EECE301 NETWORK ANALYSIS II **Dr. Charles Kim**

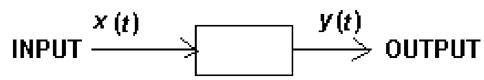
Class Note 05: System Level *s*-domain Analysis and Transfer Function

A. System Consideration

So far we analyzed circuits using s-domain analysis and now it's time to broaden our analysis to the systems level. In the system level analysis, mathematical Input-Output Relationship is more important than the circuit details. This system analysis will allow you to deal with many of the basic concepts of control and communication systems.

B. Transfer Function

In our circuit analysis, we found load voltage (or current) of a circuit excited by sources. In the system level analysis, the details of the circuit could be replaced by just a box (or "black box"). Then, the excitation is the INPUT to the box, and the response, the OUTPUT from the box. A means to describe the "black box" using x(t) and y(t), without knowing the details inside the box is a function called, "Transfer Function," which literally transfers x(t) to y(t). In the diagram below, the transfer function could be a function, *g*(t). Then the output y(t) is derived by the convolution (not the scope of this course) of the input and the transfer function: $y(t) = x(t) * g(t)$.

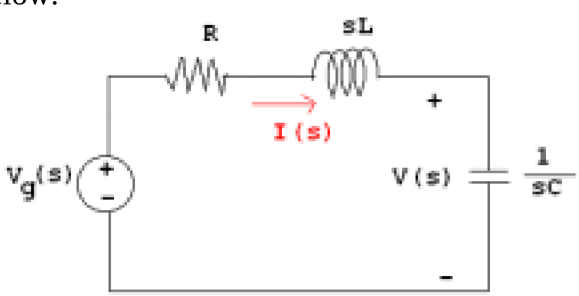


In **s**-domain, with $X(s) = L\{x(t)\}$ and $Y(s) = L\{y(t)\}$, the diagram above changes to:

Then, in s-domain, the output Y(s) can be derived by the simple multiplication of the input and the transfer function: $Y(s) = X(s)H(s)$. This is one beauty of the *s*-domain analysis. However, s-domain and time-domain equations are actually equivalent.

Note: In transfer function we assume that all initial conditions are zero.
Note: When multiple input sources are involved, we apply superposition for the response.

<u>Example:</u>
Find the transfer function for the output V and input Vg for the following s-domain circuit shown below.



By definition: $H(s) = \dfrac{V(s)}{V_g}$

Applying voltage-divider:

$$H(s) = \frac{V_g(s)\cdot\dfrac{1/sC}{R + sL + 1/sC}}{V_g(s)} = \frac{1}{s^2LC + sRC + 1}$$

Example:
For the same circuit above, find the transfer function for the output I(s) and input Vg.

(Sol)

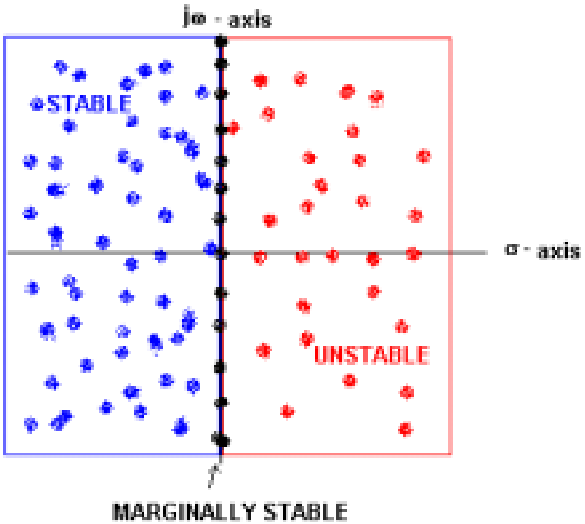
C. Poles and Zeros

The transfer function H(s) is usually expressed by the polynomials of numerator and denominator: $H(s) = \frac{N(s)}{D(s)}$. Then **Poles** are defined as the roots of D(s) and the **zeros** are the roots of N(s). In other words, **poles** are the values of *s* that will cause the transfer function to be infinity (∞), while **zeros** cause it to be zero (0).

Pole Location
The location of Poles (marked by "x") in complex s-plane indicate the *system behavior* (or "stability") while that of zeros (marked by "o") does not ordinarily affect. So let's discuss about the system stability with pole locations in the s-plane.

