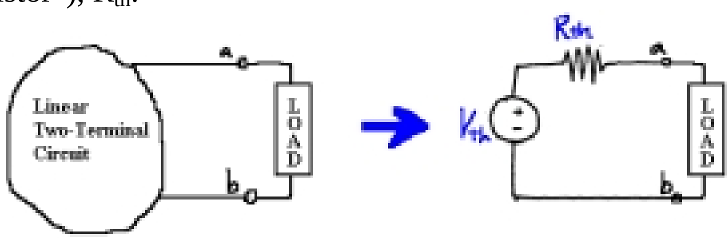
EECE202 Network Analysis I **Dr. Charles J. Kim**

Class note 12: Thevenin's Theorem

A. Introduction

1. Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source ("Thevenin voltage"), V_{th}, in series with a resistor ("Thevenin resistor"), R_{th}.



2. The theorem was developed in 1883 by Leon Thevenin, a French telegraph engineer.
3. The circuit to the left of the terminals *a* and *b* is known as the *Thevenin Equivalent Circuit*.

B. How to draw V_{th} and R_{th}

1. Thevenin voltage V_{th} is the open-circuit voltage at the terminals.
   Method: Find the voltage at the terminals which are opened.
2. R_{th} is the equivalent resistance at the terminals. (3 methods to choose)

(a) ***Input Resistance Method***
i). Use this method when all the sources are independent ones.
ii). Deactivate all the independent sources (by replacing a voltage source by short circuit, and a current source by open circuit).
iii). Find the equivalent resistance seen from the terminals --> R_{th}

(b) ***Short Current Method***
i). Use this method in any circuit situation except when there are only dependent sources.
ii). Short the terminals. Note that this action may bring a dramatic change in the circuit elements. For example, a resistor in parallel with the terminals has to be changed to an open circuit when the terminals are shorted, since all current will flow through the shorted path (R=0).
iii). Find the short circuit current (I_{sc}) through the shorted terminals.
iv). Note that there should not be source deactivation.
v). $R_{th} = \frac{V_{th}}{I_{sc}}$

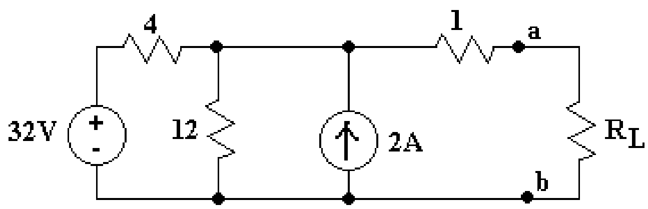
(b) ***Test Voltage Method***
i). Use this method in any circuit situation. No restriction.
ii). Deactivate all independent sources.
iii). Apply a test voltage (V_T) to the terminals of the circuit.

iv). Find the current flowing to the circuit from the test voltage source (I_T). Note that the test current should be found in terms of the test voltage. Since the Thevenin resistance is the ratio of the test voltage and the test current.

v). $R_{th} = \dfrac{V_T}{I_T}$

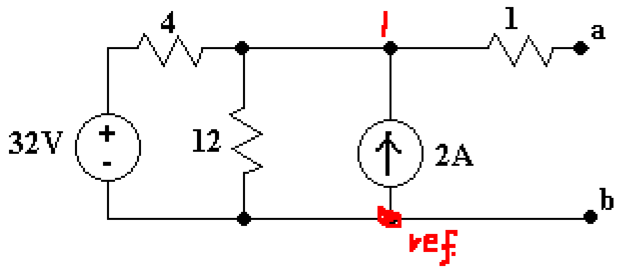
C. "Input Resistance Method" application Example

Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals ***a*** and ***b***. Then, find the current through the load resistor R_L = 6 Ω.



