EECE 301 NETWORK ANALYSIS II **Dr. Charles J. Kim**

Class Notes 07: Frequency Response and Bode Plot

A. Steady-State Frequency Response

As we discussed before, the variable *s* is a complex number, with damping (transient) component and the steady-sate component. If we are interested only in the state-steady condition of a sinusoidal system, we can change the variable *s* to be a complex number without real part. Therefore, the transfer function of steady-state can be equated as: $H(s) = H(jw)$. By doing this, we suddenly found us in the frequency-domain, and the steady-state frequency response world.

B. Frequency Response Plot

The most popular plot for amplitude and phase curves is "**Bode plot**" which is a "break-point approximation" plot and is simple and convenient to see curve for frequency-dependent steady-state quantity. Bode plot is with semi-log scale: regular scale on *y* (amplitude) axis and log scale on *x* (frequency) axis. Let's practice a few bode plots of steady-state transfer functions. In this note, our primary interest is the amplitude of the response, not the phase angle. However, the phase angle can easily be achieved.

1. Transfer Function: $H(s) = s$

The steady-state transfer function is: $H(jw) = jw$

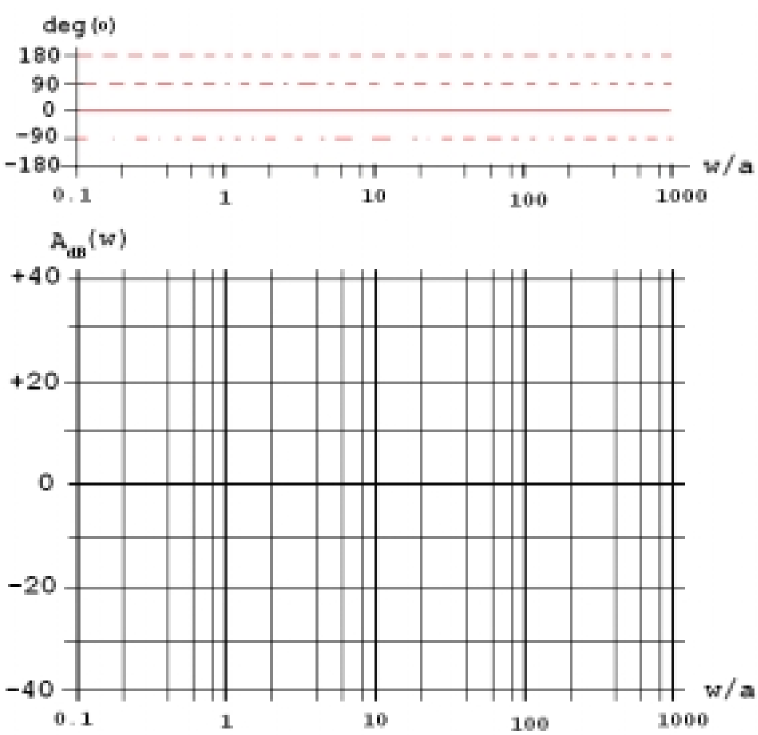
Then, the magnitude of the steady-state transfer function is: $A(w) = w$.

And the phase angle of the transfer function is: $\theta(w) = \angle(jw) = 90^\circ$.

The Bode Plot is to express it with dB level.

So, the amplitude in dB is: $A_{dB}(w) = 20\log(A(w)) = 20\log w$ (referenced to 1.0)

Now, can you draw a bode plot?



Do you see that the slope of the relative dB amplitude is 20 dB/decade or 6dB/Octave?

Octave: The interval between the first eight notes in a major scale [in music]. It is characterized by the frequency ratio of 2:1.

Octave 1	Octave 2
C ------>B	C----> B
32Hz--->65Hz	65Hz---->123Hz
f_0 $2f_0$	$2f_0$ $4f_0$

2. Transfer Function: $H(s) = \frac{1}{s}$

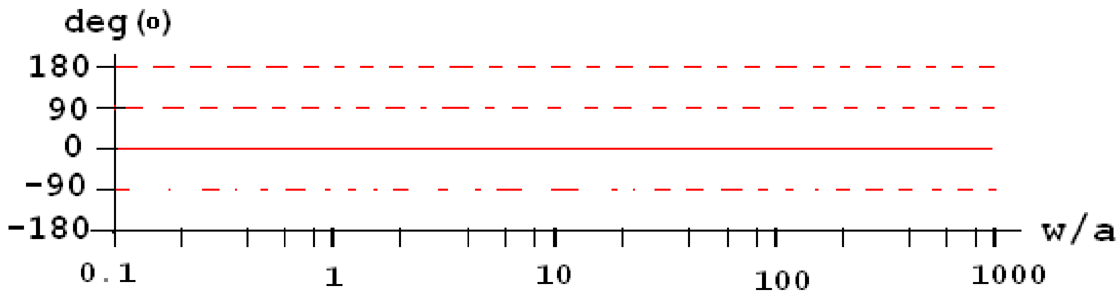
The steady-state transfer function is: $H(jw) = \frac{1}{jw}$