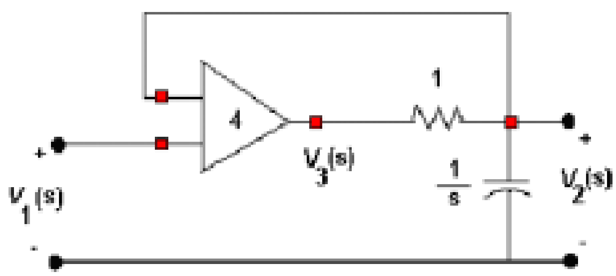
Pole Location	**F(s)**	**f(t)**	**System Stability**
On negative real axis	$\frac{A}{s+a}$	Ae^{-at}	Stable
On negative real plane	$\frac{A}{[s+a+jw][s+a-jw]} = \frac{A}{(s+a)^2+w^2}$	$Ae^{-at}\sin wt$	Stable
On positive real axis	$\frac{A}{s-a}$	Ae^{at}	Unstable
On positive real plane	$\frac{A}{(s-a)^2+w^2}$	$Ae^{at}\sin wt$	Unstable
At the origin	$\frac{A}{s}$	$Au(t)$	Marginally Stable
On jw-axis	$\frac{A}{s^2+w^2}$	$B\sin wt$	Marginally Stable

Pole Location/System Stability Diagram



Stability Check Example

Show the following active amplifier circuit (in *s*-domain) is unstable by the poles locations. The OP Amp behaves as $V_3(s) = 4[V_1(s) + V_2(s)]$.



SOLUTION

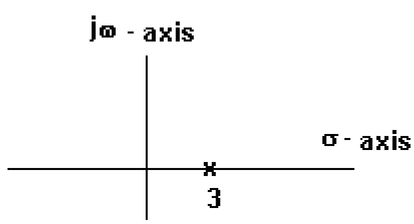
Since current does not flow to the Op Amp, the node voltage equation at the output terminal is:

$$\frac{V_2(s) - V_3(s)}{1} + \frac{V_2(s)}{1/s} = 0$$

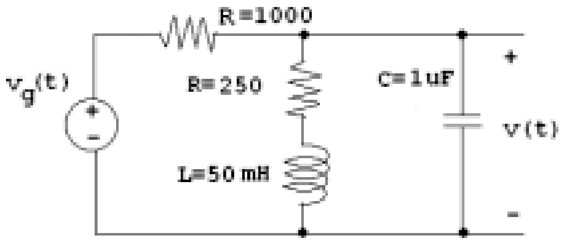
Substituting the relationship of $V_3(s) = 4[V_1(s) + V_2(s)]$ yields, $V_2(s)(s-3) = 4V_1(s)$

Therefore, the transfer function is: $H(s) = \frac{V_2(s)}{V_1(s)} = \frac{4}{s-3}$

The pole location is on the positive real axis and the time domain function $g(t) = 4e^{3t}$ is exponentially increasing. System is unstable.

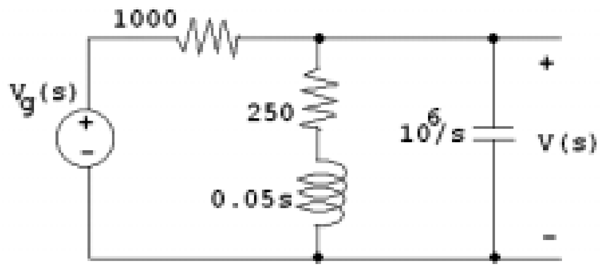


Example:



For the circuit shown left, find the transfer function for the output voltage v(t), and locate the poles and zeros of the transfer function.

(Solution) The s-domain circuit then can be obtained as shown right:



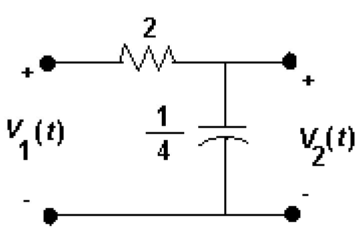
Draw Your Zero/Pole Location Here:

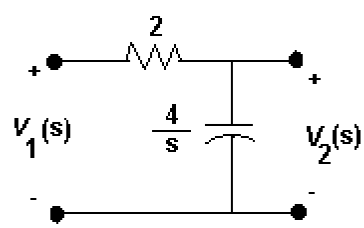
D. Sinusoidal Response (Full Response: Transient + Steady-State)

Transfer Function Example

(a) Determine Transfer function of the circuit for $V_2(t)$ (below left).

(b) Using the transfer function, determine the response due to an input source $v_1(t) = 5\sin 2t$.





SOLUTION

(a) Convert the circuit to s-domain (above right)

Transfer function: $H(s) = \dfrac{V_2(s)}{V_1(s)} = \dfrac{V_1(s)\left\{\dfrac{4/s}{2+4/s}\right\}}{V_1(s)} = \dfrac{4/s}{2+4/s} = \dfrac{2}{s+2}$

The time domain transfer function is the inverse transform of H(s): $h(t) = 2.e^{-2t}$, t>0.

