

Fault Location $d := 0.95$

$\delta := 0.001$ **Voltage Phase Angle**

$es := 12000 \cdot e^{j \cdot \delta \cdot \text{deg}}$ **Source S Voltage**

$er := 12000$ $er := 0$ $es = 1.2 \times 10^4 + 0.209i$

$Ssir := .3 \cdot e^{j \cdot -5 \cdot \text{deg}}$ **Source S SIR(Soure Impedance Ratio)**

$Ssir = 0.299 - 0.026i$

Source R Voltage

$Rsir := 0.2$ **Source R SIR**

$Z1L := 4 \cdot e^{j \cdot 1 \cdot \text{deg}}$ **Positive Sequence Line Impedance**

$Z1L := 1.4 + 1.6i$

$Z1L = 1.4 + 1.6i$

$INF := 10^{10}$

$Z0L := 3 \cdot Z1L$ **Zero Sequence Line Impedance**

$Z1S := Ssir \cdot Z1L$ **Source S Positive Sequence Impedance**

$Z1S := 4.2 + 5.1i$

$Z0S := 5 \cdot e^{j \cdot -5 \cdot \text{deg}} \cdot Z1S$ **Source S Zero Sequence Impedance**

$Z0S = 2.485 + 1.999i$

$Z1R := Z1S - \text{Re}(Z1S)$ **Source R Positive Sequence Impedance**

$Z1R = 0.442i$

$Z0R := 3 \cdot Z1R$

$Z1R := 20Z1S$

Source S Zero Sequence Impedance

$$Z_{OR} = 1.325i$$

$$Z_{OR} = 1.325i$$

Creelman

$$\underline{Z}_{1L} := 10 + 13i$$

$$\underline{Z}_{0L} := 30 + 39i$$

$$\underline{Z}_{1S} := 0.042 + 0.513i$$

$$\underline{Z}_{0S} := 0.013 + 1.225i$$

$$\underline{Z}_{1R} := 0.42 + 0.5i$$

$$\underline{Z}_{0R} := 0$$

$$Z_{1R} := 4.2 + 5i$$

$$Z_{1R} := 0$$

$$Z_{0R} := 3 \cdot Z_{1R}$$

$$INF = 1 \times 10^{10}$$

$$Z_{1R} := 400 + .01i$$

$$Z_{0R} := 0.42 + 0.5i$$

$$Z_{1R} := 0$$

Fault Impedances (for AG fault case)

$$Z_{FA} := 0 + j \cdot 0$$

$$Z_{FB} := INF + j \cdot 0$$

$$Z_{FC} := INF + j \cdot 0$$

Fault Resistance

$$Z_{FG} := 0.85 + j \cdot 0$$

$$Z_{FG} := 30 + j \cdot 0$$

$$Z_{FG} := 1$$

CONSTANTS

$$\underline{rad} := 1 \quad \underline{deg} := \frac{\pi}{180} \cdot \underline{rad}$$

$$a := -0.5 + j \cdot 0.8660254$$

$$\underline{BAL} := \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix}$$

$$\underline{one} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underline{zero} := \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Three phase voltages at S and R

$$ES := es \cdot \underline{BAL}$$

$$ER := er \cdot \underline{BAL}$$

$$ES = \begin{pmatrix} 1.2 \times 10^4 + 0.209i \\ -6 \times 10^3 - 1.039i \times 10^4 \\ -6 \times 10^3 + 1.039i \times 10^4 \end{pmatrix} \quad ER = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Conversion of positive sequence and zero sequence impedances to Self and Mutual impedances

$$z_s(z_0, z_1) := \frac{2 \cdot z_1 + z_0}{3}$$

$$z_m(z_0, z_1) := \frac{z_0 - z_1}{3}$$

Conversion Matrix Format

$$Z(z_0, z_1) := \begin{pmatrix} z_s(z_0, z_1) & z_m(z_0, z_1) & z_m(z_0, z_1) \\ z_m(z_0, z_1) & z_s(z_0, z_1) & z_m(z_0, z_1) \\ z_m(z_0, z_1) & z_m(z_0, z_1) & z_s(z_0, z_1) \end{pmatrix}$$

Now Conversion

$$\begin{aligned} ZS &:= Z(Z0S, Z1S) & ZL &:= Z(Z0L, Z1L) & ZR &:= Z(Z0R, Z1R) & ZR_{2,1} &:= 0 \\ & & & & ZR_{0,1} &:= 0 & ZR_{0,2} &:= 0 & ZR_{1,0} &:= 0 & ZR_{1,2} &:= 0 & ZR_{2,0} &:= 0 \\ ZS &= \begin{pmatrix} 0.032 + 0.75i & -9.667 \times 10^{-3} + 0.237i & -9.667 \times 10^{-3} + 0.237i \\ -9.667 \times 10^{-3} + 0.237i & 0.032 + 0.75i & -9.667 \times 10^{-3} + 0.237i \\ -9.667 \times 10^{-3} + 0.237i & -9.667 \times 10^{-3} + 0.237i & 0.032 + 0.75i \end{pmatrix} & ZR &= \begin{pmatrix} 0.28 + 0.333i & -0.14 - 0.167i & -0.14 - 0.167i \\ -0.14 - 0.167i & 0.28 + 0.333i & -0.14 - 0.167i \\ -0.14 - 0.167i & -0.14 - 0.167i & 0.28 + 0.333i \end{pmatrix} \end{aligned}$$

