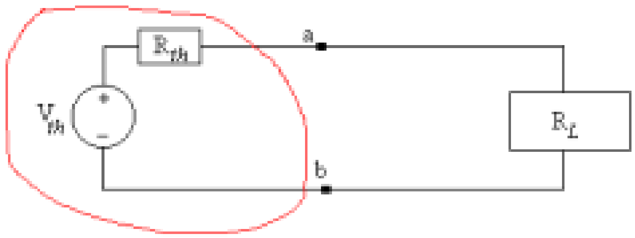
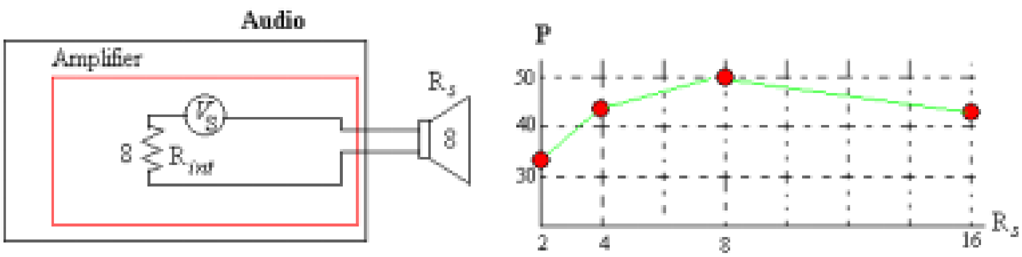
EECE 301 NETWORK ANALYSIS II Dr. Charles Kim

Note 14: Maximum Power Transfer in the context of Sinusoidal Steady-State Network

1. We learned that, maximum power can be delivered to a resistive load from a circuit when the resistor load is same as the Thevenin resistance. In other words, the condition for maximum power transfer from the source to the load is: $R_L = R_{th}$

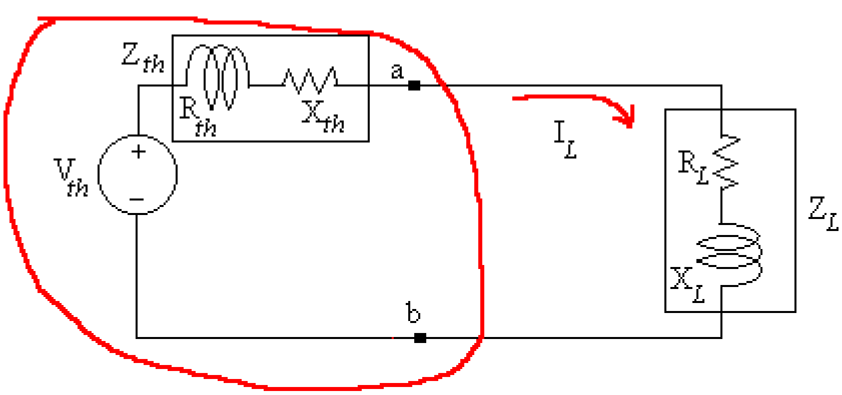


2. Thus, we are pretty much sure that, when we are told to use a 8Ω speaker (if you look at the back of the speaker cone, you can find the resistance value marking) not 16Ω one, the internal amplifier impedance is close to 8Ω. As shown below, with maximum amplifier voltage is 40V, the output power is maximized when the speaker (i.e., load) impedance is same as the internal amplifier impedance.



3. Now, we want to expand the maximum power transfer condition in the context of sinusoidal steady-state network. **<u>Note</u>** that the only power consumed is Real Power, so the term 'power' in the 'maximum power transfer' in the context of sinusoidal circuit, indicates *real power only*, as was in the constant source circuit.

4. Let's assume that a Thevenin circuit (with phasor form) is as shown below and now we want to find the load impedance which meets the maximum power transfer condition.



5. The only difference between two above circuits is that the sinusoidal circuit has complex elements. Let's assume the Thevenin voltage is with the phasor value of $\overline{V}_{th} = V\angle 0$. Then, the current in the circuit is: $\bar{I}_L = \frac{V}{Z_{th} + Z_L} = \frac{V}{(R_{th} + R_L) + j(X_{th} + X_L)}$

The magnitude of the current, then, is: $I_L = \frac{V}{\sqrt{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}}$

6. From real power equation ($P=I^2R$), the real power delivered to (or consumed at) the load is:

$$P_L = I_L^2 R_L = \frac{V^2 \cdot R_L}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}$$

7. Note that Thevenin impedance is given, and the only variables are those at the load, i.e, R_L and X_L in the real power equation above. Therefore the real power equation can be rewritten for clarification purpose as: $P_L(R_L, X_L) = \frac{V^2 \cdot R_L}{(R_{th} + R_L)^2 + (X_{th} + X_L)^2}$.

8. And now we will find the values of the variables which maximize the real power. So, this is a function derivative problem. So, let's do a (partial) derivative of the function, first, with respect to R_L.

$$\frac{\partial P}{\partial R_L} = \frac{V^2\{(R_{th} + R_L)^2 + (X_{th} + X_L)^2\} - 2V^2 R_L (R_{th} + R_L)}{\{(R_{th} + R_L)^2 + (X_{th} + X_L)^2\}^2}$$

9. Therefore, the condition for maximum leads to: $(R_{th} + R_L)^2 + (X_{th} + X_L)^2\} = 2R_L(R_{th} + R_L)$

i.e. $R_L = \sqrt{(X_{th} + X_L)^2 + R^2{}_{th}}$ --------------(1)

10. The (partial) derivative with respect to the second variable is:

$\frac{\partial P}{\partial X_L} = \frac{0 - 2V^2 R_L (X_{th} + X_L)}{\{(R_{th} + R_L)^2 + (X_{th} + X_L)^2\}^2}$, and the condition goes: $X_L = -X_{th}$ --------(2)

11. If we plug the condition (2) to the condition (1), then (1) becomes: $R_L = R_{th}$ --------(1)'

12. If we combine (1)' and (2), the maximum (real) power transfer condition in the context of sinusoidal steady-state network becomes:

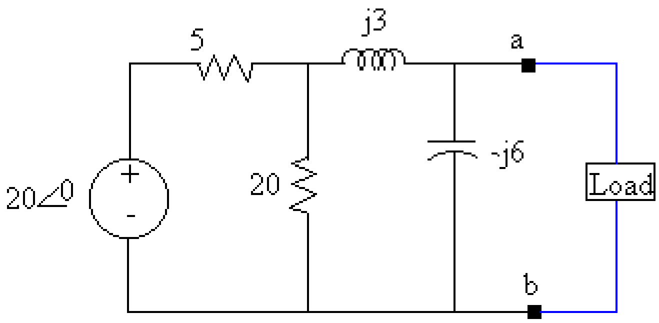
$$\underline{Z_L = Z_{th}^*}$$

13. When the above condition is met, the total impedance of the circuit is purely resistive with the value of $Z = R_{th} + R_L = 2R_L$. In this case, the magnitude of the current through the circuit is:

$$I_L = \frac{V^2}{2R_L}$$

Then the value of maximum real power at the load is: $\max(P_L) = I_L^2 \cdot R_L = \dfrac{V^2}{4 \cdot R_L}$

14. EXAMPLE Problem: A circuit to which you connect a load is given below. Now decides your load impedance so that maximum power is delivered from the circuit to your load.



SOLUTION