EECE301 NETWORK ANALYSIS II **Dr. Charles Kim**

Class Note 06: Application of Laplace Transformation to Integrodifferential Equations

The Laplace transform is useful in solving linear integrodifferential equations by following:
1. Using the differential and integration properties, each term in the integrodifferetial equation is transformed. Initial conditions must be taken into account, though.
2. We solve the resulting algebraic equation in the *s* domain.
3. We then convert the solution back to the time domain by using the inverse Laplace transformation

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Example 1:

Solve the differential equation using the Laplace transformation.

$$\frac{d^2v(t)}{dt^2} + 6\frac{dv(t)}{dt} + 8v(t) = 2u(t), \text{ subject to } v(0) = 1, v'(0) = -2.$$

Solution:

By taking Laplace transform of each term,

$$[s^2V(s) - sv(0) - v'(0)] + 6[sV(s) - v(0)] + 8V(s) = \frac{2}{s}$$

substituting $v(0) = 1, v'(0) = -2$,

$$s^2V(s) - s + 2 + 6sV(s) - 6 + 8V(s) = \frac{2}{s}$$

or, $$(s^2 + 6s + 8)V(s) = s + 4 + \frac{2}{s}$$

Hence, $$V(s) = \frac{s^2 + 4s + 2}{s(s^2 + 6s + 8)} = \frac{s^2 + 4s + 2}{s(s+2)(s+4)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4}$$

By residue or algebraic method, we have: A=0.25, B=0.5, and C=0.25

Therefore, $v(t) = 0.25[u(t) + 2e^{-2t} + e^{-4t}]$, t>0

Example 2:

Solve for y(t) in the following integrodifferntial equation.

$$\frac{dy(t)}{dt} + 5y(t) + 6\int_0^t y(x)dx = u(t), \quad y(0)=2.$$

Solution:
