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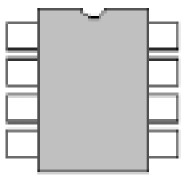
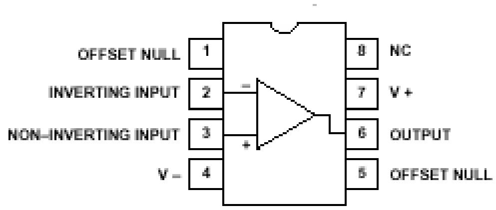
EECE301 Network Analysis II **Dr. Charles Kim**

Note09: Active Filters ---Part 1

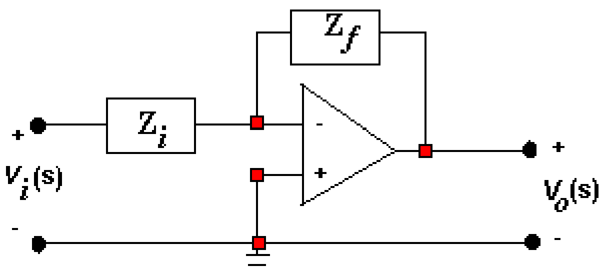
1. Active circuits employing OP Amps, resistors, and capacitors
2. Amplification gain
3. No load effect (Remember in passive filters with load at the filter terminals?)
4. Topics
    (i) Simple LPF
    (ii)Simple HPF
    (iii)Prototype Filter and Scaling Concept
    (iv)Higher Order Filter
        * Cascading Identical Filters
        * Butterworth Filters





0. General First-Order Active Filter

Circuit Configuration:



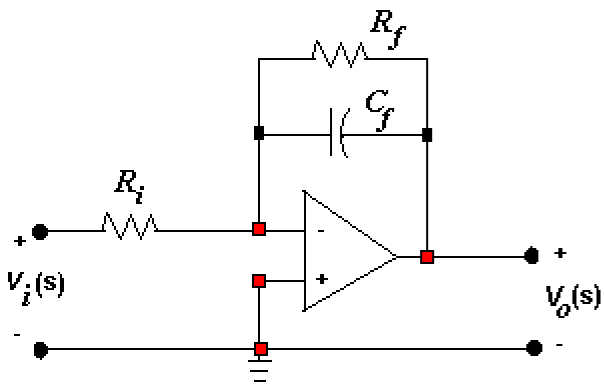
Node voltage equation at the (-) node: Remember that there is no current to the Op-Amp.

$$\frac{0-V_i}{Z_i} = \frac{0-V_o}{Z_f} \quad \text{----->} V_o = -V_i \cdot \frac{Z_f}{Z_i}$$

s-domain Transfer Function: $H(s) = \dfrac{V_o(s)}{V_i(s)} = \dfrac{-V_i(s)\dfrac{Z_f}{Z_i}}{V_i(s)} = -\dfrac{Z_f}{Z_i}$

1. First-Order Low Pass Filter

Circuit Configuration:



s-domain Transfer Function:

$$Z_i = R_i \quad \text{and} \quad Z_f = R_f // C_f \Rightarrow \frac{\frac{R_f}{sC_f}}{R_f + \frac{1}{sC_f}}$$

Therefore,

$$H(s) = -\frac{Z_f}{Z_i} = -\frac{\frac{\frac{R_f}{sC_f}}{R_f + \frac{1}{sC_f}}}{R_i} = \frac{R_f / sC_f}{R_i(R_f + 1/sC_f)} = -\frac{R_f}{R_i(sR_fC_f + 1)} = -\frac{R_f}{R_i} \cdot \frac{1}{(1 + R_fC_fs)}$$

Steady-State Transfer Function: $H(jw) = -\frac{R_f}{R_i} \cdot \frac{1}{1 + jR_fC_fw}$

Pass Band Gain: $A(0) = -\frac{R_f}{R_i}$

Relative dB Amplitude:

$$A_{dB}(w) = 20\log(R_f / R_i) - 10\log[1 + \left(\frac{w}{1/R_fC_f}\right)^2]$$

$$= 20\log(R_f / R_i) - 10\log[1 + \left(\frac{w}{w_c}\right)^2]$$

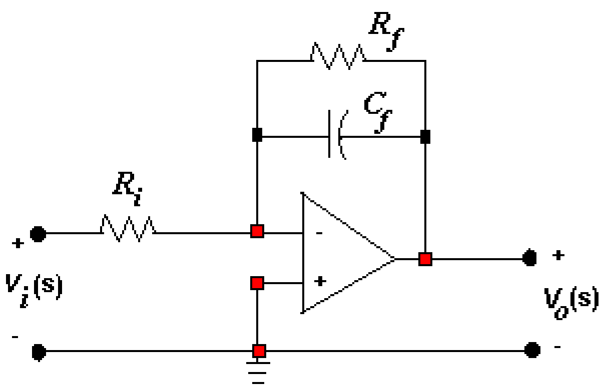
where w_c= cutoff frequency in [rad/sec]

Bode-Plot:

What is the cut-off frequency? Is it $\frac{w_c}{1/R_fC_f} = 1.0$?