(b) The input of $v_1(t) = 5\sin 2t$ in s-domain is: $V_1(s) = \dfrac{10}{s^2+4}$

Therefore, output is: $V_2(s) = V_1(s)H(s) = \frac{2}{s+2} \cdot \frac{10}{s^2+4}$

Let's inverse transform of the output:

$$V_2(s) = \frac{2}{s+2} \cdot \frac{10}{s^2+4} = \frac{A}{s+2} + \frac{Bs+C}{s^2+4}$$

By residue method: A=5/2

By algebraic method: C=5, and B= - 5/2

Therefore,

$$V_2(s) = \frac{2}{s+2} \cdot \frac{10}{s^2+4} = \frac{5/2}{s+2} + \frac{-5/2s+5}{s^2+4} = \frac{5}{2}\left[\frac{1}{s+2} - \frac{s}{s^2+4} + \frac{2}{s^2+4}\right]$$

Finally, the inverse transform gives:

$$v_2(t) = \frac{5}{2}\left[e^{-2t} - \cos t2t + \sin 2t\right] = \frac{5}{2}e^{-2t} + \frac{5}{\sqrt{2}}\cos(2t+45)\text{, t>0}$$

(c) Observation of the output

The output has two terms.

The first term is an exponential decaying **transient** one,

and the second one is **steady-state**. The transient term is due to the circuit (or "**natural behavior of the circuit**") and the steady-state term is **due to the input** source.

(c) Steady-State Sinusoidal Response

If you compare the steady-state output $v_2(t) = \frac{5}{\sqrt{2}}\cos(2t+45)$ and the input $v_1(t) = 5\sin 2t$, we see that the original frequency is not changed; however, the magnitude and phase angle are changed.

E. Steady-State Sinusoidal Response

This section further expands the discussion on steady-state sinusoidal response.

(a) Think about a sinusoidal input x(t): $x(t) = A\cos(wt+\phi)$

(b) For calculation purpose, we express the input in terms of exponential function:

$$x(t) = \Re\{A\cos(wt+\phi) + jA\sin(wt+\phi)\} = \Re\{Ae^{j(wt+\phi)}\} = \Re\{Ae^{j\phi}e^{jwt}\}$$

(c) Then the s-domain expression of x(t) is: $X(s) = \frac{Ae^{j\phi}}{s-jw}$

(d) Then, the output Y(s) can be obtained by: $Y(s) = X(s)H(s) = \frac{Ae^{j\phi} \cdot H(s)}{s-jw}$

(e)The H(s) contains both transient (with real zeros) and steady-state (with imaginary zeros) components. Then, Y(s) can be expressed as:

$Y(s) = \frac{Ae^{j\phi} \cdot H(jw)}{s-jw} + \{partial\_expansion\_comes\_from\_transient\_H(s)\}$ (See (l) below)

(f) $H(jw)$ is then called the "steady-state" transfer function.

(f) If we are interested only in the steady-state response, we keep only the first term. Therefore, the steady-response becomes:

$Y(s) = X(s)H(jw) = \dfrac{Ae^{j\phi}H(jw)}{s - jw}$

(g) If we convert the H(jw) (it is a complex number) into magnitude and angle form, $H(jw) = |H(jw)|\angle\theta(w) = |H(jw)|e^{j\theta(w)}$

(h) Finally, the steady-state response is: $Y(s) = \dfrac{Ae^{j\phi}|H(jw)|e^{j\theta(w)}}{s - jw} = \dfrac{Ae^{j[\phi+\theta(w)]}|H(jw)|}{s - jw}$

(i) Then, the inverse transform will give us the time domain steady-state response:
$y(t) = A|H(jw)|e^{j[\phi+\theta(w)]} \cdot e^{jwt} = A|H(jw)|e^{j[wt+\phi+\theta(w)]}$

(j) If we get only the real component: $y(t) = A|H(jw)|\cos(wt + \phi + \theta(w))$

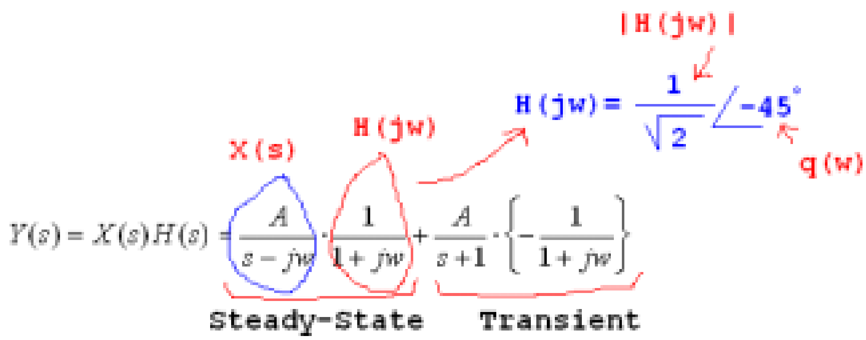
(k) If we compare the input $x(t) = A\cos(wt + \phi)$ and the output $y(t) = A|H(jw)|\cos(wt + \phi + \theta(w))$, as we discussed before, there are changes in magnitude and in angle, determined by the magnitude and the phase angle of the steady-state transfer function.

