} = 0.768$

Effective fault location $mEffective = 0.774$

$abs\_error := \left| F_{100} - mEffective \right|$ $abs\_error = 5.787 \times 10^{-3}$

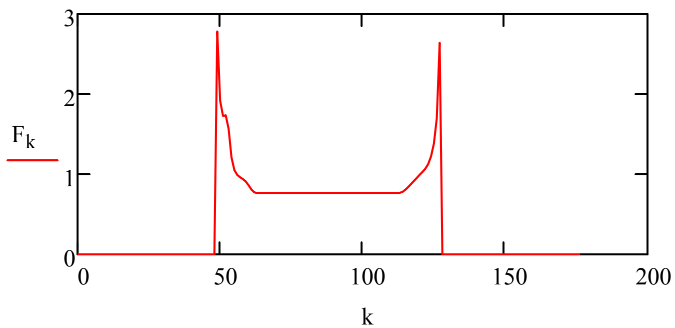
$\%\_error := \dfrac{\left| F_{100} - mEffective \right|}{mEffective} \cdot 100$ $\%\_error = 0.748$

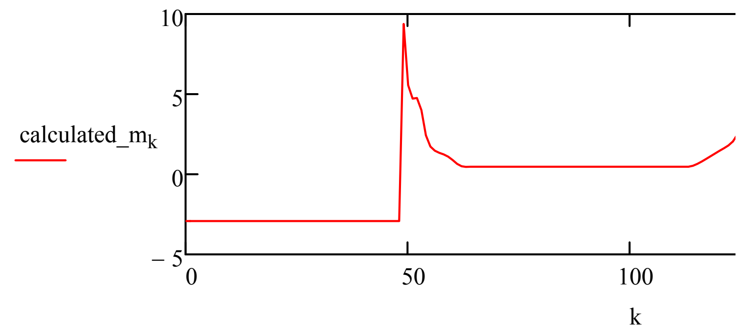
Calculated fault location $calculated\_m_{100} = 0.474$

Real fault location $m = 0.5$

$abs\_error\_true := \left| calculated\_m_{100} - m \right|$ $abs\_error\_true = 0.026$

$\%\_error\_true := \dfrac{\left| calculated\_m_{100} - m \right|}{m} \cdot 100$ $\%\_error\_true = 5.114$





Example results of the fault location calculations with different fault resistances, values of alpha and fault locations:

Fault resistance	alpha	Real fault location	Calculated fault location	Absolute error	Relative error	Effective fault location	Fault location by Pr
ZFG	alpha	m	$calculated\_m_{100}$	abs_error_true	%_error_true	mEffective	F_{100}
0	π/4	0.01	0.996	0.986	9864	0.663	0.88
0	π/4	0.10	1.044	0.944	944.068	0.683	0.89
0	π/4	0.50	1.183	0.683	136.613	0.774	0.92
0	π/4	0.75	1.209	0.459	61.255	0.83	0.93
0	π/4	0.98	1.193	0.213	21.69	0.882	0.93
0	π/3.4	0.01	-0.006228	0.016	162.278	0.663	0.65
0	π/3.4	0.10	0.081	0.019	18.742	0.683	0.67
0	π/3.4	0.50	0.429	0.071	14.122	0.774	0.75
0	π/3.4	0.75	0.613	0.137	18.233	0.83	0.79
0	π/3.4	0.98	0.76	0.22	22.497	0.882	0.83

Fault resistance	alpha	Real fault location	Calculated fault location	Absolute error	Relative error	Effective fault location	Fault location by Pr
ZFG	alpha	m	calculated_m_{100}	abs_error_true	%_error_true	mEffective	F_{100}
0	π/4	0.01	0.996	0.986	9864	0.663	0.88
0	π/4	0.10	1.044	0.944	944.068	0.683	0.89
0	π/4	0.50	1.183	0.683	136.613	0.774	0.92
0	π/4	0.75	1.209	0.459	61.255	0.83	0.93
0	π/4	0.98	1.193	0.213	21.69	0.882	0.93
0	π/3.4	0.01	-0.006228	0.016	162.278	0.663	0.65
0	π/3.4	0.10	0.081	0.019	18.742	0.683	0.67
0	π/3.4	0.50	0.429	0.071	14.122	0.774	0.75
0	π/3.4	0.75	0.613	0.137	18.233	0.83	0.79
0	π/3.4	0.98	0.76	0.22	22.497	0.882	0.83
0.85	π/4	0.01	1.077	1.067	10670	0.663	0.90
0.85	π/4	0.10	1.125	1.025	1025	0.683	0.91
0.85	π/4	0.50	1.264	0.764	152.786	0.774	0.94
0.85	π/4	0.75	1.291	0.541	72.072	0.83	0.95
0.85	π/4	0.98	1.274	0.294	29.994	0.882	0.94
0.85	π/3.4	0.01	0.039	0.029	285.294	0.663	0.66
0.85	π/3.4	0.10	0.126	0.026	26.068	0.683	0.68
0.85	π/3.4	0.50	0.474	0.026	5.114	0.774	0.76
0.85	π/3.4	0.75	0.658	0.092	12.207	0.83	0.81
0.85	π/3.4	0.98	0.805	0.175	17.873	0.882	0.84
10	π/4	0.01	1.942	1.932	19320	0.663	1.1
10	π/4	0.10	1.991	1.891	1891	0.683	1.11
10	π/4	0.50	2.134	1.634	326.89	0.774	1.14
10	π/4	0.75	2.164	1.414	188.517	0.83	1.15
10	π/4	0.98	2.15	1.17	119.373	0.882	1.14
10	π/3.4	0.01	0.52	0.51	5103	0.663	0.77
10	π/3.4	0.10	0.608	0.508	508.423	0.683	0.79
10	π/3.4	0.50	0.959	0.459	91.862	0.774	0.87
10	π/3.4	0.75	1.145	0.395	52.653	0.83	0.92
10	π/3.4	0.98	1.293	0.313	31.912	0.882	0.95

Conclusion

The patent claims that it can locate faults independent of faulted phase, fault type, fault resistance and zero-sequence (ground current) coupling to adjacent transmission line, if any.

