4. Renewable Energy Sources

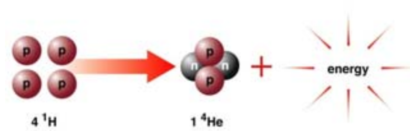
Part B1: Solar Electricity

Charles Kim, “Lecture Note on Analysis and Practice for Renewable Energy Micro Grid Configuration,” 2013. www.mwftr.com

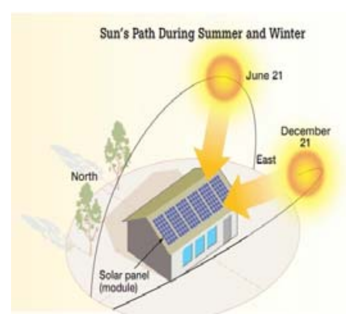
Brief on Solar Energy

- Solar Energy: Radiant energy from the sun that travels to Earth in electromagnetic waves of rays.
- Solar energy is produced in the sun’s core when hydrogen atoms combine [“fusion” process] to produce helium. During the fusion, radiant energy is emitted.

$$\boxed{4\ {}^{1}\mathrm{H} + 2\ \mathrm{e} \dashrightarrow {}^{4}\mathrm{He} + 2\ \text{neutrinos} + 6\ \text{photons}}$$



- We capture solar energy with solar collectors [Photovoltaic Cells] that turn radiant energy into electricity
- Clean and renewable energy source
- Solar Radiation Information Critical
- Intermittent source



Solar Energy Resources

- http://eosweb.larc.nansa.org/cgi-bin/sse/sse.cgi
- NASA's Surface Meteorology and Solar Energy

Surface meteorology and Solar Energy

A renewable energy resource web site (release 6.0)

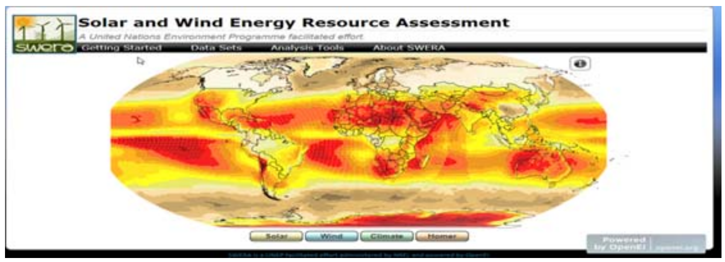
sponsored by NASA's Applied Science Program in the Science Mission Directorate

developed by POWER: Prediction of Worldwide Energy Resource Project

- over 200 satellite-derived meteorology and solar energy parameters
- monthly averaged from 22 years of data
- data tables for a particular location

