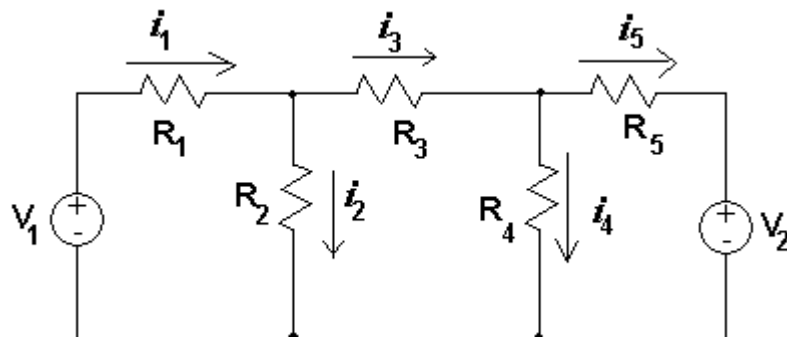
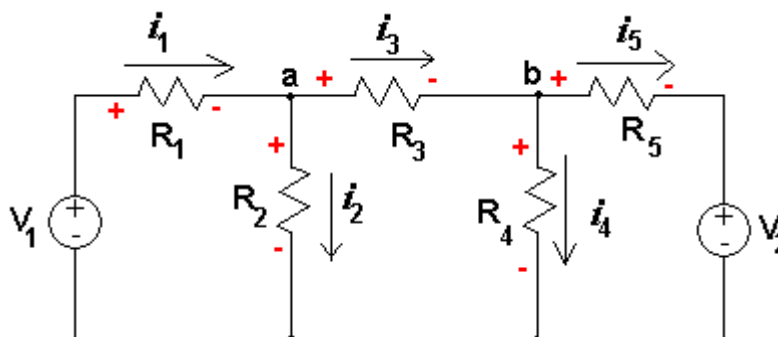


Note 09: Why Do We Need Additional Methods in Circuit Analysis**-1. Node Voltage Method****A. Why the 3 laws are not enough?**

1. We learned about the most fundamental laws in electrical engineering: Ohm's Law, Kirchhoff's Current Law (KCL), and Kirchhoff's Voltage Law (KVL).
2. We also learned about the Passive Convention in current flow direction, voltage polarity, and power calculation.
3. From homework #1 and exam #1, we had a lot practice time to apply the above for important concepts, and we could solve all the problems using only the laws and the convention.
4. There is no doubt that we can solve all circuit problems however complex or big they are.
5. However, when there are a lot of variables involved in analysis, a direct application of the laws and the convention may cause some practical problems: too many equations to be solved.
6. To see the argument of 'inconvenience,' let's consider the following problem.
From the circuit below, we want to find all the currents (i_1 , i_2 , i_3 , i_4 , and i_5) given all the resistors and two known voltage values.



7. Let's apply the Laws and the convention directly to solve the problem.
8. Since the current directions are given, following the convention, we have the voltage polarities as shown below.

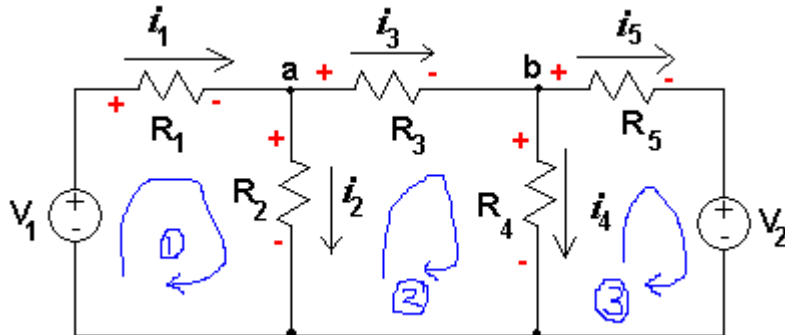


9. Since we have 5 variables, we need 5 independent equations. So starting from KCL:

@ node a: $i_1 = i_2 + i_3$ -----(1)

@ node b: $i_3 = i_4 + i_5$ -----(2)

10. Since we need two more equation, we go with KVL:



In the loop 1: $-V_1 + i_1 R_1 + i_2 R_2 = 0$ -----(3)

loop 2: $-i_2 R_2 + i_3 R_3 + i_4 R_4 = 0$ -----(4)

loop 3: $-i_4 R_4 + i_5 R_5 + V_2 = 0$ -----(5)