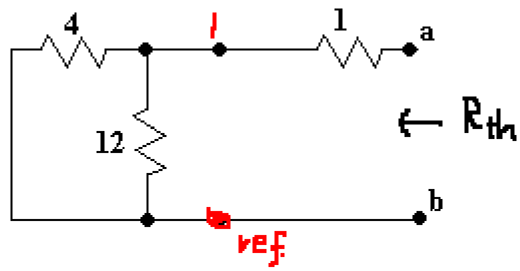
Solution:

(a) Finding V_{th} : Open-circuit voltage. Since two terminals *a* and *b* are open, there is no current flowing through 1 Ω resistor. If we apply the node-voltage method, the open circuit voltage is the same as the node voltage V_1.



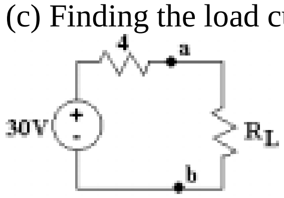
Therefore, @node 1: $\dfrac{V_1 - 32}{4} + \dfrac{V_1}{12} - 2 = 0$ ---> V_1=30 V ----->V_{th}=30 V

(b) Finding R_{th}: After deactivating independent sources, we have,



Therefore, $R_{th}=R_{ab}$=1 + (4//12) = 1+3=4 Ω

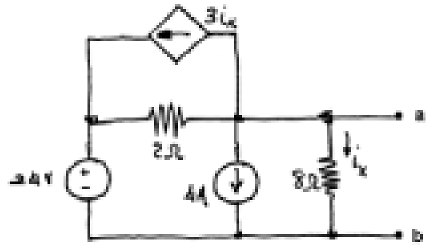
(c) Finding the load current: The final equivalent circuit with the load is reduced to:



Therefore, $I_L = \dfrac{30}{4+6} = 3\,[\text{A}]$

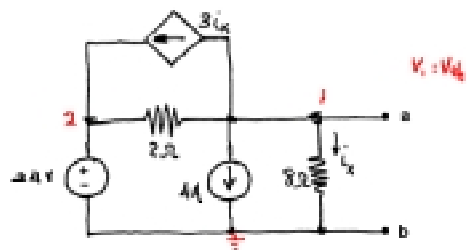
D. "Short Current Method" application Example

Find the Thevenin equivalent circuit of the following circuit.



Solution:

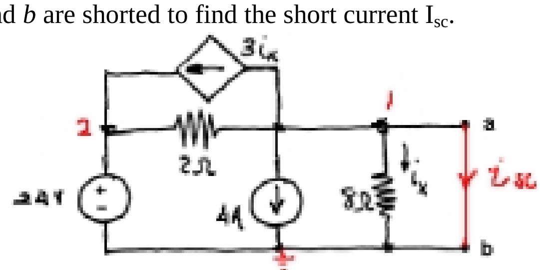
(a) **Derivation of V_{th}.**



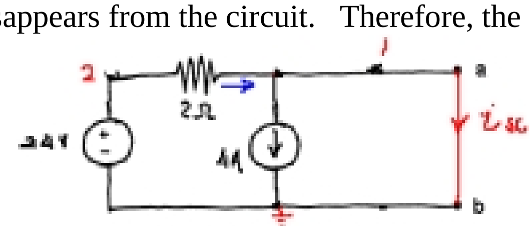
Constraints: $V_2 = 24\,;\ i_x = \dfrac{V_1}{8}$

@ node 1: $\dfrac{V_1 - 24}{2} + 4 + \dfrac{V_1}{8} + 3 \cdot \dfrac{V_1}{8} = 0$ ------------>V_1=8 [V]=V_{th}

(b) First, two terminals *a* and *b* are shorted to find the short current I_{sc}.



When and b are shorted out, there is no current through 8 resistor, therefore, i_x=0. Hence, the dependent source disappears from the circuit. Therefore, the circuit has changed to:

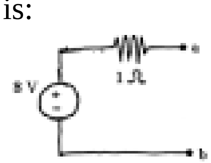


The circuit is very weird, but somehow we may apply node-voltage equation like:

$\dfrac{24}{2} = 4 + I_{sc}$, so I_{sc}=8.