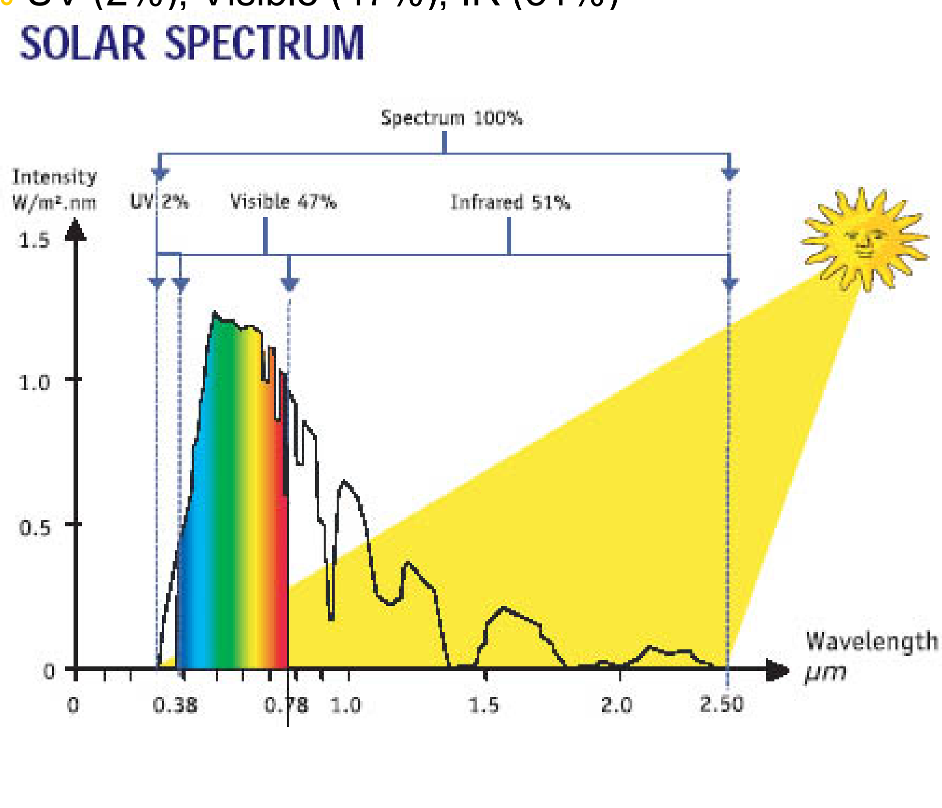
HIGHLIGHTS

- SWERA



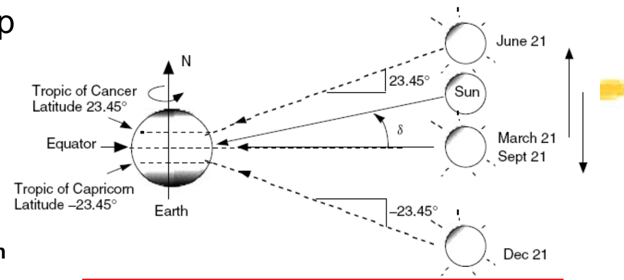
Solar Spectrum

- UV (2%); Visible (47%); IR (51%)



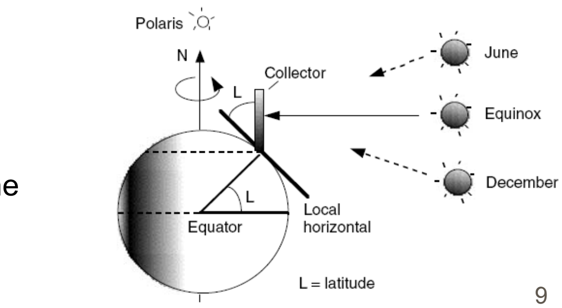
Solar Declination

- Fixed Earth and Sun Moving Up and Down View
- Solar Declination: Angle between the sun and the equator
- Solar declination: "**angle between the sun's rays and the earth's equatorial plane, the latitude at which the sun is directly overhead at midday**. Declination values are positive when the sun is north of the equator (March 21 to September 23) and negative when the sun is south of the equator. Maximum and minimum values are +0.409 radians (+23.45 degrees) and -0.409 radians (-23.45 degrees)."
- A good rule of thumb of solar panel
  - Face it south
  - Tilt it up at an angle equal to the local latitude



$$\delta = 23.45° \sin\left[\frac{360}{365}(n-81)\right]$$

$$\delta = \sin^{-1}\left\{\sin(23.45°)\sin\left[\frac{360}{365}(n-81)\right]\right\}$$

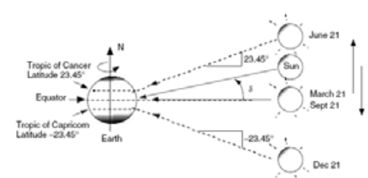


Solar Declination Angle

- Angle between the sun's rays and the earth's equatorial plane
- The latitude at which the sun is directly overhead at midday.
- Declination values are positive when the sun is north of the equator (March 21 to September 23) and negative when the sun is south of the equator. Maximum and minimum values are +0.409 radians (+23.45 degrees) and -0.409 radians (-23.45 degrees)."

$$\delta = 23.45° \sin\left[\frac{360}{365}(n-81)\right]$$

$$\delta = \sin^{-1}\left\{\sin(23.45°)\sin\left[\frac{360}{365}(n-81)\right]\right\}$$



- EXCEL

	A	B	C
1	n	δ	δ
2	1	-23.0084804	-22.98290396
3	31	-17.7755991	-17.55717467
4	61	-7.91105916	-7.715532161
5	91	4.01480801	3.906791706
6	121	14.8945658	14.64128411
7	151	21.8933548	21.81023387
8	181	23.1875515	23.17215181
9	211	18.4399363	18.23595278
10	241	8.88756089	8.674823443
11	271	-2.98058167	-2.899381994

=23.45*(SIN((360/365)*(A2-81)*3.14/180))

=ASIN(SIN(23.45*3.14/180)*SIN((360/365)*(A2-81)*3.14/180))*180/3.14

MathCAD

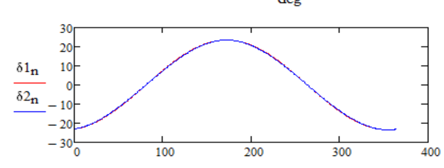
Solar Declination.xmcd Charles Kim 2013

$n := 1..365$ $\quad 180 \times deg = 3.1416$

$\delta 1_n := 23.45 \times deg \times \sin\left[\frac{360}{365} \times (n-81) \times deg\right]$