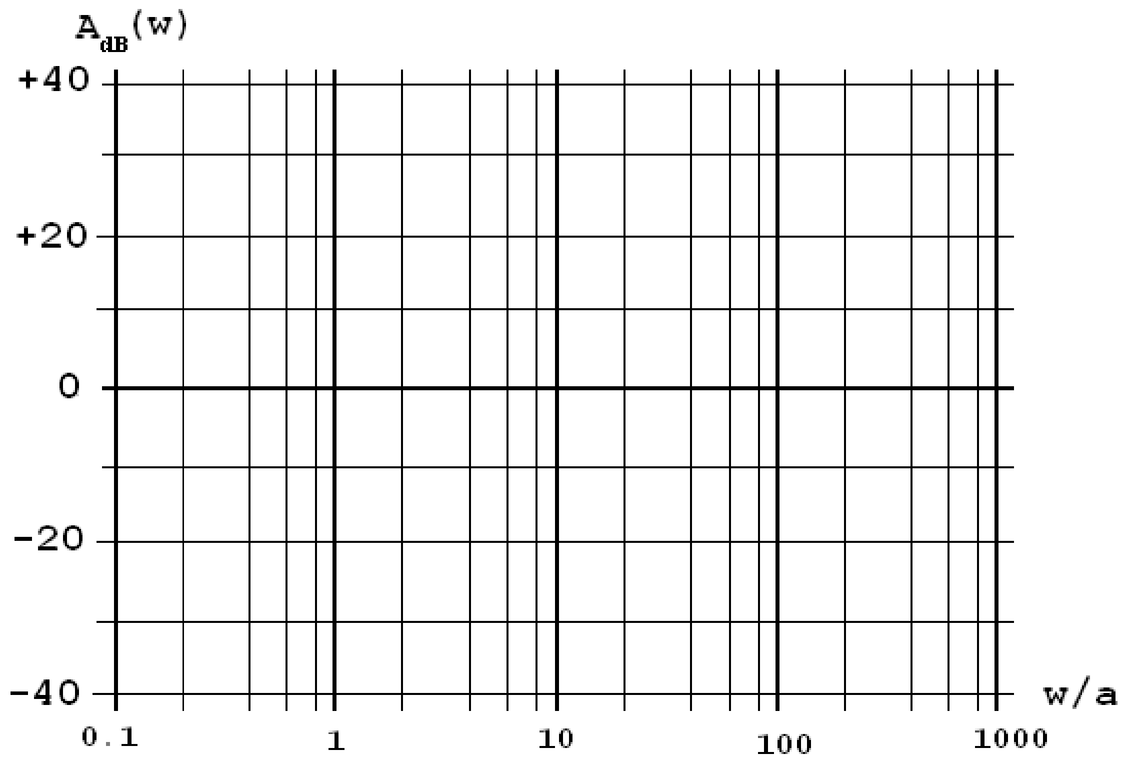
Phase Angle: $\theta(w) = \frac{\angle 0}{\angle 90} = -90^{\circ}$

Amplitude: $A(w) = \frac{1}{w}$

Then, $A_{dB}(w) = 20\log(A(w)) = -20\log w$ (referenced to 1.0)

Now, let's draw another bode plot.





Can you see the slope decreasing by 20dB/decade or 6dB/octave ?

3. Transfer Function: $H(s) = 1 + \frac{s}{a}$

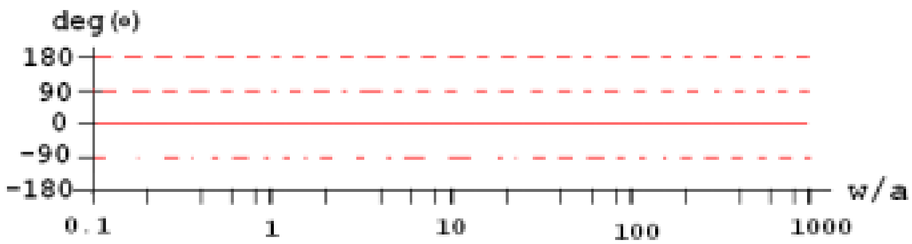
The steady-state transfer function is: $H(jw) = 1 + \frac{jw}{a}$