What is the cut-off frequency and passband gain with R_i=400, R_f=1600, C_f=0.2uF ?

<u>Example Problem:</u> Using the circuit below, design a low-pass filter with a pass-band gain of 15 dB and a cut-off frequency of 10 kHz. Assume a 5 nF capacitor is available.
(a) Specify the values of the resistors.
(b) Draw a Bode Plot



<u>Solution</u>

Given Information:

(1) Pass band gain (i.e. $G_{dB}(0)$=15dB)
(2)f_c=10000 (i.e., $w_c=2\pi(10000)$ rad/s)
(3) C=5nF

What's the question, then? -->**Find R_1 and R_2.**

SOLUTION:

<u>Bode Plot:</u>

2. Scaling

1. Drawing a transfer function would be much easier with values of 1Ω, 1F, and 1H than with realistic values of elements.

2. From the LPF, the transfer function of $H(jw) = -\frac{R_f}{R_i} \cdot \frac{1}{1 + jR_f C_f w}$, can then become $H(jw) = - \cdot \frac{1}{1+w}$. And the cutoff frequency is $\frac{1}{R_2 C} = \frac{1}{1 \cdot 1} = 1$ [rad/sec]. This filter with all convenient values with w_c=1 [rad/sec] is called a ***prototype filter***.

3. After making computation for a transfer function using the convenient values of R, L, and C, we can transform the convenient values to realistic values using the process known as ***scaling***.

4. Two types of scaling:

(a) magnitude scaling (k_m: scale factor for magnitude)

R, L, C: convenient values

R', L', C': actual (realistic values)

Then, $R' = k_m R$, $L' = k_m L$, and $C' = C/k_m$

(b) frequency scaling (k_f: scale factor for frequency)

Then, $k_f = k_w = \frac{w_c'}{w_c}$, $R' = R$, $L' = L/k_f$, and $C' = C/k_f$

(since $wL = w'L'$ and $\frac{1}{wC} = \frac{1}{w'C'}$)

(c)Combination of two scale factors:

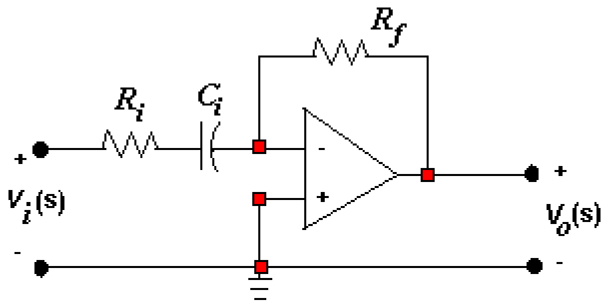
$$R' = k_m R,\quad L' = \frac{k_m}{k_f} L, \text{ and } C' = \frac{1}{k_m k_f} C$$

5. **Scaling Example**: use the prototype active LPF, along with magnitude and frequency scaling, to compute the resistor values with a gain of 5, a cutoff frequency of 1000 Hz, and a feedback capacitor of 0.01 uF.

Solution:

3. First Order High Pass Filter

Circuit Configuration:



$Z_i = R_i + 1/sC_i$ and $Z_f = R_f$

s-domain Transfer Function:

$$H(s) = -\frac{Z_f}{Z_i} = -\frac{R_f}{R_i + \frac{1}{sC_i}} = -\frac{R_f}{R_i} \cdot \frac{1}{(1 + \frac{1}{R_i C_i s})} = -\frac{R_f}{R_i} \cdot \frac{R_i C_i s}{(1 + R_i C_i s)} = -\frac{R_f}{R_i} \cdot \frac{\frac{s}{1/R_i C_i}}{[1 + \frac{s}{1/R_i C_i}]}$$

Steady-State: $H(jw) = -\frac{R_f}{R_i} \cdot \frac{j\frac{w}{1/R_i C_i}}{1 + j\frac{w}{1/R_i C_i}} = -\frac{R_f}{R_i} \cdot \frac{j\frac{w}{w_c}}{1 + j\frac{w}{w_c}}$, with $w_c = \frac{1}{R_i C_i}$.

Pass band Gain: $A(\infty) = -\frac{R_f}{R_i}$

Relative dB Amplitude: $A_{dB}(w) = 20\log\left(\frac{R_f}{R_i}\right) + 20\log[\left(\frac{w}{1/R_i C_i}\right)] - 10\log[1 + \left(\frac{w}{1/R_i C_i}\right)^2]$

What is the cut-off frequency? Is it $\frac{w_c}{1/R_i C_i} = 1.0$?

What is the cut-off frequency and DC gain with R_i=800, R_f=4000, and C_i=0.1uF?