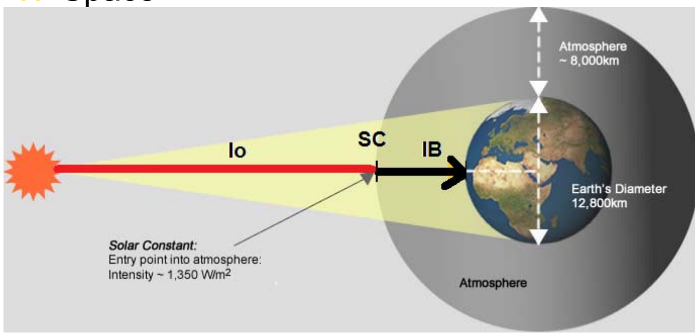
$\delta 2_n := \operatorname{asin}\left[\sin(23.45 \times deg) \times \sin\left[\frac{360}{365} \times (n-81) \times deg\right]\right]$

$\delta 1_n := \frac{\delta 1_n}{deg} \qquad \delta 2_n := \frac{\delta 2_n}{deg}$



Solar Radiation in Space and on Earth Surface

- Space



- Units:
  - ✓ kWh per square meter (preferred)
  - ✓ British Thermal Units
  - ✓ Kilocalories
  - ✓ Langleys

1 kW/m^2	=	316.95 $Btu/h\text{-}ft^2$
	=	1.433 langley/min
1 kWh/m^2	=	316.95 Btu/ft^2
	=	85.98 langleys
	=	3.60×10^6 $joules/m^2$
1 langley	=	1 cal/cm^2
	=	41.856 $kjoules/m^2$
	=	0.01163 kWh/m^2
	=	3.6878 Btu/ft^2

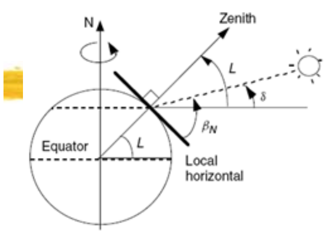
- Earth Surface - 3 components:
  - Beam Radiation: I_B
  - Diffuse Radiation: I_D
  - Reflected Radiation: I_R

Clear Sky Beam Radiation

- Extraterrestrial Solar Insolation (I_o)

$$I_0 = \mathrm{SC} \cdot \left[1 + 0.034 \cos\left(\frac{360n}{365}\right)\right] \quad (\mathrm{W/m^2})$$

degree°



  - SC: Solar constant $1.377\ \mathrm{kW/m^2}$
  - n: day number

January	$n = 1$	July	$n = 182$
February	$n = 32$	August	$n = 213$
March	$n = 60$	September	$n = 244$
April	$n = 91$	October	$n = 274$
May	$n = 121$	November	$n = 305$
June	$n = 152$	December	$n = 335$

- Portion of the beam reaching the earth surface (I_B)

$$I_B = Ae^{-km}$$

  - A: Apparent extraterrestrial flux

$$A = 1160 + 75 \sin\left[\frac{360}{365}(n - 275)\right] \quad (\mathrm{W/m^2})$$

  - k : optical depth

$$k = 0.174 + 0.035 \sin\left[\frac{360}{365}(n - 100)\right]$$

  - m: air mass ratio

$$m = \frac{1}{\sin\beta}$$

  - β: altitude angle of the sun

Beam on Earth Surface [Example Calculation]

- Question: Find the direct beam solar radiation normal to the sun's rays at solar noon on a clear day in Atlanta (latitude 33.7 degrees) on May 21. (solar declination table)

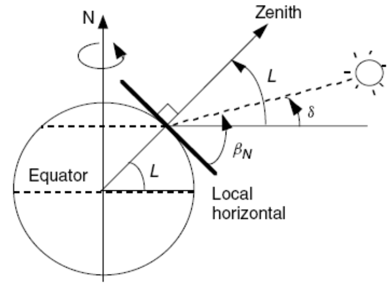
Month:	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sept	Oct	Nov	Dec
δ:	−20.1	−11.2	0.0	11.6	20.1	23.4	20.4	11.8	0.0	−11.8	−20.4	−23.4

$$I_B = Ae^{-km}$$

Air mass ratio $m = \dfrac{1}{\sin\beta}$

$$A = 1160 + 75\sin\left[\frac{360}{365}(n-275)\right] \quad (\text{W/m}^2)$$

$$k = 0.174 + 0.035\sin\left[\frac{360}{365}(n-100)\right]$$



altitude angle of the sun at solar noon.

$$\beta_N = 90^\circ - L + \delta$$

Solution

- Find the direct beam solar radiation normal to the sun's rays at solar noon on a clear day in Atlanta (latitude 33.7 degrees) on May 21.
- SOLUTION
  - n=141 for May 21