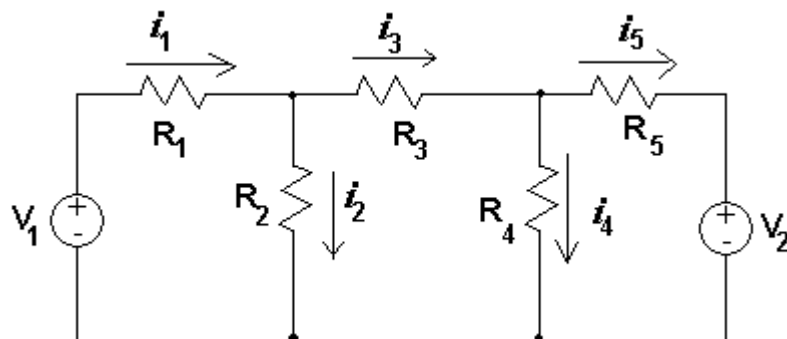
11. So we have 5 equations, but we have not solved problem yet: we have to solve the equation.
12. With 5 variables, it's not easy to find solution, even with Cramer's Method. (*see Appendix A of the textbook for discussions on this method and other matrix related solution methods.*)
13. Apparently, we need some easier application of the Law for somewhat complex circuit problems.
14. We will have detailed methods derived from KCL and KVL later, but the this discussion serves the answer to "why the laws are not enough?"
15. One of the "methods" we can apply is "a disguised KCL" at two nodes. This "KCL in disguise" is called "node voltage method." The other is called "mesh current method" which is, I would say, "KVL in disguise."
16. Before we explore the node voltage method, let's do some terminology study.

B. Terminology

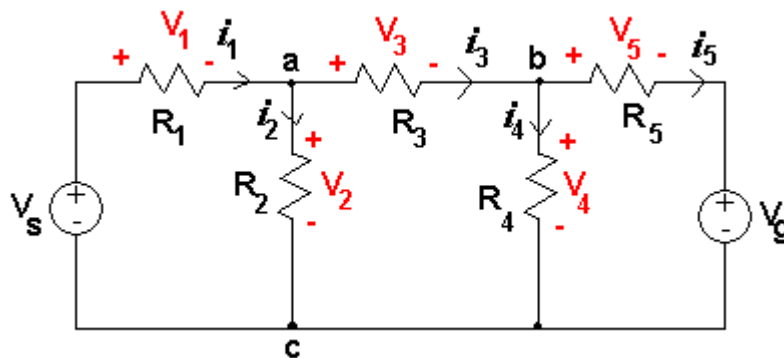
1. Node: A point where two or more elements join
2. **Essential node**: A node where three or more elements join
3. Path: A trace of adjoining elements with no elements included more than once
4. Branch: A path that connects two nodes via element(s)
5. Essential branch: A path which connects two essential nodes without passing through an essential node
6. Loop: A path whose last node is the same as the starting node
7. **Mesh**: A loop that does not enclose any other loops
8. Independent loop: Mesh

C. Node Voltage Method

1. This method is as I mentioned above "KCL in disguise."
2. That is, the node voltage method applies KCL at a node.
3. However, instead of directly using the KCL like $\sum I_{in} = \sum I_{out}$, we rephrase the currents (by applying Ohm's Law) like $\sum \left(\frac{V}{R}\right)_{in} = \sum \left(\frac{V}{R}\right)_{out}$. Here V is the voltage across a resistor.
4. Then, why do we use 'node voltage' instead of just 'voltage'? It's all because of equation complexities and the desire to minimize the number of unknown variables.
5. By the way, *voltage* is a potential different between two points. That's why we use the term 'voltage across' a resistor.
6. *Node voltage* is a *voltage at a node* referred to a *designated reference node* (like ground).
7. Let's see why we need this new concept of *node voltage*. Let's go back to the original circuit problem.



8. Let's apply KCL in "voltage/resistor" form, instead of "current" form. To do this we have to define all 5 voltages across the resistors.



9. With just *Voltage* (or voltage across):

$$\text{KCL @ a: } \frac{V_1}{R_1} = \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

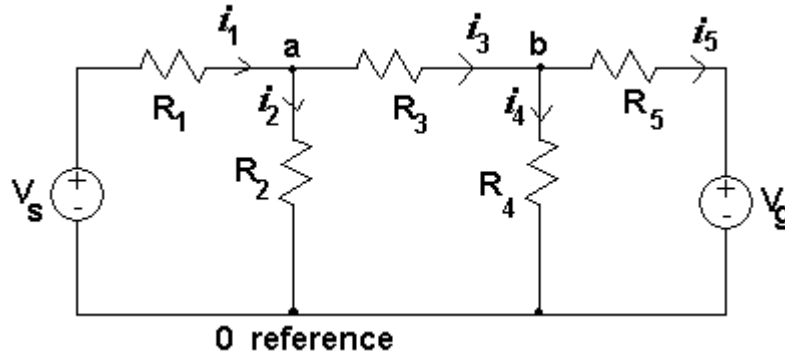
$$\text{KCL @ b: } \frac{V_3}{R_3} = \frac{V_4}{R_4} + \frac{V_5}{R_5} \quad \text{and} \quad \text{KCL @ c: } \frac{V_2}{R_2} + \frac{V_4}{R_4} + \frac{V_5}{R_5} = \frac{V_1}{R_1}$$

We have 5 variables and we have three equations, so we need 2 more equations from applying KVL in 2 loops.

Basically, for number of variables involved, the direct KCL (with current form) and the KCL in disguise ("voltage/resistor") are the same!

10. Let's go with *node voltage*:

In this method, we have to designate a node as the reference. So we pick the bottom node as the reference.