Source and Line Impedances to the Fault

$$ZSS := ZS + d \cdot ZL \quad ZSS = \begin{pmatrix} 15.866 + 21.334i & 6.324 + 8.471i & 6.324 + 8.471i \\ 6.324 + 8.471i & 15.866 + 21.334i & 6.324 + 8.471i \\ 6.324 + 8.471i & 6.324 + 8.471i & 15.866 + 21.334i \end{pmatrix}$$

$$ZRR := ZR + (1 - d) \cdot ZL \quad ZRR = \begin{pmatrix} 1.113 + 1.417i & 0.193 + 0.267i & 0.193 + 0.267i \\ 0.193 + 0.267i & 1.113 + 1.417i & 0.193 + 0.267i \\ 0.193 + 0.267i & 0.193 + 0.267i & 1.113 + 1.417i \end{pmatrix}$$

Build System Part of the Impedance Matrix —see (2) of page 3 in the paper.

$$ZTOP := \text{augment}(\text{augment}(ZSS, \text{zero}), \text{one}) \quad ZTOP = \begin{pmatrix} 15.866 + 21.334i & 6.324 + 8.471i & 6.324 + 8.471i & 0 & 0 & 0 & 1 & 0 & 0 \\ 6.324 + 8.471i & 15.866 + 21.334i & 6.324 + 8.471i & 0 & 0 & 0 & 0 & 1 & 0 \\ 6.324 + 8.471i & 6.324 + 8.471i & 15.866 + 21.334i & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$ZMID := \text{augment}(\text{augment}(\text{zero}, ZRR), \text{one}) \quad ZMID = \begin{pmatrix} 0 & 0 & 0 & 1.113 + 1.417i & 0.193 + 0.267i & 0.193 + 0.267i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.193 + 0.267i & 1.113 + 1.417i & 0.193 + 0.267i & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.193 + 0.267i & 0.193 + 0.267i & 1.113 + 1.417i & 0 & 0 & 1 \end{pmatrix}$$

$$ZSYS := \text{stack}(ZTOP, ZMID) \quad ZSYS = \begin{pmatrix} 15.866 + 21.334i & 6.324 + 8.471i & 6.324 + 8.471i & 0 & 0 & 0 & 1 & 0 & 0 \\ 6.324 + 8.471i & 15.866 + 21.334i & 6.324 + 8.471i & 0 & 0 & 0 & 0 & 1 & 0 \\ 6.324 + 8.471i & 6.324 + 8.471i & 15.866 + 21.334i & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1.113 + 1.417i & 0.193 + 0.267i & 0.193 + 0.267i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.193 + 0.267i & 1.113 + 1.417i & 0.193 + 0.267i & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.193 + 0.267i & 0.193 + 0.267i & 1.113 + 1.417i & 0 & 0 & 1 \end{pmatrix}$$

Pre-fault conditions:

$$ZPRE := ZS + ZL + ZR$$

$$ZPRE = \begin{pmatrix} 16.979 + 22.75i & 6.517 + 8.737i & 6.517 + 8.737i \\ 6.517 + 8.737i & 16.979 + 22.75i & 6.517 + 8.737i \\ 6.517 + 8.737i & 6.517 + 8.737i & 16.979 + 22.75i \end{pmatrix}$$

$$IPRE := ZPRE^{-1} \cdot (ES - ER)$$

$$IPRE = \begin{pmatrix} 410.529 - 549.85i \\ -681.448 - 80.603i \\ 270.92 + 630.453i \end{pmatrix}$$

$$VSP := ES - ZS \cdot IPRE$$

$$VSP = \begin{pmatrix} 1.17 \times 10^4 - 187.298i \\ -6.013 \times 10^3 - 1.004i \times 10^4 \\ -5.688 \times 10^3 + 1.023i \times 10^4 \end{pmatrix} \quad ES = \begin{pmatrix} 1.2 \times 10^4 + 0.209i \\ -6 \times 10^3 - 1.039i \times 10^4 \\ -6 \times 10^3 + 1.039i \times 10^4 \end{pmatrix}$$

Voltage at the relay near bus S? Yes, Prefault :