Month:	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sept	Oct	Nov	Dec
δ:	−20.1	−11.2	0.0	11.6	20.1	23.4	20.4	11.8	0.0	−11.8	−20.4	−23.4

$$A = 1160 + 75\sin\left[\frac{360}{365}(n-275)\right] = 1160 + 75\sin\left[\frac{360}{365}(141-275)\right]$$
$$= 1104 \text{ W/m}^2$$

$$k = 0.174 + 0.035\sin\left[\frac{360}{365}(n-100)\right]$$
$$= 0.174 + 0.035\sin\left[\frac{360}{365}(141-100)\right] = 0.197$$

January	$n = 1$	July	$n = 182$
February	$n = 32$	August	$n = 213$
March	$n = 60$	September	$n = 244$
April	$n = 91$	October	$n = 274$
May	$n = 121$	November	$n = 305$
June	$n = 152$	December	$n = 335$

  - Altitude angle of the sun at solar noon
  - Air mass ratio:
  - Clear Sky Beam Radiation at the earth surface

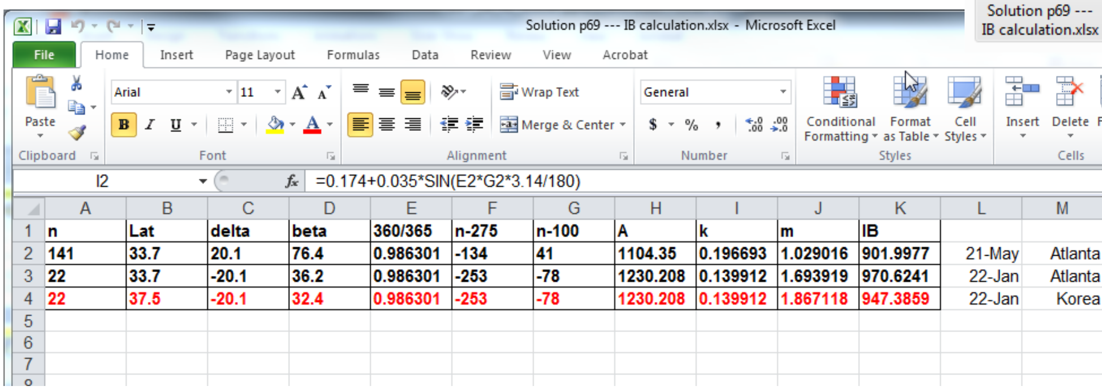
$$\beta_N = 90^\circ - L + \delta = 90 - 33.7 + 20.1 = 76.4^\circ$$

$$m = \frac{1}{\sin\beta} = \frac{1}{\sin(76.4^\circ)} = 1.029$$

$$I_B = Ae^{-km} = 1104\ e^{-0.197\times 1.029} = 902 \text{ W/m}^2$$

Excel Solution

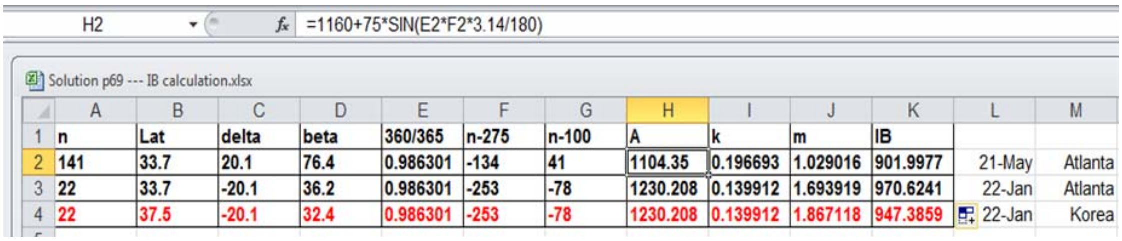
Solution p69 --- IB calculation.xlsx



Solution p69 --- IB calculation.xlsx - Microsoft Excel

I2 f_x =0.174+0.035*SIN(E2*G2*3.14/180)

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	n	Lat	delta	beta	360/365	n-275	n-100	A	k	m	IB		
2	141	33.7	20.1	76.4	0.986301	-134	41	1104.35	0.196693	1.029016	901.9977	21-May	Atlanta
3	22	33.7	-20.1	36.2	0.986301	-253	-78	1230.208	0.139912	1.693919	970.6241	22-Jan	Atlanta
4	22	37.5	-20.1	32.4	0.986301	-253	-78	1230.208	0.139912	1.867118	947.3859	22-Jan	Korea



H2 f_x =1160+75*SIN(E2*F2*3.14/180)

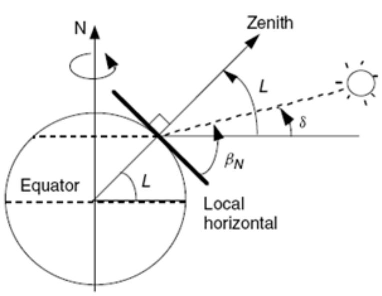
Solution by MathCad

Clear Sky Beam Radiation.xmcd   Charles Kim 2013

Q. Find the direct beam solar radiation normal to the sun's days at solar noon on a clear day in Atlanta (Latitude 33.7 degrees N) on May 21.

