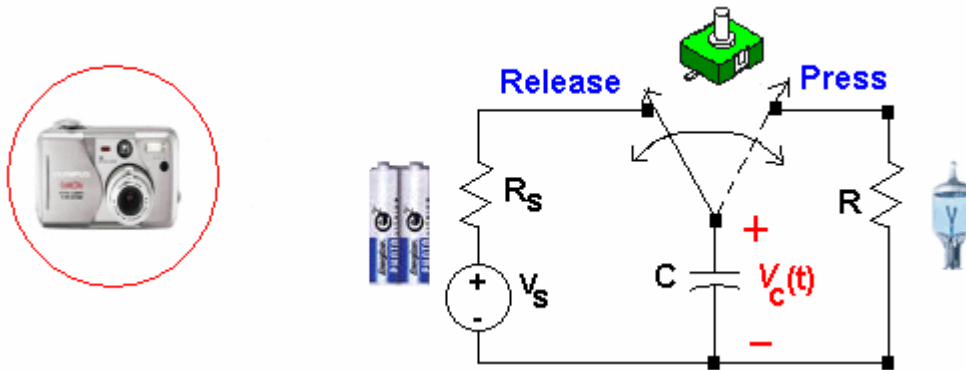
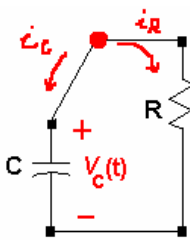


Mobile Studio (MS)-07 - RC Circuit with Time Constant (τ)

1. RC Circuit: Consider the flash light of a camera. The operation of the flash circuit, from a user standpoint, involves depressing the push button on the camera that triggers both the shutter and the flash and then waiting a few milli-seconds before repeating the process to take the next picture. This operation can be modeled using the circuit below. The voltage source V_s and the resistor R_s model the battery that power the camera and flash. The capacitor models the energy storage, the switch models the push button. And the resistor R models the xenon flash lamp.



The capacitor is charged when push button is in the released position, up to the battery voltage level. When the button is pressed, the battery is disconnected from the circuit, and the capacitor energy is released through the xenon lamp, producing the flash. In practice, this energy release or discharge time is determined by the elements (R and C) in the circuit. Let's further investigate the discharging of the capacitor, with the following circuit for the discharging process:



Applying the node-voltage method (or KCL) at the node, we have the following equation:

$$i_c + i_R = 0, \text{ which is}$$

$$C \frac{dv_c(t)}{dt} + \frac{v_c(t)}{R} = 0, \text{ which is rearranged as}$$

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = 0 \quad (1) \text{ --- We will come back to this equation later.}$$

2. First order differential equation: For an ordinary first order differential equation using a general variable $x(t)$

$$\frac{dx(t)}{dt} + ax(t) = 0,$$

the complete solution for $x(t)$ is given by:

$$x(t) = x(\infty) + [x(0) - x(\infty)]e^{-at} \quad (2)$$

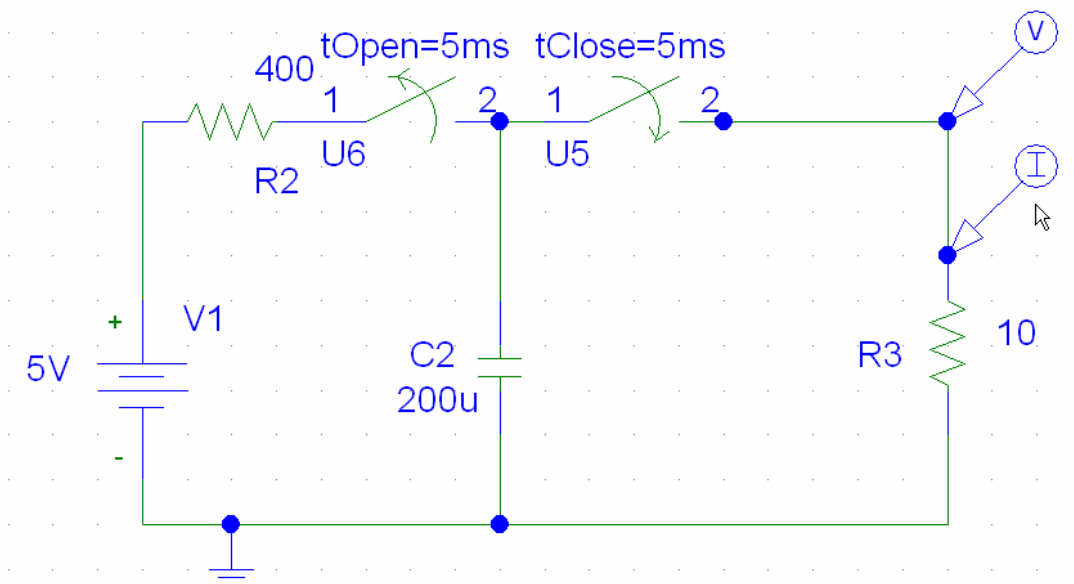
Applying the solution form (2) of the first order differential equation to the camera flash equation (1), we have the following voltage equation:

$$v_c(t) = [v_c(0) - v_c(\infty)]e^{-t/RC} \quad (3)$$

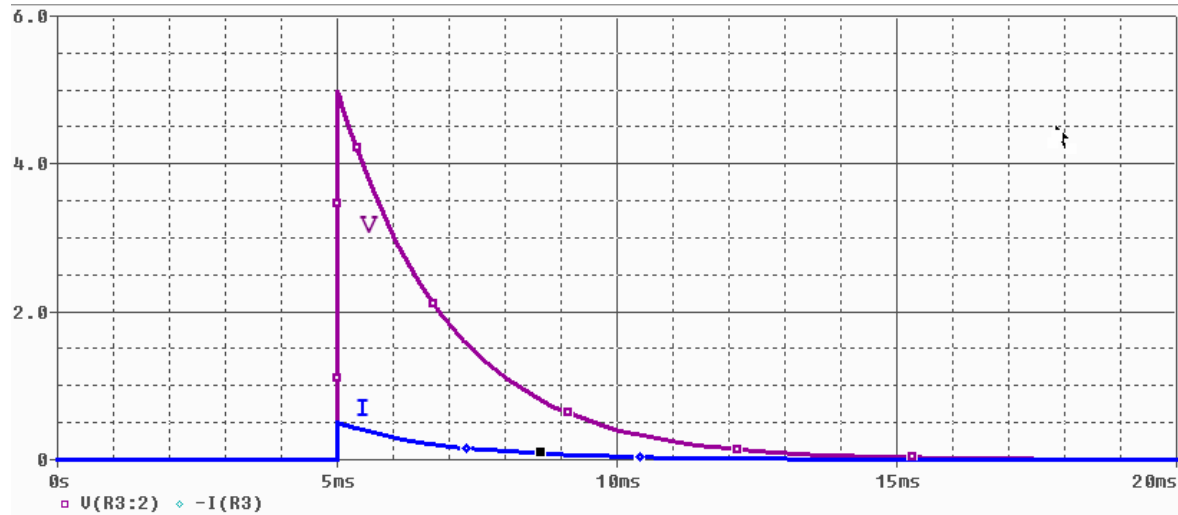
Since the initial voltage (charged voltage level before the battery is disconnected) is $v_c(0) = V_s$, and the final voltage at time $t=\infty$ is zero ($v_c(\infty) = 0$) since the energy would be all discharged and will die eventually because there is no voltage source in the circuit but a consuming resistor, the solution (3) becomes:

