

Note 15: 3-phase system -- SUMMARY

1. Balanced 3-φ system is characterized by:

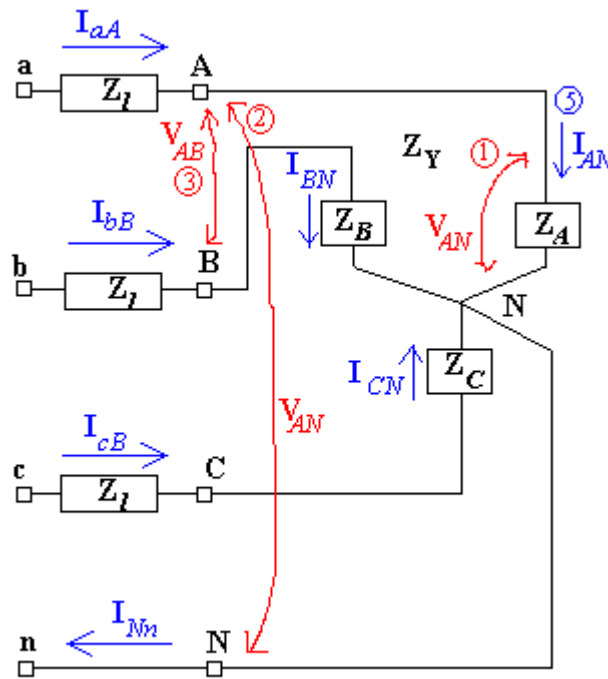
- 3 voltages are with same magnitude and 120° phase shift
- 3 load impedances are same
- Therefore, the currents are balanced with same magnitude and 120° apart

2. There are two types of voltages and two types of currents:

- Phase Voltage ( $V_\phi$ )= “voltage across a phase impedance”
- Line Voltage ( $V_l$ )= “voltage between a (phase) line and another (phase) line”
- Phase Current ( $I_\phi$ )= “current through a phase impedance”
- Line Current ( $I_l$ )= “current through a (phase) line”

3. Above definitions have different meaning at different load formation, Y or Δ

Y-load case: As shown below, the three phase impedances ( $Z_A, Z_B, Z_C$ ) form the letter “Y”. In the figure, the line connecting 3-phase source to the Y-load is represented by a line impedance,  $Z_l$ .



$V_\phi = V_{AN}, V_{BN}, \text{ and } V_{CN}$  (1)

i.e., phase voltage is same as the voltage between a (phase) line and the neutral (marked as “N”) (2)

$V_l = V_{AB}, V_{BC}, \text{ and } V_{CA}$  (3)

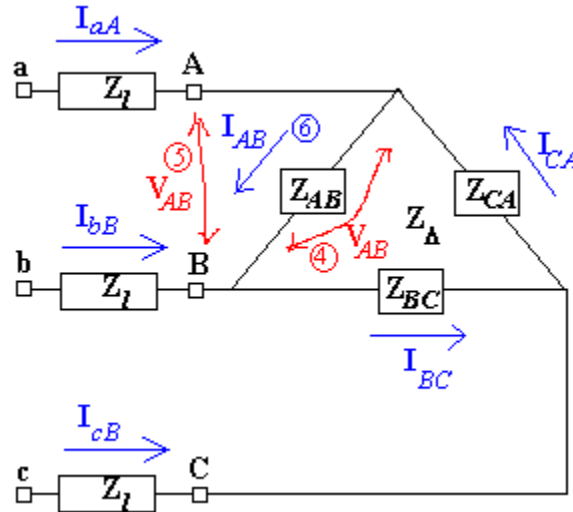
$I_\phi = I_{AN}, I_{BN}, \text{ and } I_{CN}$

$I_l = I_{aA}, I_{bB}, \text{ and } I_{cC}$  (also,  $I_{AN} = I_{aA}$ , etc )

Conclusion of Y-load:

(i)  $I_\phi = I_l$     (ii)  $V_\phi \neq V_l$  (instead,  $V_l = \sqrt{3}V_\phi \angle 30^\circ$ )

*Δ Load Case:* As shown below, the 3 phase loads ( $Z_{AB}$ ,  $Z_{BC}$ , and  $Z_{CA}$ ) form a Delta shape. As in Y-load, the line connecting 3-phase source to the Y-load is represented by a line impedance,  $Z_l$ . Note that there is no neutral point.