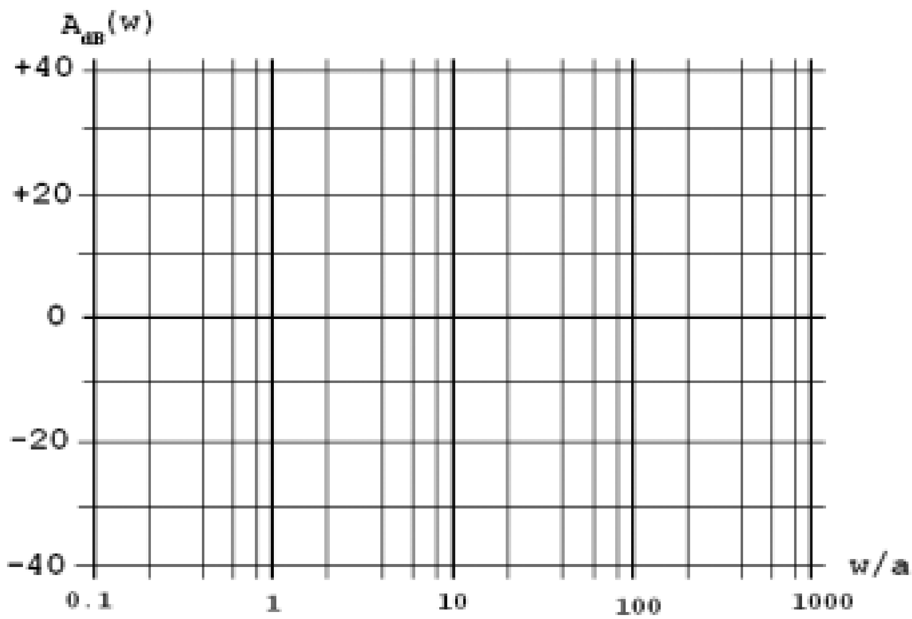
Amplitude: $A(w) = |H(jw)| = \sqrt{1^2 + \left(\frac{w}{a}\right)^2}$

Phase Angle: $\theta(w) = \frac{\angle(a + jw)}{\angle a} = \angle \tan^{1}(\frac{w}{a})$

Then, $A_{dB}(w) = 20\log\sqrt{1 + \left(\frac{w}{a}\right)^2} = 10\log\left(1 + \left(\frac{w}{a}\right)^2\right)$ (referenced to 1.0)

Now, can you draw a bode plot?





Note that the breakdown approximation (at $\frac{w}{a} = 1$) has about 3dB difference from the actual (precise) curve. So we can say that, the breakdown approximation is accurate for $w << a$ and $w >> a$.

Do you see that the slope of the relative dB amplitude is 20 dB/decade or 6dB/Octave?

Q?: Is this a **high pass filter**?

4. <u>Transfer Function:</u> $H(s) = \dfrac{1}{1 + s/a}$

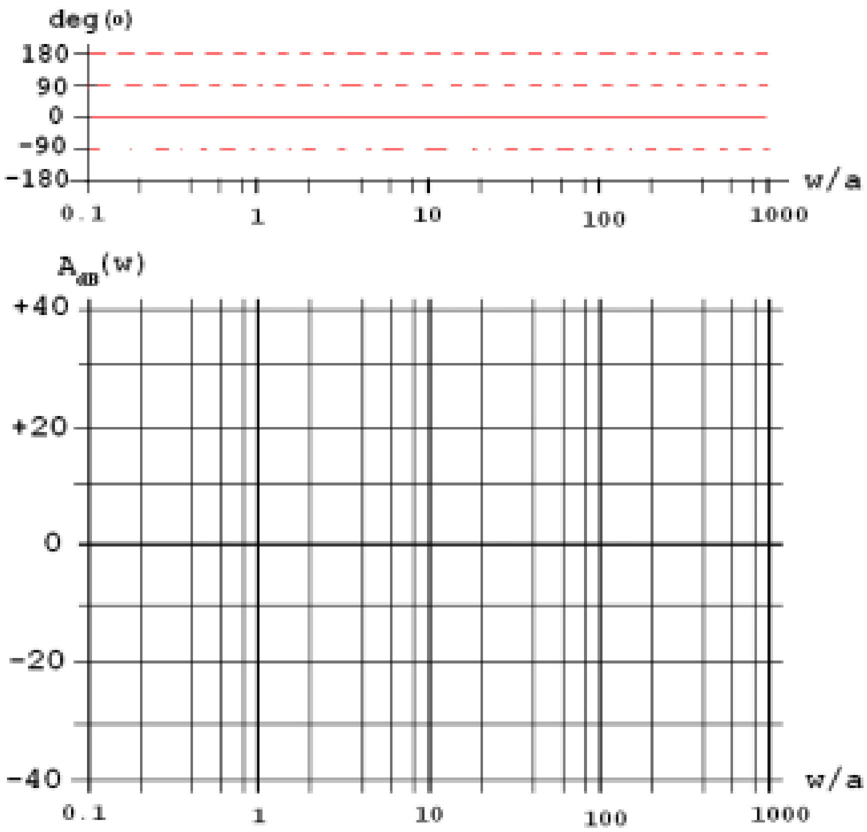
The steady-state transfer function is: $H(jw) = \dfrac{1}{1 + jw/a}$