11. Then, voltage at node *a* referenced to the reference is V_{a0} or just V_a .

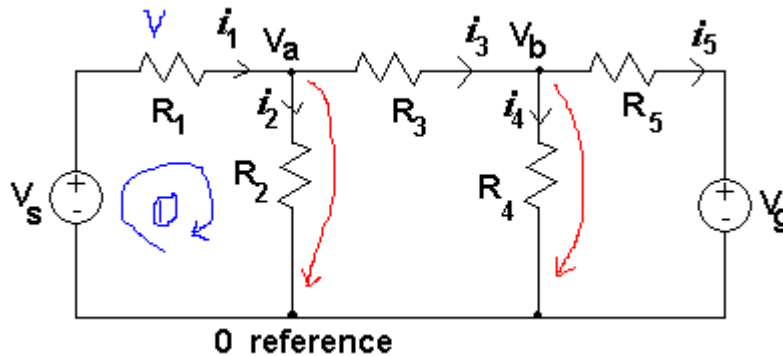
12. In other words, the node voltage V_a is a voltage between node *a* and the reference node.

13. Since we have only two essential nodes, we will express any voltage in the circuit in terms of these two node voltages.

14. For the voltages across the resistors R_1 and R_2 are already given with the node voltages,

since the two resistors are between node and the reference. Therefore, $i_2 = \frac{V_a}{R_2}$ and $i_4 = \frac{V_b}{R_4}$

15. For the resistors R_1 , R_3 , and R_5 , We apply KVL to express the "voltage across" in terms of the two node voltages.



$$\text{From Loop 1: } -V_s + V + V_a = 0 \Rightarrow V = V_s - V_a \Rightarrow i_1 = \frac{V_s - V_a}{R_1}$$

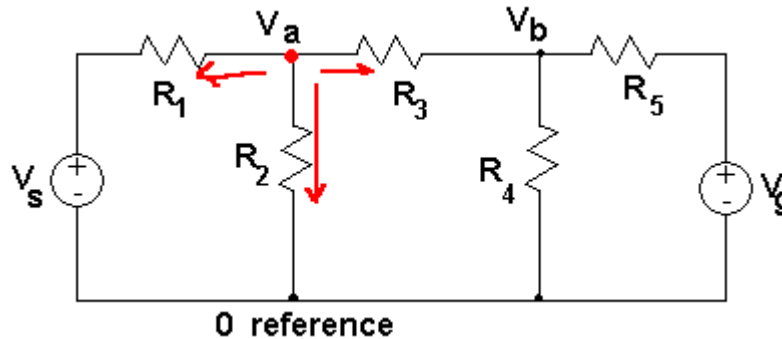
$$\text{Similarly, we have: } i_3 = \frac{V_a - V_b}{R_3} \text{ and } i_5 = \frac{V_b - V_g}{R_5}$$

16. Finally, back to KCL:

$$\text{KCL @ a: } i_1 = i_2 + i_3 \Rightarrow \frac{V_s - V_a}{R_1} = \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3}$$

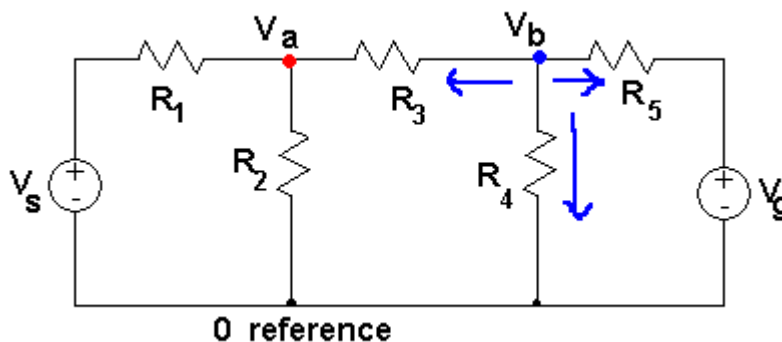
$$\text{KCL @ b: } i_3 = i_4 + i_5 \Rightarrow \frac{V_a - V_b}{R_3} = \frac{V_b}{R_4} + \frac{V_b - V_g}{R_5}$$

17. So, we have two variables (V_a and V_b) and two equations. Case closed!
 18. In actual node voltage method application, we do not assume the current flows in the circuit. Instead, at every node we assume our current flow direction in our convenience.
 19. Let's explore the above argument again with the original circuit.



At node a , we have our currents all out of the node. Then, node voltage equation at the node becomes: $\frac{V_a - V_s}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0$

20. At node b , we again have all our current of the node.



Then, the node voltage equation becomes: $\frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} + \frac{V_b - V_g}{R_5} = 0$

21. The above two equations are neater than those we derived earlier. However, the two sets of equations are the same.
 22. Here is the order of node voltage method application:
 a. Select an essential node as the reference node
 b. A "node voltage" is a voltage between a node and the reference node
 c. Assign node voltages to the remaining essential nodes
 d. At each of the nodes, express the branch currents in terms of node voltages (using Ohm's Law)
 e. Apply KCL to each node
 f. Solve the resulting simultaneous equations to obtain the unknown node voltages.