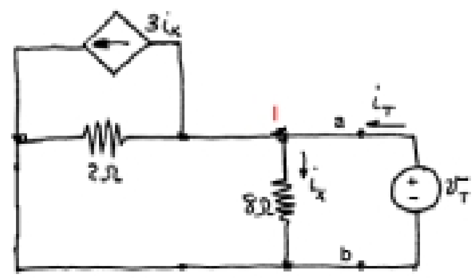
Therefore R_{th}=8/8=1 [Ω]

So, the Thevenin equivalent circuit is:



E. "Test Voltage Method" Application (from the above example)

Derivation of R_{th} by Test Voltage Method: After deactivation of the independent sources, we have the following circuit.

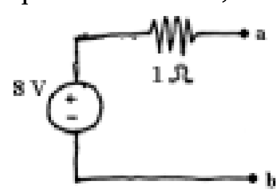


Constraint: $i_x = \frac{V_1}{8}$, $V_1=V_T$.

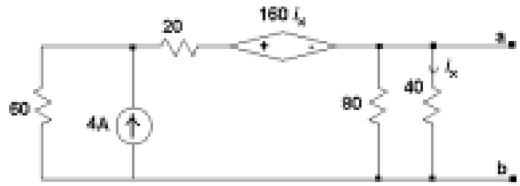
Applying **KCL** at node 1: $I_T = i_x + \frac{V_T}{2} + 3i_x = \frac{8V_T}{8} = V_T$

Therefore, $R_{th} = \frac{V_T}{I_T} = 1$

So we have the same Thevenin equivalent circuit, like this.

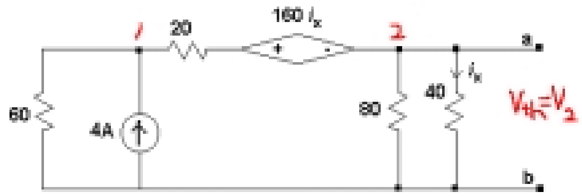


F. Another Thevenin equivalent circuit problem.



SOLUTION

(a) V_{th} derivation: Open-circuit voltage

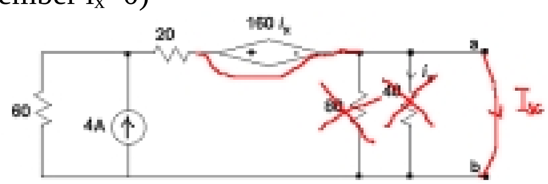


Constraint: $i_x = \frac{V_2}{40}$, therefore $160i_x=4V_2$

@ node 1: $\frac{V_1}{60} - 4 + \frac{V_1 - V_2 - 4V_2}{20} = 0$ ------(1)

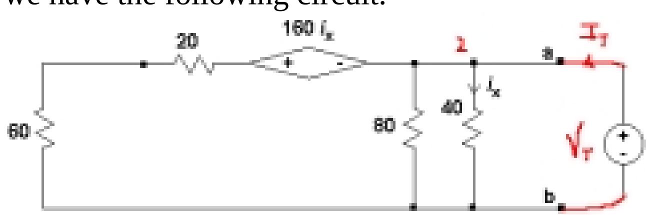
@ node 2: $\frac{V_2}{80} + \frac{V_2}{40} + \frac{V_2 - V_1 + 4V_2}{20} = 0$ -----(2) From (1) and (2): $V_2=V_{th}=30$ [V]

(b) Derivation of R_{th} by Short Current Method: If you short the terminal, then the circuit becomes like below: (Remember $i_x=0$)



By current-division, we have: $I_{sc} = 4 \cdot \frac{60}{60+20} = 3$ Therefore, R_{th} =30/3=10 [Ω]

(c) Derivation of R_{th} by Test Voltage Method: After deactivation of the independent source and applying a test voltage, we have the following circuit.



Constraint: $i_x = \frac{V_T}{40}$, therefore $160i_x=4V_T$

Applying KCL @ node 2: $I_T = \frac{V_T}{80} + \frac{V_T}{40} + \frac{V_T + 4V_T}{80}$, from this $\frac{V_T}{I_T} = 10 = R_{th}$

(d) Final Thevenin equivalent circuit?