$$V_\phi = V_{AB}, V_{BC}, \text{ and } V_{CA} \quad (4)$$

$$V_l = V_{AB}, V_{BC}, \text{ and } V_{CA} \quad (5)$$

$$I_\phi = I_{AB}, I_{BC}, \text{ and } I_{CA}$$

$$I_l = I_{aA}, I_{bB}, \text{ and } I_{cC}$$

**Conclusion of  $\Delta$ -load:**

$$(i) I_\phi \neq I_l \quad (\text{instead } I_l = \sqrt{3}I_\phi \angle -30^\circ) \quad (ii) V_\phi = V_l$$

#### 4. 3-Phase Power Calculations

*Y-Load Case:*

$$P_{3\phi} = 3V_\phi I_\phi \cos \theta_\phi = 3\left(\frac{V_l}{\sqrt{3}}\right) I_l \cos \theta_\phi = \sqrt{3}V_l I_l \cos \theta_\phi$$

$$Q_{3\phi} = 3V_\phi I_\phi \sin \theta_\phi = \sqrt{3}V_l I_l \sin \theta_\phi$$

*Δ Load Case:*

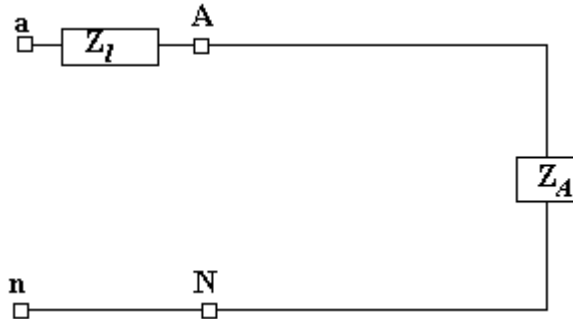
$$P_{3\phi} = 3V_\phi I_\phi \cos \theta_\phi = 3V_l \left(\frac{I_l}{\sqrt{3}}\right) \cos \theta_\phi = \sqrt{3}V_l I_l \cos \theta_\phi$$

$$Q_{3\phi} = 3V_\phi I_\phi \sin \theta_\phi = \sqrt{3}V_l I_l \sin \theta_\phi$$

## 5. Single-Phase Equivalent Circuit

- In a balanced 3-phase system, voltage and current magnitudes are same.
- In a balanced 3-phase system, voltage and current are  $120^\circ$  apart from each other
- Therefore, once a phase value is known, the other two are also known
- **NOTE:** Single-phase equivalent circuit is formed so that a phase impedance is connected *between a phase (line) and the neutral*.

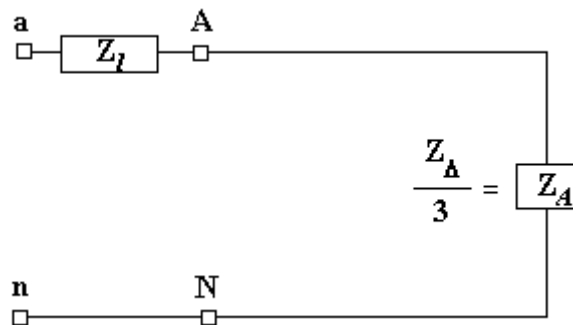
*Y-load case:* From the Y-load figure, let's delete two phases (B, and C), then the remaining circuit looks like below:



Note:  $V_\phi = V_{AN}$ , and

$$I_\phi = I_{aA}$$

*Δ-Load Case:* There is a slight problem here, since there is no neutral point. So we have to convert the load to Y-load equivalent. By the usual Δ-Y Transformation, we could get the Y impedance, in terms of Delta-load, as  $Z_Y = \frac{Z_\Delta}{3}$ . Then the single-phase circuit looks like this:



**NOTE:** The voltage across the impedance in this single-phase circuit is **not** the actual phase voltage across the impedance.  $V_{AB}$  is the actual voltage across a impedance. So we have to convert the voltage, after your calculation of  $V_{AN}$ , to  $V_{AB}$  for a delta-load phase voltage.

$$V_\phi = V_{AB} = \sqrt{3}V_{AN} \angle 30^\circ \text{ and,}$$

$$I_\phi = I_{AB} = \frac{I_{aA}}{\sqrt{3}} \angle 30^\circ$$