Build the voltage Vector: Again, see (2) of page 3

$$\text{null} := (0 \ 0 \ 0)$$

$$E := \text{stack}(\text{stack}(ES, ER), \text{null}^T)$$

$$TS := \text{augment}(\text{augment}(\text{one}, \text{zero}), \text{zero})$$

$$TR := \text{augment}(\text{augment}(\text{zero}, \text{one}), \text{zero})$$

$$\text{TVF} := \text{augment}(\text{augment}(\text{zero}, \text{zero}), \text{one})$$

$$E = \begin{pmatrix} 1.2 \times 10^4 + 0.209i \\ -6 \times 10^3 - 1.039i \times 10^4 \\ -6 \times 10^3 + 1.039i \times 10^4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$TS = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$TR = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{TVF} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Building Fault Part of the Impedance Matrix:

$$ZFAG := ZFA + ZFG$$

$$ZFBG := ZFB + ZFG$$

$$ZFCG := ZFC + ZFG$$

$$ZFAG = 30$$

$$ZFBG = 1 \times 10^{10}$$

$$ZF := \begin{pmatrix} ZFAG & ZFG & ZFG \\ ZFG & ZFBG & ZFG \\ ZFG & ZFG & ZFCG \end{pmatrix}$$

$$FABCG := \text{augment}(\text{augment}(-ZF, -ZF), \text{one})$$

$$ZF = \begin{pmatrix} 30 & 30 & 30 \\ 30 & 1 \times 10^{10} & 30 \\ 30 & 30 & 1 \times 10^{10} \end{pmatrix}$$

$$FABCG = \begin{pmatrix} -30 & -30 & -30 & -30 & -30 & -30 & 1 & 0 & 0 \\ -30 & -1 \times 10^{10} & -30 & -30 & -1 \times 10^{10} & -30 & 0 & 1 & 0 \\ -30 & -30 & -1 \times 10^{10} & -30 & -30 & -1 \times 10^{10} & 0 & 0 & 1 \end{pmatrix}$$

$$ZABCG := \text{stack}(ZSYS, FABCG)$$

$$ZABCG = \begin{pmatrix} 15.866 + 21.334i & 6.324 + 8.471i & 6.324 + 8.471i & 0 & 0 & 0 & 1 & 0 & 0 \\ 6.324 + 8.471i & 15.866 + 21.334i & 6.324 + 8.471i & 0 & 0 & 0 & 0 & 1 & 0 \\ 6.324 + 8.471i & 6.324 + 8.471i & 15.866 + 21.334i & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1.113 + 1.417i & 0.193 + 0.267i & 0.193 + 0.267i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0.193 + 0.267i & 1.113 + 1.417i & 0.193 + 0.267i & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.193 + 0.267i & 0.193 + 0.267i & 1.113 + 1.417i & 0 & 0 & 1 \\ -30 & -30 & -30 & -30 & -30 & -30 & 1 & 0 & 0 \\ -30 & -1 \times 10^{10} & -30 & -30 & -1 \times 10^{10} & -30 & 0 & 1 & 0 \\ -30 & -30 & -1 \times 10^{10} & -30 & -30 & -1 \times 10^{10} & 0 & 0 & 1 \end{pmatrix}$$

$$YABCG := ZABCG^{-1}$$

Fault Current $I_{ABCG} := Y_{ABCG} \cdot E$

$$E = \begin{pmatrix} 1.2 \times 10^4 + 0.209i \\ -6 \times 10^3 - 1.039i \times 10^4 \\ -6 \times 10^3 + 1.039i \times 10^4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad I_{ABCG} = \begin{pmatrix} 412.874 - 550.098i \\ -681.827 - 80.552i \\ 270.541 + 630.505i \\ -380.434 + 547.628i \\ 681.827 + 80.552i \\ -270.541 - 630.505i \\ 973.187 - 74.096i \\ -541.231 - 865.986i \\ -482.767 + 883.41i \end{pmatrix}$$

S - End Fault Currents: $I_S := T_S \cdot I_{ABCG}$

R - End Fault Currents: $I_R := T_R \cdot I_{ABCG}$

$$I_S = \begin{pmatrix} 412.874 - 550.098i \\ -681.827 - 80.552i \\ 270.541 + 630.505i \end{pmatrix} \quad I_R = \begin{pmatrix} -380.434 + 547.628i \\ 681.827 + 80.552i \\ -270.541 - 630.505i \end{pmatrix}$$

VF fault voltages:

$V_F := T_{VF} \cdot I_{ABCG}$

$$V_F = \begin{pmatrix} 973.187 - 74.096i \\ -541.231 - 865.986i \\ -482.767 + 883.41i \end{pmatrix}$$

S - End Voltages

$V_S := E_S - Z_S \cdot I_S$

$$V_S = \begin{pmatrix} 1.17 \times 10^4 - 188.869i \\ -6.013 \times 10^3 - 1.004i \times 10^4 \\ -5.688 \times 10^3 + 1.023i \times 10^4 \end{pmatrix}$$