SOLUTION

$n := 141$ for May 21 $L := 33.7$ Try to change L to Seoul 37.5



$$\delta(x) := 23.45 \times \sin\left[\frac{360}{365} \times (x - 81) \times deg\right]$$

$$\delta(n) = 20.1380$$

$$\beta := 90 - L + \delta(n) = 76.4380$$

Air mass ratio (m) : $m := \dfrac{1}{\sin(\beta \times deg)} = 1.0287$

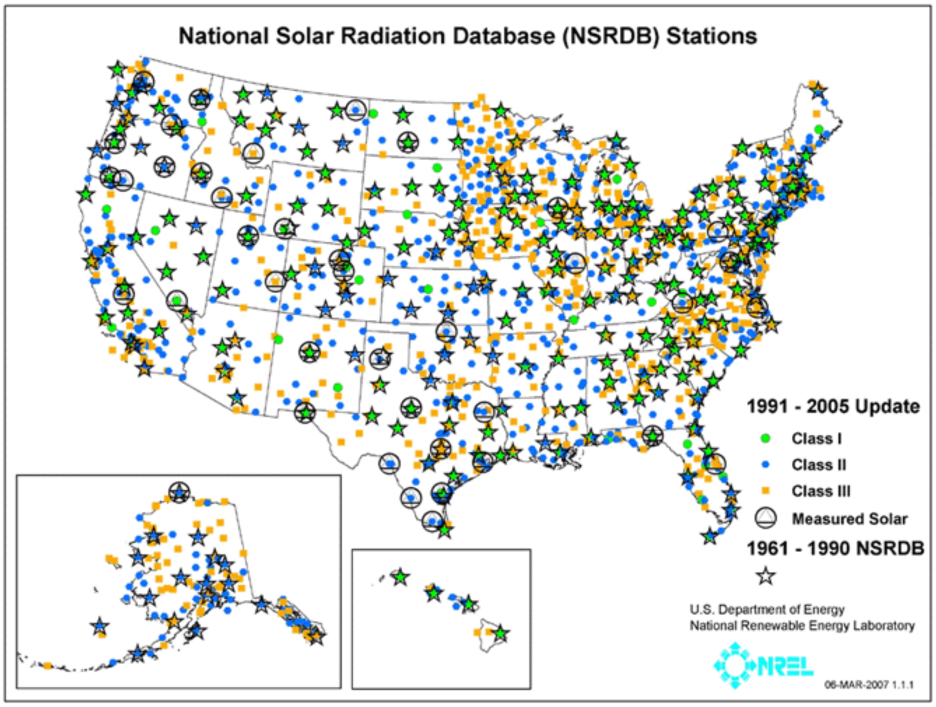
Apparent Extraterrestrial flux (A) : $A := 1160 + 75 \times \sin\left[\frac{360}{365} \times (n - 275) \times deg\right] = 1.1044 \times 10^3$

Optical Depth (k) : $k := 0.174 + 0.035 \times \sin\left[\frac{360}{365} \times (n - 100) \times deg\right] = 0.1967$

Clear Sky Beam Radiation at the Earth Surface (IB) : $IB := A \times e^{-k \times m} = 902.0961$ W / m^2

Solar Radiation Measurement Stations

- 239 National Solar Radiation Database Stations



Pyranometer and Pyrheliometer

- Pyranometer: measures total radiation arriving from all directions, direct and diffuse compoenst
- Pyrheliometer: measures only direct radiation

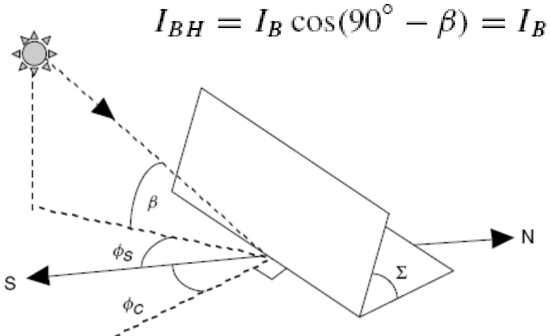
Radiation on collector

- Collector Surface:
  - Beam radiation: I_{BC}
  - Diffuse radiation: I_{DC}
  - Reflected radiation: I_{RC}