(l) Illustration: Consider an input $X(s) = \dfrac{A}{s - jw}$, and the transfer function $H(s) = \dfrac{1}{s+1}$.

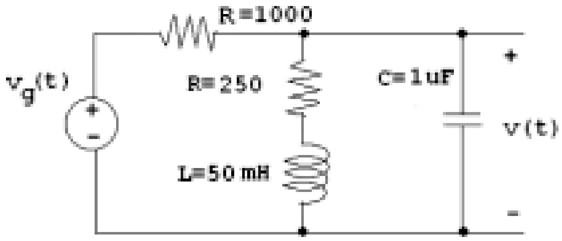
Then the response is $Y(s) = X(s)H(s) = \dfrac{A}{s - jw} \cdot \dfrac{1}{s+1} = \dfrac{K_1}{s - jw} + \dfrac{K_2}{s+1}$

By residue/algebraic method we get: $K_1 = \dfrac{1}{1 + jw}$ and $K_2 = -\dfrac{A}{1 + jw}$

Therefore, $Y(s) = X(s)H(s) = \dfrac{A}{s - jw} \cdot \dfrac{1}{s+1} = \dfrac{A}{s - jw} \cdot \dfrac{1}{1 + jw} + \dfrac{A}{s+1} \cdot \left\{ -\dfrac{1}{1 + jw} \right\}$

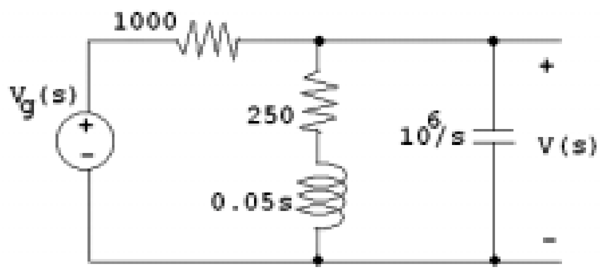


Example #1 for Steady-State Response:



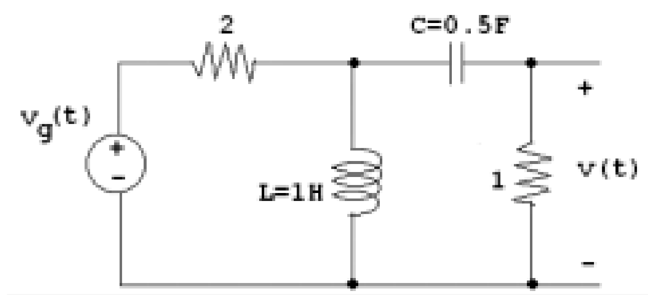
For the circuit shown left, find the steady-state response for v(t) if the input $v_g(t)=120\cos(5000t+30°)$.

(Solution) The s-domain circuit then can be obtained as shown right, and the transfer function:



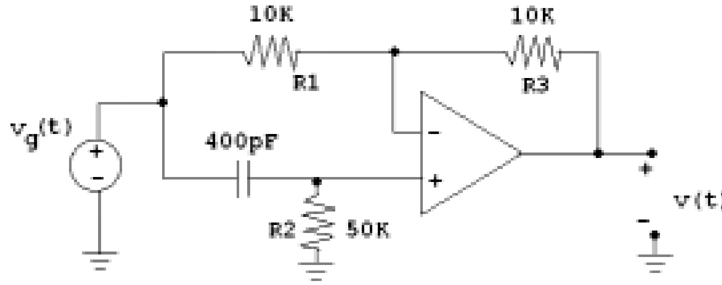
Solution:______________________________

Example #2 for steady-state sinusoidal response:
For the circuit show right,
Find the steady-state voltage v(t) for t>0.
The input $v_g(t)=10\cos 2t$.



Solution:

Example #3 for steady-state sinusoidal response:
For the circuit shown below,
(a)find the steady state voltage v(t) when $v_g(t)$=10cos50000t.
(b) Replace the 50K resistor with a variable resistor and computer the value of resistance necessary to cause v(t) to lead $v_g(t)$ by 120°.



(SOL)

(a)

(b)