Line Prefault Load Currents from S Bus

$$I_a := IPRE_0 \quad |I_a| = 686.199 \quad \frac{\arg(I_a)}{\text{deg}} = -53.254 \quad 0.32 \cdot \frac{180}{3.14} = 18.344$$

$$I_b := IPRE_1 \quad |I_b| = 686.199 \quad \frac{\arg(I_b)}{\text{deg}} = -173.254 \quad IPRE = \begin{pmatrix} 410.529 - 549.85i \\ -681.448 - 80.603i \\ 270.92 + 630.453i \end{pmatrix}$$

$$I_c := IPRE_2 \quad |I_c| = 686.199 \quad \frac{\arg(I_c)}{\text{deg}} = 66.746$$

Line Prefault Voltages at S Bus

$$V_a := VSP_0 \quad |V_a| = 1.17 \times 10^4 \quad \frac{\arg(V_a)}{\text{deg}} = -0.917$$

$$V_b := VSP_1 \quad |V_b| = 1.17 \times 10^4 \quad \frac{\arg(V_b)}{\text{deg}} = -120.917$$

$$V_c := VSP_2 \quad |V_c| = 1.17 \times 10^4 \quad \frac{\arg(V_c)}{\text{deg}} = 119.083$$

$$VSP = \begin{pmatrix} 1.17 \times 10^4 - 187.298i \\ -6.013 \times 10^3 - 1.004i \times 10^4 \\ -5.688 \times 10^3 + 1.023i \times 10^4 \end{pmatrix}$$

Line Fault Currents from S Bus

$$I_{asf} := IS_0 \quad |I_{asf}| = 687.802 \quad \frac{\arg(I_{asf})}{\text{deg}} = -53.11$$

$$I_{bsf} := IS_1 \quad |I_{bsf}| = 686.569 \quad \frac{\arg(I_{bsf})}{\text{deg}} = -173.262$$

$$I_{csf} := IS_2 \quad |I_{csf}| = 686.097 \quad \frac{\arg(I_{csf})}{\text{deg}} = 66.776$$

$$IS = \begin{pmatrix} 412.874 - 550.098i \\ -681.827 - 80.552i \\ 270.541 + 630.505i \end{pmatrix}$$

Line Fault Currents from R Bus

$$I_{arf} := IR_0 \quad |I_{arf}| = 666.803 \quad \frac{\arg(I_{arf})}{\text{deg}} = 124.788$$

$$I_{brf} := IR_1 \quad |I_{brf}| = 686.569 \quad \frac{\arg(I_{brf})}{\text{deg}} = 6.738$$

$$I_{crf} := IR_2 \quad |I_{crf}| = 686.097 \quad \frac{\arg(I_{crf})}{\text{deg}} = -113.224$$

$$IR = \begin{pmatrix} -380.434 + 547.628i \\ 681.827 + 80.552i \\ -270.541 - 630.505i \end{pmatrix}$$

Line Fault Voltages at S Bus

$$V_{af} := VS_0 \quad |V_{af}| = 1.17 \times 10^4 \quad \frac{\arg(V_{af})}{\text{deg}} = -0.925$$

$$V_{bf} := VS_1 \quad |V_{bf}| = 1.17 \times 10^4 \quad \frac{\arg(V_{bf})}{\text{deg}} = -120.917$$

$$V_{cf} := VS_2 \quad |V_{cf}| = 1.17 \times 10^4 \quad \frac{\arg(V_{cf})}{\text{deg}} = 119.083$$

$$I_r := \sum_{mm=0}^2 IS_{mm} \quad I_r = 1.588 - 0.144i$$

$$IS = \begin{pmatrix} 412.874 - 550.098i \\ -681.827 - 80.552i \\ 270.541 + 630.505i \end{pmatrix}$$

$$d = 0.95$$

$$T1 := \frac{\arg\left[\frac{ZOS + ZOR + ZOL}{ZOR + (1-d)ZOL}\right]}{\text{deg}} = 0.841$$

$$T1 = 0.841$$

$$ZOS = 0.013 + 1.225i$$

$$\left[\frac{ZOS + ZOR + ZOL}{ZOR + (1-d)ZOL}\right] = 20.398 + 0.299i$$

$$ZOR = 0$$

$$ZOR = 0$$

$$Z1R = 0.42 + 0.5i$$

$$ZOL = 30 + 39i$$

$$k0 := \frac{ZOL - Z1L}{3 \cdot Z1L} \quad k0 = 0.667$$

$$e^{j \cdot T1} = 0.667 + 0.745i$$

$$ZOS + ZOR + ZOL = 30.013 + 40.225i$$

$$ZOL = 30 + 39i$$

distance L (which is supposed to be same as m)