$$v_c(t) = [V_s - 0]e^{-t/RC} = V_s e^{-t/\tau}, \text{ where } \tau=RC.$$

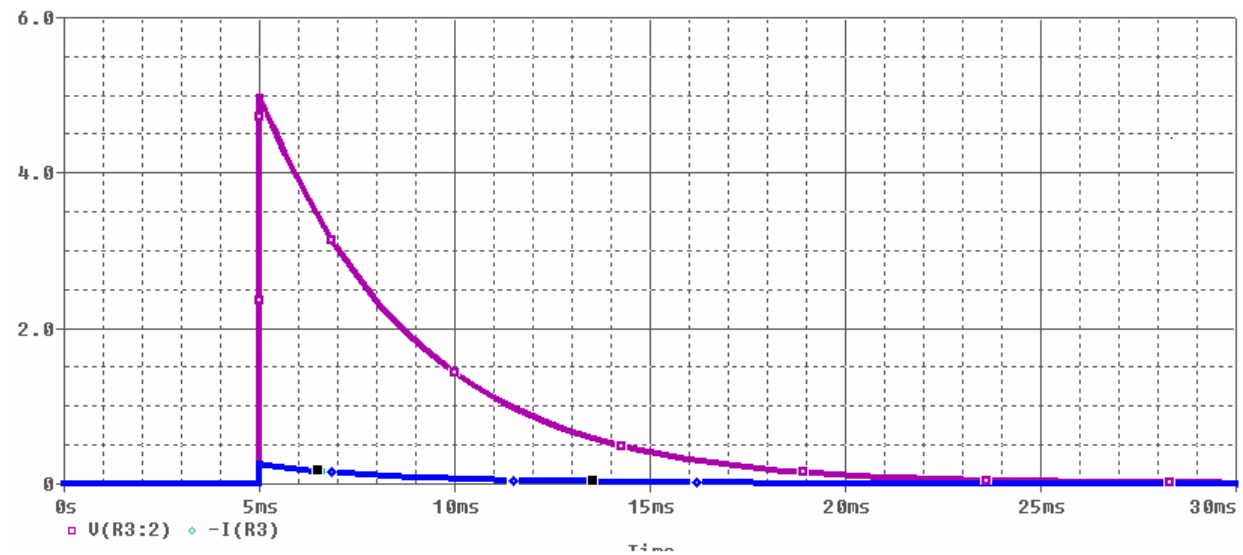
3. PSpice Simulation: A simulation would give you much better picture on how the discharge proceeds and how the voltage across the lamp would die down. The figure below shows the schematic for 5V battery with 200uF capacitor and 10-ohm lamp. The capacitor is charged for 5ms, and then instantaneously, the battery is disconnected, and the energy stored in the capacitor is discharged through the lamp.



The transient analysis of the above schematic is shown below. As expected the voltage at time 5ms is at the fully charged level of 5V. Then, the voltage is dying with exponential decaying $e^{-t/RC}$, and after 10ms, it almost disappears. It's apparent that the value of RC changes the decaying rate of the discharge: bigger RC would mean slower rate.



Now, let's change the lamp resistor to 20-ohm, and see the same discharge behavior. The figure bellows shows that with RC doubled, the decaying rate is also doubled (slowed), taking about 20ms to disappear.



4. Time constant: As discussed above, the rate of the decay is determined by the constant RC in $e^{-t/RC}$. Let's replace RC by τ , "**time constant**", and find the values of the decaying term $v(t)$ at time $t=0$ and $t=\tau$:

$$v(0) = v(0) \cdot e^{-0/\tau} = 1 \cdot v(0)$$

$$v(\tau) = v(0) \cdot e^{-\tau/\tau} = v(0) \cdot e^{-1} = 0.3768 \cdot v(0)$$

By comparing the values of the decaying term, we have: $\frac{v(\tau)}{v(0)} = \frac{v(0)e^{-1}}{v(0)} = e^{-1} = 0.3678$.

The time constant of a circuit then is defined as the time required for the response to decay by a factor of 1/e of its initial value. In other words, the time constant is the time required to decay to the 37% of the initial value.

Further, the value of $\frac{v(t)}{v(0)} = \frac{v(0)e^{-t/\tau}}{v(0)}$ for several t values in terms of τ is tabulated for a graph:

The value of $\frac{v(t)}{v(0)} = e^{-t/\tau}$	
time $t=$	$e^{-t/\tau}$
0	$e^0=1.00000$
τ	$e^{-1}=0.36788$
2τ	$e^{-2}=0.13534$
3τ	$e^{-3}=0.04979$
4τ	$e^{-4}=0.01832$
5τ	$e^{-5}=0.00674$

From the table, it is evident that the value $v(t)$ is less than 1% of the initial value after 5τ (i.e., five time constants). Thus, it is customary to assume that it takes 5τ for the circuit to reach its final (or steady) state -- "**5 Tau rule**"

PRE-LAB -07: RC Circuit and Time Constant

Name: _____

1. Study the tutorial on *Transient Analysis using Pspice*, (webpage)
2. Decide all the values of the attributes of VPULSE if we want to generate a pulse of:
2ms pulse width, 4ms pulse period, 0.75 V On-pulse voltage, 0 V off-pulse voltage.