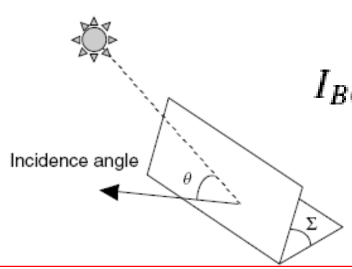
Beam Radiation on **Collector**

$$\boxed{\begin{array}{l}\text{incidence angle } \theta \\ \text{collector azimuth angle } \phi_C \\ \text{altitude angle } \beta \\ \text{solar azimuth angle } \phi_S \\ \text{tilt angle } \Sigma\end{array}}$$

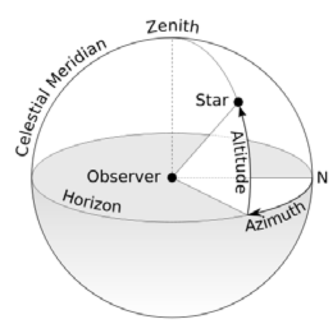
$$I_{BH} = I_B \cos(90^\circ - \beta) = I_B \sin\beta$$



- Solar altitude: β
- Normal to vertical surface: ϕ_c



$$I_{BC} = I_B \cos\theta$$



$$\boxed{\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma}$$

An **azimuth** (/ˈæzɪməθ/; from Arabic السمت *as-samt*, meaning "a way, a part, or quarter"[1]) is an angular measurement in a spherical coordinate system. The

Diffuse Radiation on Collector

- Sky diffuse factor (C)

$$C = 0.095 + 0.04 \sin\left[\frac{360}{365}(n - 100)\right]$$

- n: day number

January	$n = 1$	July	$n = 182$
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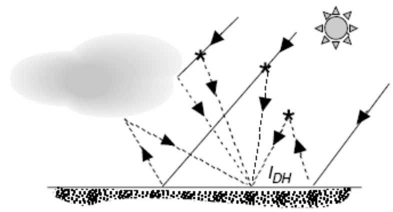
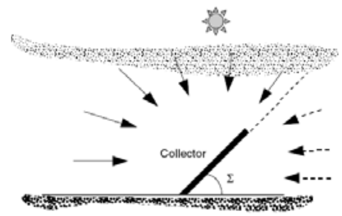
- Diffuse insolation on a Horizontal surface is proportional to the direct radiation

$$I_{DH} = C\, I_B$$

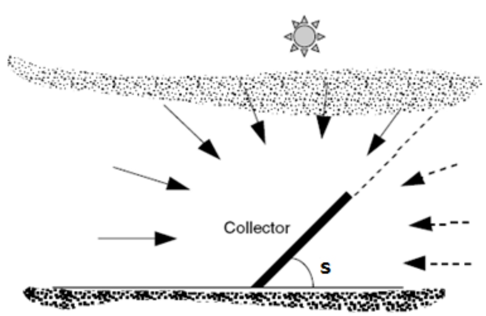


- Diffuse Radiation on collector

$$I_{DC} = I_{DH}\left(\frac{1+\cos\Sigma}{2}\right) = C\, I_B\left(\frac{1+\cos\Sigma}{2}\right)$$



I_{DC}



I_{DH} all the surface 180 degrees

I_{DC} Only the portion of collector surface (180 - s) degrees

If S=0; Collector gets full diffuse radiation.

If S=90; collecter gets 1/2 of the diffuse radiation.

mathematical expression $\frac{1 + \sin(90-S)}{2}$

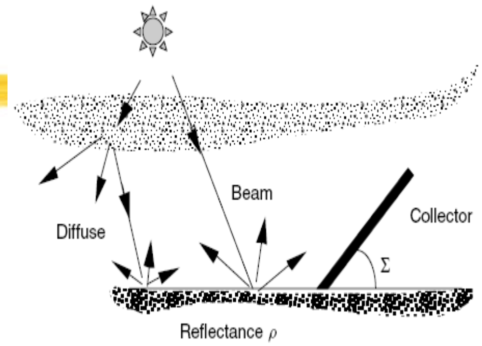
sin (X-Y)=sinX*cosY - cosX*sinY

sin (90-S)=sin90*cosS - cos90*sinS =cosS

Finally, $I_{DC} = \frac{1 + \cos S}{2}$

Reflected Radiation on Collector

- Reflection from ground with reflectance



$$I_{RC} = \rho(I_{BH} + I_{DH})\left(\frac{1-\cos\Sigma}{2}\right)$$

$$I_{RC} = \rho I_B(\sin\beta + C)\left(\frac{1-\cos\Sigma}{2}\right)$$

- **Combination** of all three: Radiation striking a collector on a clear day

$$\boxed{\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma}$$

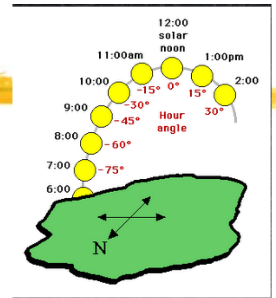
$$I_C = I_{BC} + I_{DC} + I_{RC}$$

$$I_C = Ae^{-km}\left[\cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma + C\left(\frac{1+\cos\Sigma}{2}\right) + \rho(\sin\beta + C)\left(\frac{1-\cos\Sigma}{2}\right)\right]$$