$$L := \frac{\text{Im}\left[VS_0 \cdot \left(I_r \cdot e^{j \cdot T1 \cdot \text{deg}}\right)\right]}{\text{Im}\left[Z1L \cdot \left(IS_0 + k0 \cdot I_r\right) \cdot \left(I_r \cdot e^{j \cdot T1 \cdot \text{deg}}\right)\right]}$$

$$L = 0.95$$

$$e^{j \cdot T1} = 0.667 + 0.745i$$

$$IPRE_0 = 410.529 - 549.85i$$

$$IS_0 = 412.874 - 550.098i$$

$$|IS_0| = 687.802$$

$$VS_0 = 1.17 \times 10^4 - 188.869i$$

$$\frac{|IS_0|}{|IPRE_0|} = 1.002$$

$$VSP_0 = 1.17 \times 10^4 - 187.298i$$

$$VSP_0 = 1.17 \times 10^4 - 187.298i$$

$$VS_0 = 1.17 \times 10^4 - 188.869i$$

$$\frac{VSP_0}{IPRE_0} - Z1L = 0.42 + 0.5i$$

$$|VSP_0| = 1.17 \times 10^4 \frac{\arg(VSP_0)}{\text{deg}} = -0.917$$

$$|VS_0| = 1.17 \times 10^4$$

$$\frac{\arg(VS_0)}{\text{deg}} = -0.925$$

$$\frac{VS_0 - VSP_0}{IS_0 - IPRE_0} = -0.033 - 0.673i$$

So How do I generate digital signals of Voltage and Current of the Simulation 4 Cycles

$$|VS_1| = 1.17 \times 10^4$$

$$k := 0..511$$

$$\text{delT} := 0.0001302$$

$$\text{Van}_k := \left| \text{VSP}_0 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{VSP}_0\right)\right)$$

$$t1_k := k \cdot \text{delT}$$

$$t2_k := 512 \cdot \text{delT} + k \cdot \text{delT}$$

$$t3_k := 1024 \cdot \text{delT} + k \cdot \text{delT}$$

$$\text{Vbn}_k := \left| \text{VSP}_1 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{VSP}_1\right)\right)$$

$$\text{Vcn}_k := \left| \text{VSP}_2 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{VSP}_2\right)\right)$$

$$\text{Vaf}_k := \left| \text{VS}_0 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{VS}_0\right)\right)$$

$$\text{Vbf}_k := \left| \text{VS}_1 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{VS}_1\right)\right)$$

$$\text{Vcf}_k := \left| \text{VS}_2 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{VS}_2\right)\right)$$

$$\text{Ian}_k := \left| \text{IPRE}_0 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{IPRE}_0\right)\right)$$

$$\text{Ibn}_k := \left| \text{IPRE}_1 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{IPRE}_1\right)\right)$$

$$\text{Icn}_k := \left| \text{IPRE}_2 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{IPRE}_2\right)\right)$$

$$\text{Iaf}_k := \left| \text{IS}_0 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{IS}_0\right)\right)$$

$$\text{Ibf}_k := \left| \text{IS}_1 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{IS}_1\right)\right)$$