ANSWER: _____

3. Simulate the following circuit using Pspice.

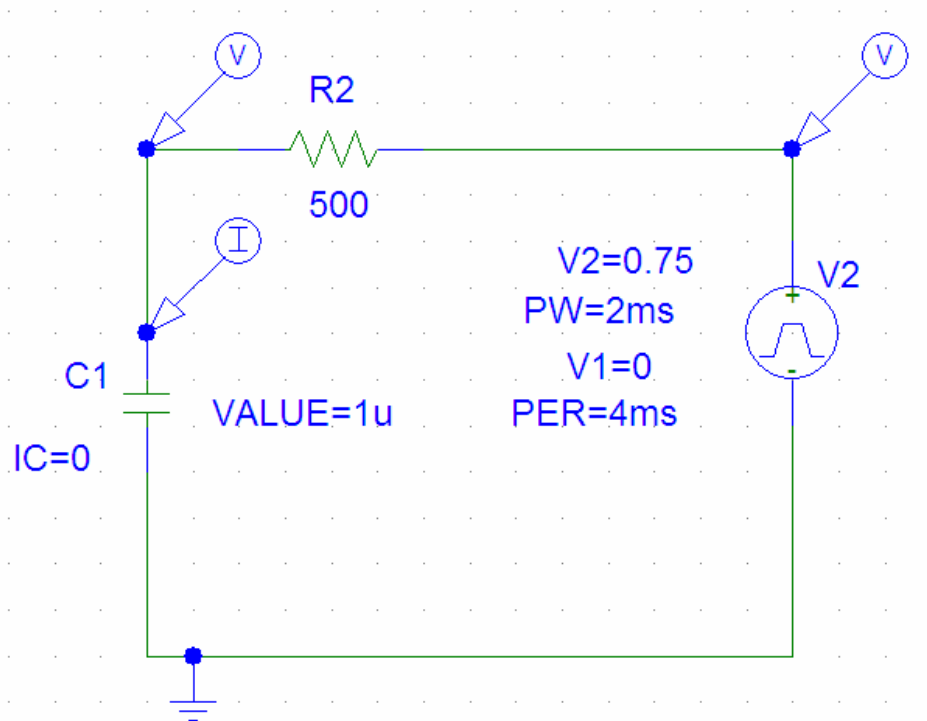


FIG.1

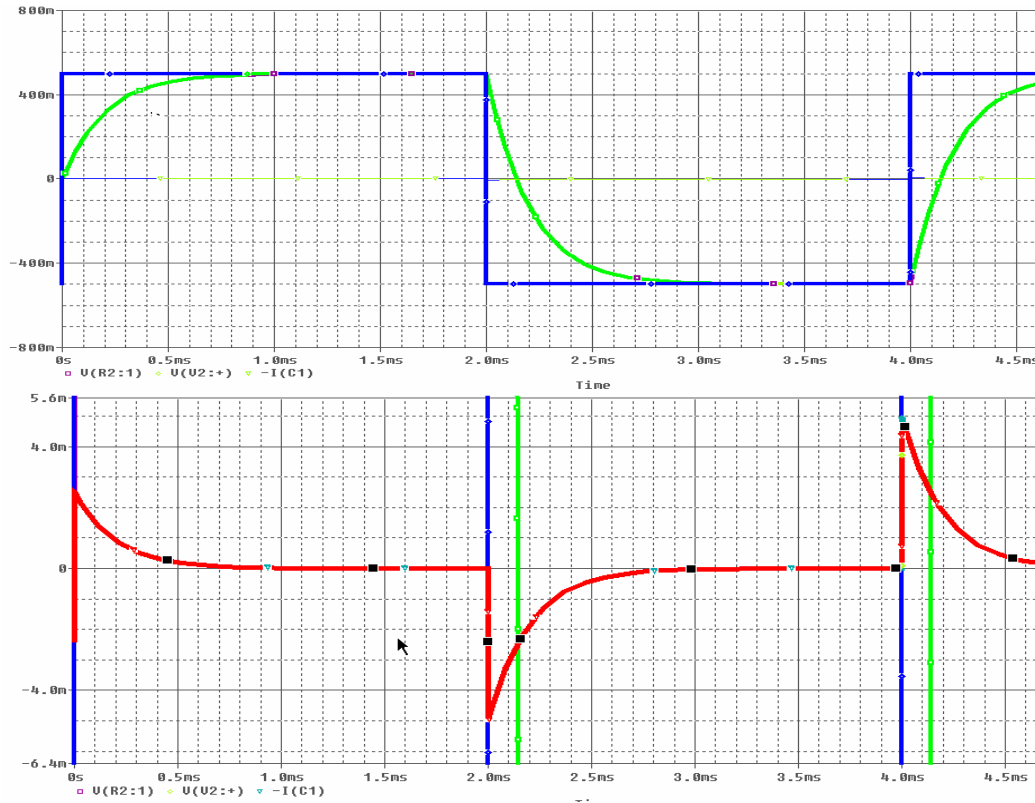
4. Attach your simulation output. When does the circuit reach the steady state?