Average Monthly Insolation

- Estimate of average insolation that strikes a tilted collector under real conditions at a particular site
- $I_C = I_{BC} + I_{DC} + I_{RC}$ (direct + Diffuse + reflection) on collector surface
- Working on horizontal insolation first (since primary measurement data is on horizontal insolation I_H)
- $I_H = I_{DH} + I_{BH}$ (Horizontal Insolation = Horizontal Diffuse + Horizontal Beam)
- $I_{DC} \leftarrow I_{DH}$ & $I_{RC} \leftarrow I_H$ (already discussed)
- Question is how to get I_{BC} from I_H

Decomposition of Total Horizontal Insolation (I_H)



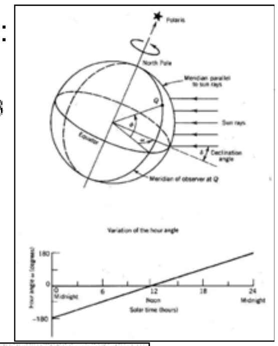
- **Clearness index** (K_T): Ratio of average horizontal insolation at a site (I_H) to the extraterrestrial insolation on a horizontal surface above the site and just outside the atmosphere (I_o)

$$K_T = \frac{\overline{I}_H}{\overline{I}_0}$$

- Average value of I_o: averaging the product of normal radiation and the SIN of the solar hour angle from sunrise and sunset:

$$\overline{I}_0 = \left(\frac{24}{\pi}\right) \mathrm{SC} \left[1 + 0.034 \cos\left(\frac{360n}{365}\right)\right] (\cos L \cos\delta \sin H_{SR} + H_{SR} \sin L \sin\delta)$$

- Correlation between Clearness Index and Diffuse Radiation:



$$\frac{\overline{I}_{DH}}{\overline{I}_H} = 1.390 - 4.027 K_T + 5.531 {K_T}^2 - 3.108 {K_T}^3$$

- Diffuse and Reflected Radiation on a tilted collector surface

$$\overline{I}_{DC} = \overline{I}_{DH} \left(\frac{1 + \cos\Sigma}{2}\right) \qquad \overline{I}_{RC} = \rho \overline{I}_H \left(\frac{1 - \cos\Sigma}{2}\right)$$

$$H_{SR} = \cos^{-1}(-\tan L \tan\delta)$$

SUNRISE HOUR ANGLE - The sunrise hour angle is the hour angle, expressed in degrees, when the sun's center reaches the horizon.

Conversion to Beam Radiation on Collector

- The average beam radiation on a horizontal surface (I_{BH}) can be found by subtracting the diffuse portion (I_{DH}) from the total (I_H):

$$\overline{I}_H = \overline{I}_{DH} + \overline{I}_{BH} \longrightarrow \overline{I}_{BH} = \overline{I}_H - \overline{I}_{DH}$$

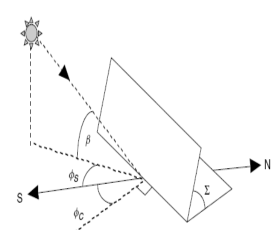
- Conversion of horizontal beam radiation (I_{BH}) to the beam radiation on collector (I_{BC}):

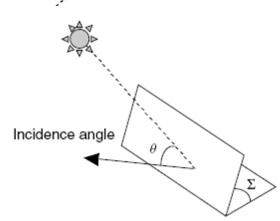
$$I_{BH} = I_B \sin\beta$$
$$I_{BC} = I_B \cos\theta \longrightarrow I_{BC} = I_{BH}\left(\frac{\cos\theta}{\sin\beta}\right) = I_{BH} R_B$$

θ is the incidence angle between the collector and beam

β is the sun's altitude angle

R_B is *beam tilt factor*





- Average value of Beam Tilt Factor (R_B):

Average value of Beam Tilt Factor (R_B)

- For South-Facing Collectors:

$$\overline{R}_B = \frac{\cos(L-\Sigma)\cos\delta\sin H_{SRC} + H_{SRC}\sin(L-\Sigma)\sin\delta}{\cos L\cos\delta\sin H_{SR} + H_{SR}\sin L\sin\delta}$$

$H_{SR} = \cos^{-1}(-\tan L\tan\delta)$ sunrise hour angle (in radians)

$H_{SRC} = \min\{\cos^{-1}(-\tan L\tan\delta), \cos^{-1}[-\tan(L-\Sigma)\tan\delta]\}$ sunrise hour angle for the collector

L is the latitude

Σ is the collector tilt angle.