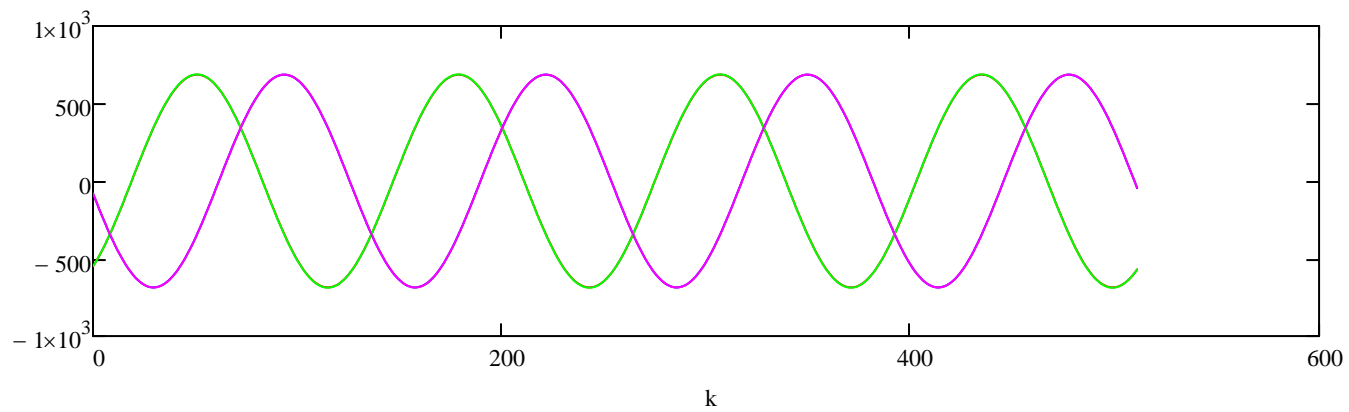
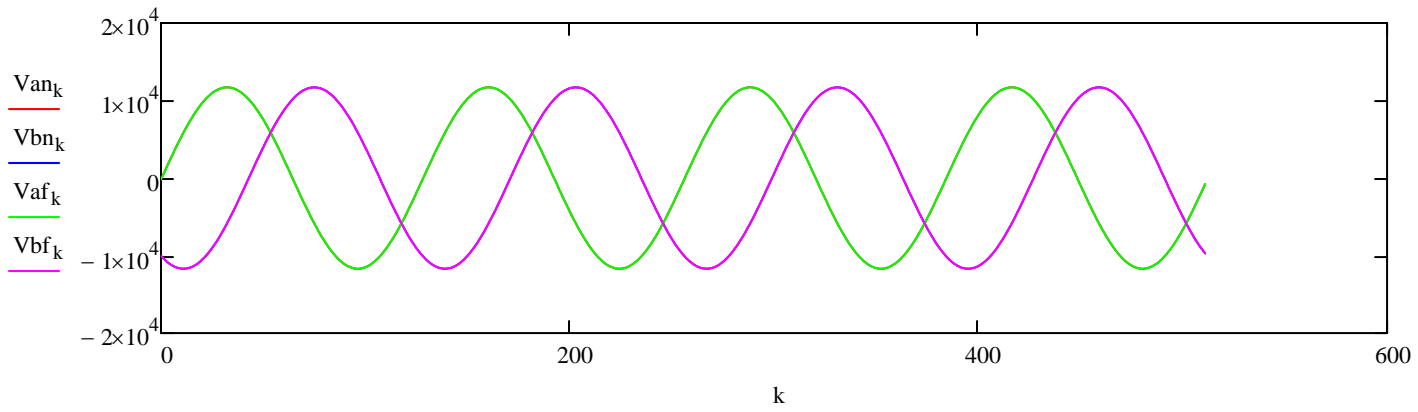
$$\text{Icf}_k := \left| \text{IS}_2 \right| \cdot \sin\left(2 \cdot \pi \cdot 60 \cdot k \cdot \text{delT} + \arg\left(\text{IS}_2\right)\right)$$

$$e^{j \cdot T1} = 0.667 + 0.745i$$

$$\left| \text{VS}_2 \right| = 1.17 \times 10^4$$

$$T1_k =$$

▪



$\min(t1) = 0$

$\max(t1) = 0.067$

$$\min(t2) = 0.067 \quad \max(t2) = 0.133$$

$$\min(t3) = 0.133 \quad \max(t3) = 0.2$$

$$\frac{1}{\frac{60}{128}} = 1.302 \times 10^{-4}$$

$$t1_{511} = 0.067 \quad t2_0 = 0.067$$

$$t2_0 - t1_{511} = 1.302 \times 10^{-4}$$

$$t3_0 - t2_{511} = 1.302 \times 10^{-4}$$

Seg1 := augment(t1, Ian, Ibn, Icn, Van, Vbn, Vcn)

Seg2 := augment(t2, Iaf, Ibf, Icf, Vaf, Vbf, Vcf)

Seg3 := augment(t3, Ian, Ibn, Icn, Van, Vbn, Vcn)

Final := stack(Seg1, Seg2, Seg3)

$$\begin{aligned} \underline{T} &:= \text{Final}^{(0)} & \underline{Ia} &:= \text{Final}^{(1)} & \underline{Ib} &:= \text{Final}^{(2)} & \underline{Ic} &:= \text{Final}^{(3)} & \underline{Va} &:= \text{Final}^{(4)} & \underline{Vb} &:= \text{Final}^{(5)} & \underline{Vc} &:= \text{Final}^{(6)} \\ \underline{Ir} &:= \underline{Ia} + \underline{Ib} + \underline{Ic} \end{aligned}$$

$$\underline{m} := \text{length}(\underline{Va}) \quad m = 1.536 \times 10^3$$

$$\underline{Ko} := .3$$

$$\underline{Ko} := \frac{1}{1 + 3 \cdot k0}$$

window := 128

wind := window - 1

End of waveform creation

EXAM WORK STARTS HERE

The exam task is based on US Patent No 4559491 (Saha) - Method and Device for Locating a Fault Point on a Three-Phase Power Transmission Line

The solution, that will provide us the fault location in the network, is presented in column "9" on the patent

$$\left. \begin{aligned} K_1 &= \frac{U_A}{I_A \times Z_L} + 1 + \frac{Z_B}{Z_L} \\ K_2 &= \frac{U_A}{I_A \times Z_L} \left(\frac{Z_B}{Z_L} + 1 \right) \\ K_3 &= \frac{I_{FA}}{I_A \times Z_L} \left[\frac{Z_A + Z_B}{Z_L} + 1 \right] \end{aligned} \right\} \quad (41)$$