The amplitude: $A(w) = \dfrac{1}{\sqrt{1 + \left(\dfrac{w}{a}\right)^2}}$.

Phase angle: $\theta(w) = \dfrac{\angle 0}{\angle \tan^{-1}\dfrac{w}{a}} = \angle - \tan^{-1}\dfrac{w}{a}$

Then, $A_{dB}(w) = -10\log[1 + \left(\dfrac{w}{a}\right)^2]$ (referenced to 1.0)

Now, let's draw a bode plot.



What's the slope?

Do we have the same 3dB error at the breakpoint?

Q: Is this a **low pass filter**?

5. Transfer Function: $H(s) = \dfrac{80s}{(s+2)(s+20)}$

First we convert the H(s) so that it has the term of s/a.

$$H(s) = \frac{80s}{(s+2)(s+20)} = \frac{2s}{(1+s/2)(1+s/20)}$$

Then, the steady-state transfer function is: $H(jw) = \dfrac{j2w}{(1+j\dfrac{w}{2})(1+j\dfrac{w}{20})}$

The Amplitude: $A(w) = \dfrac{2w}{\sqrt{1+\left(\dfrac{w}{2}\right)^2}\cdot\sqrt{1+\left(\dfrac{w}{20}\right)^2}}$

Phase Angle: $\dfrac{\angle 90}{\angle\tan^{-1}\left(\dfrac{w}{2}\right)+\angle\tan^{-1}\left(\dfrac{w}{20}\right)} = \angle[90-\tan^{-1}\left(\dfrac{w}{2}\right)-\tan^{-1}\left(\dfrac{w}{20}\right)]$

Relative dB amplitude:

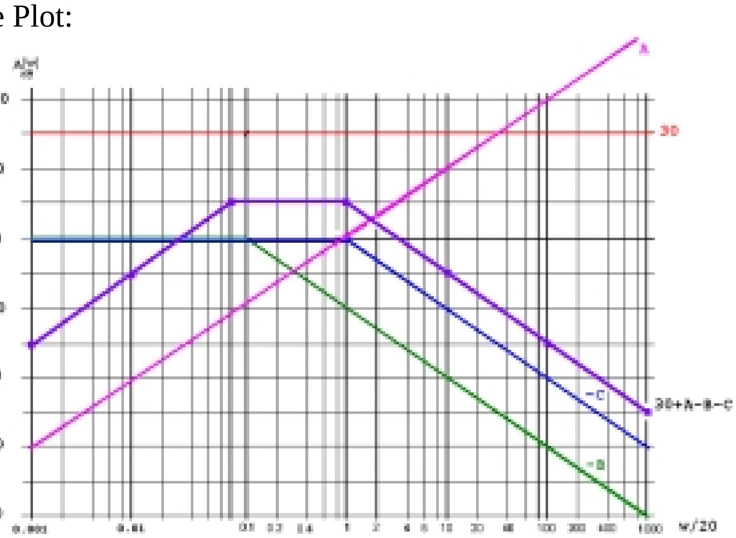
$$A_{dB}(w) = 20\log\frac{2w}{\sqrt{1+\left(\frac{w}{2}\right)^2}\cdot\sqrt{1+\left(\frac{w}{20}\right)^2}} = 20\log 2w - 10\log\left[1+\left(\frac{w}{2}\right)^2\right] - 10\log\left[1+\left(\frac{w}{20}\right)^2\right]$$

$$= 20\log\left(40\cdot\frac{w}{20}\right) - 10\log\left[1+100\cdot\left(\frac{w}{20}\right)^2\right] - 10\log\left[1+\left(\frac{w}{20}\right)^2\right]$$

$$= 20\log 40 + 20\log\left[\frac{w}{20}\right] - 10\log\left[1+100\cdot\left(\frac{w}{20}\right)^2\right] - 10\log\left[1+\left(\frac{w}{20}\right)^2\right]$$

$$\approx 30 + A - B - C$$

Bode Plot:



What is this? A band-pass filter?