Class note 12: Thevenin’s Theorem

A. Introduction

1. Thevenin’s theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source (“Thevenin voltage”), $V_{th}$, in series with a resistor (“Thevenin resistor”), $R_{th}$.

2. The theorem was developed in 1883 by Leon Thevenin, a French telegraph engineer.

3. The circuit to the left of the terminals $a$ and $b$ is known as the Thevenin Equivalent Circuit.

B. How to draw $V_{th}$ and $R_{th}$

1. Thevenin voltage $V_{th}$ is the open-circuit voltage at the terminals.
   Method: Find the voltage at the terminals which are opened.

2. $R_{th}$ is the equivalent resistance at the terminals. (3 methods to choose)

   (a) Input Resistance Method
   i). Use this method when all the sources are independent ones.
   ii). Deactivate all the independent sources (by replacing a voltage source by short circuit, and a current source by open circuit).
   iii). Find the equivalent resistance seen from the terminals --> $R_{th}$

   (b) Short Current Method
   i). Use this method in any circuit situation except when there are only dependent sources.
   ii). Short the terminals. Note that this action may bring a dramatic change in the circuit elements. For example, a resistor in parallel with the terminals has to be changed to an open circuit when the terminals are shorted, since all current will flow through the shorted path ($R=0$).
   iii). Find the short circuit current ($I_{sc}$) through the shorted terminals.
   iv). Note that there should not be source deactivation.
   v). $R_{th} = \frac{V_{th}}{I_{sc}}$

   (b) Test Voltage Method
   i). Use this method in any circuit situation. No restriction.
   ii). Deactivate all independent sources.
   iii). Apply a test voltage ($V_T$) to the terminals of the circuit.
iv). Find the current flowing to the circuit from the test voltage source ($I_T$). Note that the test current should be found in terms of the test voltage. Since the Thevenin resistance is the ratio of the test voltage and the test current.

v). $R_{th} = \frac{V_T}{I_T}$

C. "Input Resistance Method" application Example

Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals $a$ and $b$. Then, find the current through the load resistor $R_L = 6 \, \Omega$.

Solution:
(a) Finding $V_{th}$: Open-circuit voltage. Since two terminals $a$ and $b$ are open, there is no current flowing through $1 \, \Omega$ resistor. If we apply the node-voltage method, the open circuit voltage is the same as the node voltage $V_1$.

\[
\begin{align*}
\text{Therefore, } V_1 &= 30 \, \text{V} \\
\Rightarrow V_{th} &= 30 \, \text{V}
\end{align*}
\]

(b) Finding $R_{th}$: After deactivating independent sources, we have,

\[
R_{th} = R_{ab} = 1 + \frac{4}{12} = 1 + \frac{1}{3} = 4 \, \Omega
\]

(c) Finding the load current: The final equivalent circuit with the load is reduced to:

\[
I_L = \frac{30}{4 + 6} = 3 \, \text{A}
\]
D. "Short Current Method" application Example

Find the Thevenin equivalent circuit of the following circuit.

Solution:
(a) Derivation of $V_{th}$.

Constraints: $V_2 = 24 \, \Omega ; \, i_x = \frac{V_1}{8}$

@ node 1: \[ \frac{V_1 - 24}{2} + 4 \cdot \frac{V_1}{8} + 3 \cdot \frac{V_1}{8} = 0 \quad \text{----------->} V_1 = 8 \, [V] = V_{th} \]

(b) First, two terminals $a$ and $b$ are shorted to find the short current $I_{sc}$.

When $a$ and $b$ are shorted out, there is no current through $8 \, \Omega$ resistor, therefore, $i_x=0$. Hence, the dependent source disappears from the circuit. Therefore, the circuit has changed to:

The circuit is very weird, but somehow we may apply node-voltage equation like:
\[ \frac{24}{2} = 4 + I_{sc} \, , \quad \text{so} \quad I_{sc} = 8. \]

Therefore $R_{th} = 8/8 = 1 \, [\Omega]$
So, the Thevenin equivalent circuit is:
E. "Test Voltage Method" Application (from the above example)

**Derivation of $R_{th}$ by Test Voltage Method:** After deactivation of the independent sources, we have the following circuit.

![Circuit Diagram](image)

Constraint: \( i_x = \frac{V_1}{8}, \ V_1 = V_T \).

**Applying KCL at node 1:**
\[
I_T = i_x + \frac{V_T}{2} + 3i_x = \frac{8V_T}{8} = V_T
\]

Therefore, \( R_{th} = \frac{V_T}{I_T} = 1 \)

So we have the same Thevenin equivalent circuit, like this.

![Thevenin Equivalent Circuit](image)
F. Another Thevenin equivalent circuit problem.

SOLUTION

(a) $V_{th}$ derivation: Open-circuit voltage

Constraint: $i_x = \frac{V_2}{40}$, therefore $160i_x = 4V_2$

@ node 1: $\frac{V_1}{60} - 4 + \frac{V_1 - V_2 - 4V_2}{20} = 0 \quad ----- (1)$

@ node 2: $\frac{V_2}{80} + \frac{V_2 - V_1 + 4V_2}{20} = 0 \quad ----- (2)$

From (1) and (2): $V_2 = V_{th} = 30 \text{ [V]}$

(b) Derivation of $R_{th}$ by Short Current Method: If you short the terminal, then the circuit becomes like below: (Remember $i_x = 0$)

By current-division, we have: $I_{sc} = 4 \cdot \frac{60}{60 + 20} = 3 \quad \text{Therefore, } R_{th} = 30/3 = 10 \Omega$

(c) Derivation of $R_{th}$ by Test Voltage Method: After deactivation of the independent source and applying a test voltage, we have the following circuit.

Constraint: $i_x = \frac{V_T}{40}$, therefore $160i_x = 4V_T$

**Applying KCL** @ node 2: $I_T = \frac{V_T}{80} + \frac{V_T}{40} + \frac{V_T + 4V_T}{80}$, from this $\frac{V_T}{I_T} = 10 = R_{th}$

(d) Final Thevenin equivalent circuit?