Where U_A - Voltage at source observed (faulty phase) during fault

I_A - Current at source observed (faulty phase) during fault

Z_L - Impedance of the faulty section

Z_A - Source impedance of the first end (observing)

Z_B - Source impedance of the second end (not observing)

I_{FA} - Current change at point A during fault

In this model, the corresponding variables are:

$Z_A \rightarrow Z_S$

$Z_B \rightarrow Z_R$

This basically is pretty much the same as Novosel -patent algorithm

$$\left. \begin{aligned} k_1 &= \frac{v_{sf}}{I_{sf} Z_{L1}} + \frac{Z_{load}}{z_{L1}} + 1 \\ k_2 &= \frac{v_{sf}}{I_{sf} Z_{L1}} \left(\frac{Z_{load}}{Z_{L1}} + 1 \right) \\ k_3 &= \frac{\Delta I_s}{I_{sf} Z_{L1}} \left(\frac{Z_s + Z_{load}}{Z_{L1}} + 1 \right) \end{aligned} \right\}$$

Novosel algorithm is very straightforward when coming to distance solutions

Whereas the "exam" patent solution is given by

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where:

$$a = 1$$

$$b = - \left(Re(k_1) - \frac{Im(k_1) \times Re(k_3)}{Im(k_3)} \right)$$

$$c = Re(k_2) - \frac{Im(k_2) \times Re(k_3)}{Im(k_3)}$$

$$n^2 + B \times n + C = 0$$

where

$$B = \frac{K_{1V} \times K_{3H} - K_{1H} \times K_{3V}}{K_{3V}} \text{ and}$$

$$C = \frac{K_{2H} \times K_{3V} - K_{2V} \times K_{3H}}{K_{3V}}$$

In "exam" patent the "n" is the relative distance

It turns out that these 2 methods for the fault location ARE identical !!

This brings me to the next issue : what is the point of having the same methods in different patents, filed at different times?

My first guess is that the competition is tough and forces to find alternative methods to describe the solution. I do not know the patent law very deep, but it seems to me by this case, that when one could propose a different approach to the same calculation, it could be a different patent?

OK then, on to the trials of the algorithm.

In calculations, only the basic 50 Hz harmonics are used, all others rejected. Thus first FFT algorithms have to be used for extracting the first harmonic.

I am using the formulas provided in the course example

window := 128

wind := window - 1

k := 0 .. $\frac{m - \text{window}}{8}$

Isa := Ia + k0·Ir

Isb := Ib + k0·Ir

Isc := Ic + k0·Ir

Ua_k := submatrix(Va, k·8, k·8 + wind, 0, 0)

Ub_k := submatrix(Vb, k·8, k·8 + wind, 0, 0)

Uc_k := submatrix(Vc, k·8, k·8 + wind, 0, 0)

Aa_k := submatrix(Isa, k·8, k·8 + wind, 0, 0)

Ab_k := submatrix(Isb, k·8, k·8 + wind, 0, 0)

Ac_k := submatrix(Isc, k·8, k·8 + wind, 0, 0)

Pa_k := FFT(Ua_k)

Pb_k := FFT(Ub_k)

Pc_k := FFT(Uc_k)

Fa_k := FFT(Aa_k)

Fb_k := FFT(Ab_k)

Fc_k := FFT(Ac_k)

Algorithm constants k1, k2, k3 for the 3 phases

$$ka1_k := \frac{(Pa_k)_{1,0}}{(Fa_k)_{1,0} \cdot Z1L} + \frac{Z1R}{Z1L} + 1$$

$$ka2_k := \frac{(Pa_k)_{1,0}}{(Fa_k)_{1,0} \cdot Z1L} \cdot \left(\frac{Z1R}{Z1L} + 1 \right)$$

$$ka3_k := \frac{(Pa_k)_{1,0}}{(Fa_k)_{1,0} \cdot Z1L} \cdot \left(\frac{Z1S + Z1R}{Z1L} + 1 \right)$$

$$kb1_k := \frac{(Pb_k)_{1,0}}{(Fb_k)_{1,0} \cdot Z1L} + \frac{Z1R}{Z1L} + 1$$

$$kb2_k := \frac{(Pb_k)_{1,0}}{(Fb_k)_{1,0} \cdot Z1L} \cdot \left(\frac{Z1R}{Z1L} + 1 \right)$$

$$kb3_k := \frac{(Pb_k)_{1,0}}{(Fb_k)_{1,0} \cdot Z1L} \cdot \left(\frac{Z1S + Z1R}{Z1L} + 1 \right)$$

$$kc1_k := \frac{(Pc_k)_{1,0}}{(Fc_k)_{1,0} \cdot Z1L} + \frac{Z1R}{Z1L} + 1$$

$$kc2_k := \frac{(Pc_k)_{1,0}}{(Fc_k)_{1,0} \cdot Z1L} \cdot \left(\frac{Z1R}{Z1L} + 1 \right)$$