MS-07 RC Circuit and Time Constant

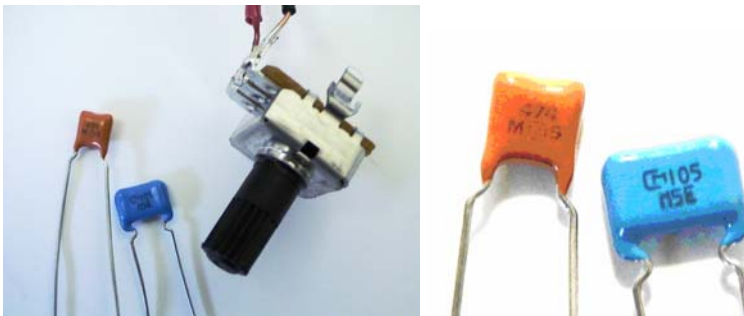
GROUP#: _____ NAME: _____

MS-07 PROCEDURE/REPORT

1. Simulate the circuit (**FIG. 1**) but with 0.5V on-pulse and -0.5V off-pulse, using Pspice. Check if you have the following voltage and current waveforms. (**Attach your result**)



2. Components: Capacitors (1 μ F and 0.47 μ F) and a variable resistor (0 - 1Kohm)



3. Implement the following circuit on the breadboard using the Instrumentation Board and the elements.

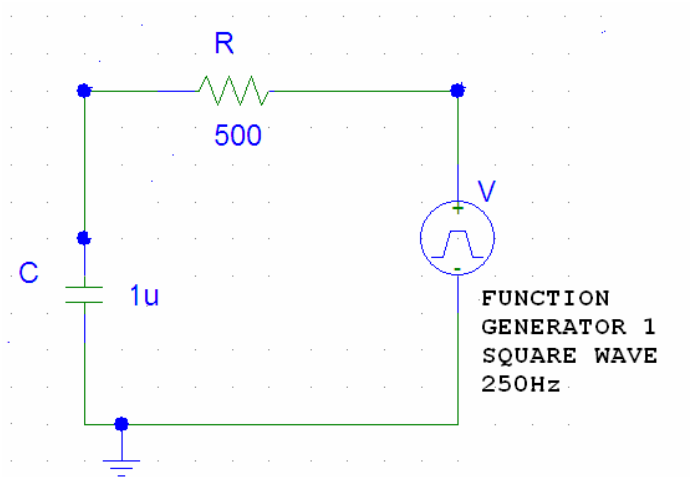
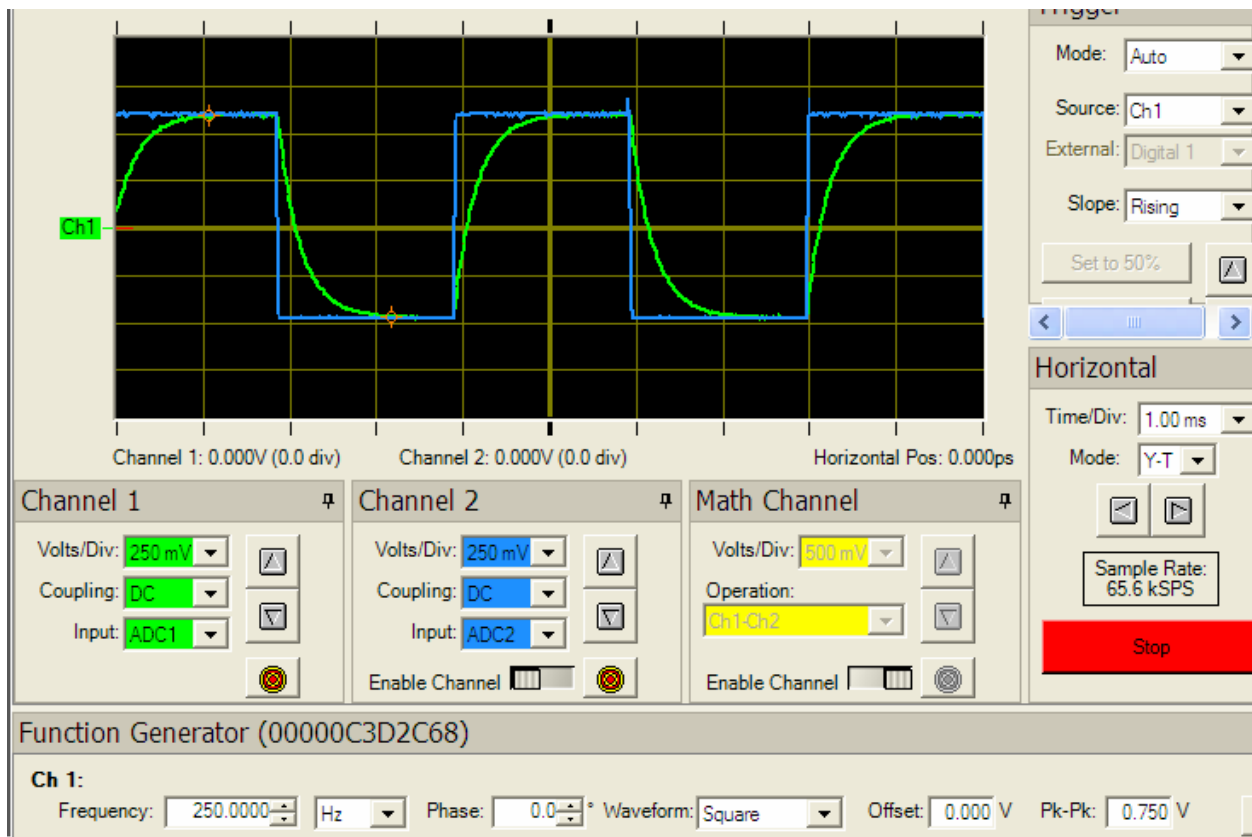


FIG.2

4. Connect your Instrumentation Board and run the Mobile Studio Desktop. Open Scope and Function Generator. Type in 250 for the frequency, select Square. Keep others intact, as shown below.



Do you see the one like above? If not, turn the wiper of the variable resistor until you have it. **(Attach the screen shot)**

5. From your screen for voltage, how long does it take to fully discharge? _____

Then, what would be the time constant of the circuit. Remember the "5 tau rule." _____

From the time constant, can you guess the resistance of the variable resistor in the current wiper position. What's your guess?: _____

Now take the variable resistor (without touching the wiper handle) from the circuit, using the DMM, measure the resistance and record here: _____

How close are the two values? _____

6. Now replace the 1uF capacitor by 0.47uF capacitor. Run the Mobile Studio.

From your screen for voltage, how long does it take to fully discharge? _____

Then, what would be the time constant of the circuit. Remember the "5 tau rule." _____

From the time constant, can you guess the resistance of the variable resistor in the current wiper position. What's your guess?: _____

Now take the variable resistor (without touching the wiper handle) from the circuit, using the DMM, measure the resistance and record here: _____

How close are the two values? _____

7. Now, in the Function Generator window, change the waveform from Square to Sinusoid, and run the Mobile Studio. What kind of output voltage do you get? Why?

8. Now, in the Function Generator window, change the waveform from Sinusoid to Triangle, and run the Mobile Studio. What kind of output voltage do you get? Why?

9. Now, from the experiments and simulations, discuss about the roles of the resistor and capacitor in a circuit.