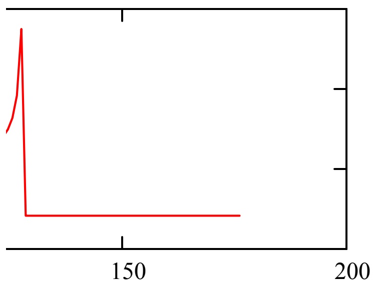
This mathcad implementation and the results above do not verify the claimed abilities. Firstly, the fault resistance value has huge effect on the accuracy of the Premelani fault location algorithm. Secondly, the value of alpha has crucial effect on the fault location results. The suggested value for alpha, π/4, doesn't give reasonable results in this single-phase-to-ground fault case. When value alpha = π/3.4 was used and the fault resistance was small, the results were tolerable even though still somewhat inaccurate.

If this algorithm was to be used in practice, the users would need to somehow find out the fault type and have a good estimate of the fault resistance before selecting a proper value for alpha. If the selected alpha is optimal for the fault case and for the fault resistance, the fault location results given by the algorithm are tolerable, even though there might still be 5-25 % relative error in the fault location.

$^{3} - 0.014i$	$-0.017 + 0.03i$	$1.169 \times 10^{-11} - 1.111i \times 10^{-12}$	$1.169 \times 10^{-11} - 1.111i \times 10^{-12}$
$^{3} - 0.014i$	$1.244 \times 10^{-3} - 3.83i \times 10^{-3}$	$-2.095 \times 10^{-11} - 1.248i \times 10^{-12}$	$1.253 \times 10^{-12} + 4.467i \times 10^{-14}$
$0.039i$	$1.244 \times 10^{-3} - 3.83i \times 10^{-3}$	$1.253 \times 10^{-12} + 4.467i \times 10^{-14}$	$-2.095 \times 10^{-11} - 1.248i \times 10^{-12}$
$^{-3} + 0.011i$	$-0.055 + 0.128i$	$3.486 \times 10^{-11} - 6.802i \times 10^{-12}$	$3.486 \times 10^{-11} - 6.802i \times 10^{-12}$
$^{-3} + 0.014i$	$-1.244 \times 10^{-3} + 3.83i \times 10^{-3}$	$-7.905 \times 10^{-11} + 1.248i \times 10^{-12}$	$-1.253 \times 10^{-12} - 4.467i \times 10^{-14}$
$0.039i$	$-1.244 \times 10^{-3} + 3.83i \times 10^{-3}$	$-1.253 \times 10^{-12} - 4.467i \times 10^{-14}$	$-7.905 \times 10^{-11} + 1.248i \times 10^{-12}$
$3.256i \times 10^{-3}$	$0.939 + 0.134i$	$-4.543 \times 10^{-11} - 6.726i \times 10^{-12}$	$-4.543 \times 10^{-11} - 6.726i \times 10^{-12}$
$9i \times 10^{-3}$	$0.405 + 0.055i$	$1.036 \times 10^{-10} + 4.251i \times 10^{-10}$	$1.807 \times 10^{-11} + 1.258i \times 10^{-10}$
$0.016i$	$0.405 + 0.055i$	$1.807 \times 10^{-11} + 1.258i \times 10^{-10}$	$1.036 \times 10^{-10} + 4.251i \times 10^{-10}$

ts.

$Clarke\_I1_k + Clarke\_I2_k = ▮$



ation by Premelani method	Absolute error	Relative error
F_{100}	abs_error	%_error
0.886	0.223	33.685
0.897	0.214	31.279
0.928	0.155	19.983
0.934	0.104	12.524
0.93	0.048	5.453
0.659	0.003673	0.554
0.679	0.004242	0.621
0.758	0.016	2.066
0.799	0.031	3.728
0.832	0.05	5.656

ation by Premelani method	Absolute error	Relative error
F_{100}	abs_error	%_error
0.886	0.223	33.685
0.897	0.214	31.279
0.928	0.155	19.983
0.934	0.104	12.524
0.93	0.048	5.453
0.659	0.003673	0.554
0.679	0.004242	0.621
0.758	0.016	2.066
0.799	0.031	3.728
0.832	0.05	5.656
0.904	0.241	36.429
0.915	0.232	33.944
0.947	0.173	22.349
0.953	0.122	14.736
0.949	0.067	7.54
0.669	0.006457	0.974
0.689	0.0059	0.864
0.768	0.005787	0.748
0.81	0.021	2.496
0.843	0.04	4.493
1.1	0.437	65.968
1.111	0.428	62.637
1.144	0.37	47.816
1.15	0.32	38.544
1.147	0.265	31.01
0.778	0.116	17.428
0.798	0.115	16.845
0.878	0.104	13.437
0.92	0.089	10.765
0.953	0.071	8.023