$$kc3_k := \frac{(Pc_k)_{1,0}}{(Fc_k)_{1,0} \cdot Z1L} \cdot \left(\frac{Z1S + Z1R}{Z1L} + 1 \right)$$

$$ba_k := - \left(\operatorname{Re}(ka1_k) - \frac{\operatorname{Im}(ka1_k) \cdot \operatorname{Re}(ka3_k)}{\operatorname{Im}(ka3_k)} \right)$$

$$ca_k := \operatorname{Re}(ka2_k) - \frac{\operatorname{Im}(ka2_k) \operatorname{Re}(ka3_k)}{\operatorname{Im}(ka3_k)}$$

$$ma_k := \frac{-ba_k - \sqrt{(ba_k \cdot ba_k - 4 \cdot 1 \cdot ca_k)}}{2 \cdot 1}$$

$$bb_k := - \left(\operatorname{Re}(kb1_k) - \frac{\operatorname{Im}(kb1_k) \cdot \operatorname{Re}(kb3_k)}{\operatorname{Im}(kb3_k)} \right)$$

$$cb_k := \operatorname{Re}(kb2_k) - \frac{\operatorname{Im}(kb2_k) \operatorname{Re}(kb3_k)}{\operatorname{Im}(kb3_k)}$$

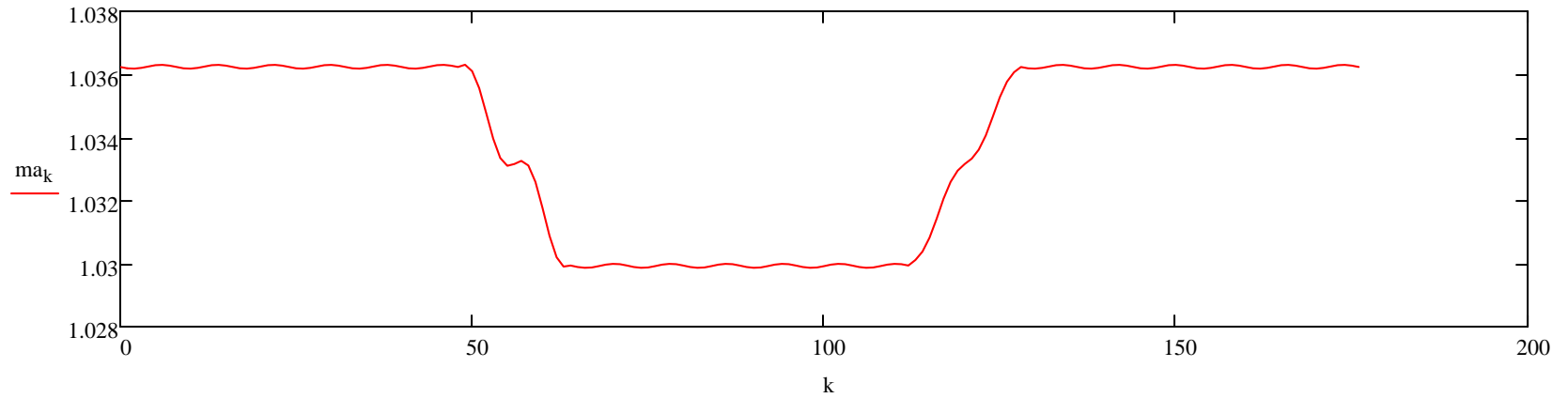
$$mb_k := \frac{-bb_k - \sqrt{(bb_k \cdot bb_k - 4 \cdot 1 \cdot cb_k)}}{2 \cdot 1}$$

$$bc_k := - \left(\operatorname{Re}(kc1_k) - \frac{\operatorname{Im}(kc1_k) \cdot \operatorname{Re}(kc3_k)}{\operatorname{Im}(kc3_k)} \right)$$

$$cc_k := \operatorname{Re}(kc2_k) - \frac{\operatorname{Im}(kc2_k) \operatorname{Re}(kc3_k)}{\operatorname{Im}(kc3_k)}$$

$$mc_k := \frac{-bc_k - \sqrt{(bc_k \cdot bc_k - 4 \cdot 1 \cdot cc_k)}}{2 \cdot 1}$$

$$ma_{80} = 1.03$$



This is pretty much as expected and as seen from the Novosel algorithm trials in class.

This method seems to perform even with 5% of the span to fault distance quite accurately, at 95% of the distance the accuracy is slightly more off. This is for the zero-resistance fault.

Of course it fails more with fault resistance not being zero. Here is an example of 30-ohm fault, which is quite common in MV distribution. Error is around 8% of the span. This though is still quite close, but it is also ideal conditions here!