

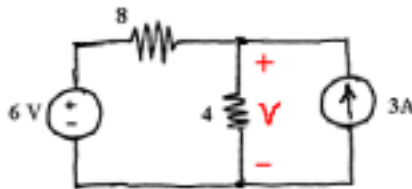
Class note 11: Superposition

A. Superposition

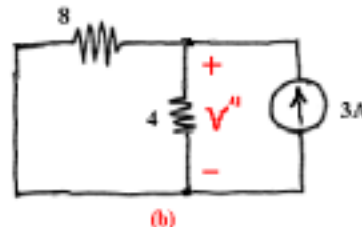
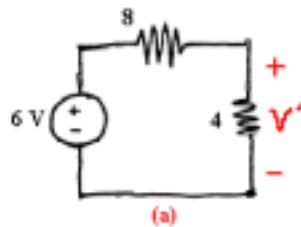
1. If a circuit has two or more independent sources, one way to analyze the circuit is to use nodal or mesh method.
2. Another way of analysis is to determine the contribution of each independent source and then add them up. This approach is called the superposition.
3. The superposition principle: “the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.”
4. Steps of Applying Superposition Principle:
 - (a) Deactivate all independent sources except one source. Find the output (V or I) due to that active source.
 - (b) Repeat step (a) for each of the other independent sources.
 - (c) Find the total contribution by adding algebraically all the contributions due to the independent sources.
5. Caveat: Some weakness and caution
 - (a) Applying the superposition may very likely involve more works.
 - (b) Since the principle is based on linearity, you cannot directly apply it to the power due to each source: power absorbed by a resistor depends on the square of the voltage or current.¹ Therefore, if power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

B. Example 1

Find V in the circuit using the superposition theorem,



Solution:



(a) Contribution by 6V voltage source: V'

$$V' = 6 \cdot \left(\frac{4}{12}\right) = 2 \text{ [V]}$$

(b) Contribution by 3A current source: V''

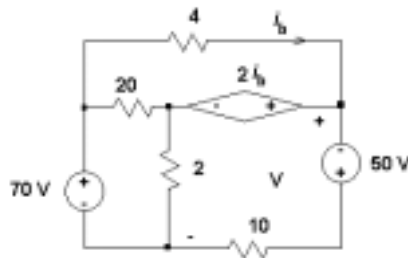
$$V'' = 4 \cdot \left(3 \cdot \left(\frac{8}{12}\right)\right) = 8 \text{ [V]}$$

(c) Therefore, $V = V' + V'' = 10 \text{ [V]}$

¹ For example, when current i_1 flows through resistor R, the power is $P_1 = R \cdot i_1^2$, and when current i_2 flows through R, the power is $P_2 = R \cdot i_2^2$. If current $(i_1 + i_2)$ flows through R, the power is $P_3 = R \cdot (i_1 + i_2)^2$, and $P_3 \neq (P_1 + P_2)$. Power relation is nonlinear.

C. Another Example

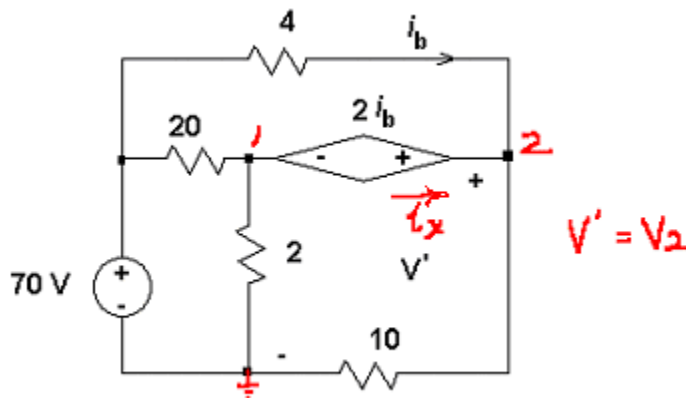
Use superposition to find voltage V in the circuit shown below.



(i) Strategy: The total (combined) voltage V is the sum of the voltage V' (with 70V source only) and the voltage V'' (with 50V source only).

(ii) Analysis Method: 3 essential nodes and the reference with a known node voltage. So we apply the node-voltage method.

(iii) 70V Source only.



With 70V only, $V'=V_2$.

Constraints: $V_2 - V_1 = 2i_b$ and $i_b = (70 - V_2)/4$. -----> $2V_2 - 2V_1 = 70 - V_2$ or $V_1 = \frac{3V_2 - 70}{2}$ ----(1)

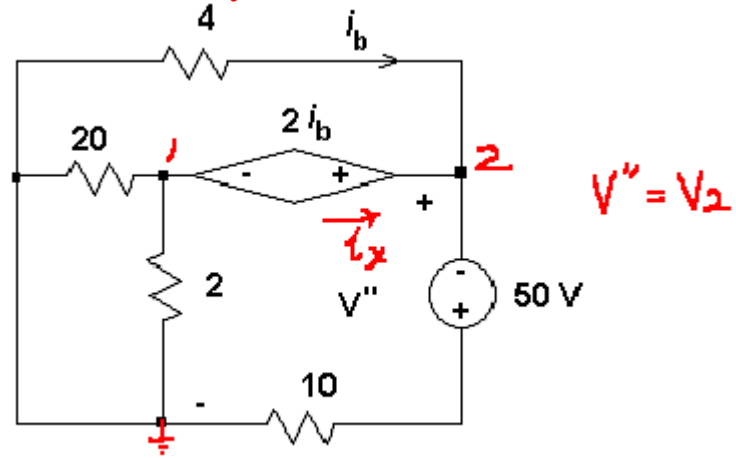
@ node 1: $\frac{V_1 - 70}{20} + \frac{V_1}{2} + i_x = 0$ with $i_x = -i_b + \frac{V_2}{10} = -\frac{70 - V_2}{4} + \frac{V_2}{10}$

Therefore, we have: $\frac{V_2 - 70}{4} + \frac{V_2}{10} + \frac{V_1 - 70}{20} + \frac{V_1}{2} = 0$

The simplified equation is: $11V_1 + 7V_2 = 420$ -----(2)

From (1) and (2): we have $V'=V_2=34.25$ [V]

(iv) 50 V Source only



With 50V only, $V''=V_2$.

(e) Finally, the combined Voltage: $V=V'+V''=34.25 + (\underline{\quad}) = (\underline{\quad})$ [V]