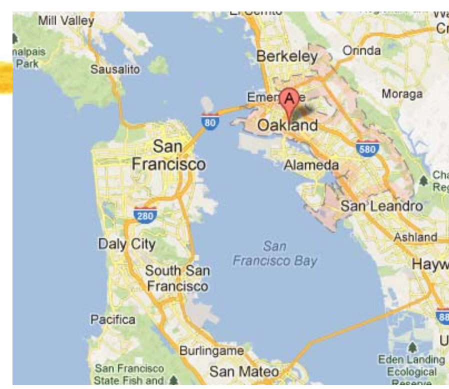
δ is the solar declination

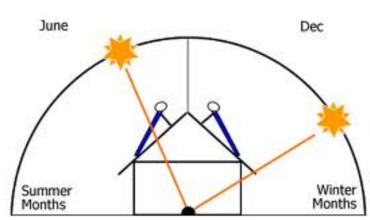
- Final Equation for Insolation striking a collector

$$\boxed{\overline{I}_C = \overline{I}_H\left(1-\frac{\overline{I}_{DH}}{\overline{I}_H}\right)\cdot\overline{R}_B + \overline{I}_{DH}\left(\frac{1+\cos\Sigma}{2}\right) + \rho\overline{I}_H\left(\frac{1-\cos\Sigma}{2}\right)}$$

Example Calculation

- Average Monthly Insolation on a Tilted Collector
- Average horizontal insolation (I_H) in Oakland, California (latitude 37.73◦N) in July is 7.32 kWh/m^2-day. Assume ground reflectivity of 0.2.
- **Question**: Estimate the insolation on a south-facing collector at a tilt angle of 30◦ with respect to the horizontal.





Solution Approach

- 0. Target $\overline{I}_C = \overline{I}_H \left(1 - \frac{\overline{I}_{DH}}{\overline{I}_H}\right) \cdot \overline{R}_B + \overline{I}_{DH}\left(\frac{1+\cos\Sigma}{2}\right) + \rho \overline{I}_H \left(\frac{1-\cos\Sigma}{2}\right)$
- 1. Sun declination (δ) for July 16 (n=197)
- 2. Sunrise Hour Angle (H_{SR}) using L=37.73°
- 3. Extraterrestrial Insolation (I_o) (with SC=1.37 kW/m²)
- 4. Clearness Index (K^T)
- 5. Horizontal Diffuse Radiation (I_{DH})
- 6. Diffuse Radiation on the Collector (I_{DC})
- 7. Reflected Radiation on the Collector (I_{RC})
- 8. Horizontal Beam Radiation (I_{BH})
- 9. Sunrise Hour Angle on the Collector (H_{SRC})
- 10. Beam Tilt Factor (R_B)
- 11. Beam Radiation on the Collator (I_{BC})
- 12. Total Insolation on the Collector (I_C)

Solution - Details

July 16 ($n = 197$):

$$\delta = 23.45 \sin\left[\frac{360}{365}(n-81)\right] = 23.45 \sin\left[\frac{360}{365}(197-81)\right]$$
$$= 21.35^\circ$$

$$H_{SR} = \cos^{-1}(-\tan L \tan \delta)$$
$$= \cos^{-1}(-\tan 37.73^\circ \tan 21.35^\circ) = 107.6^\circ = 1.878 \text{ radians}$$

$$\overline{I}_0 = \left(\frac{24}{\pi}\right) \text{SC}\left[1 + 0.034\cos\left(\frac{360n}{365}\right)\right](\cos L \cos\delta \sin H_{SR} + H_{SR}\sin L \sin\delta)$$
$$= \left(\frac{24}{\pi}\right) 1.37\left[1 + 0.034\cos\left(\frac{360\cdot 197}{365}\right)^\circ\right](\cos 37.73 \cos 21.35^\circ \sin 107.6^\circ$$
$$+ 1.878 \sin 37.73^\circ \sin 21.35^\circ)$$
$$= 11.34 \text{ kWh/m}^2\text{-day}$$

$$K_T = \frac{\overline{I}_H}{\overline{I}_0} = \frac{7.32 \text{ kWh/m}^2\cdot\text{day}}{11.34 \text{ kWh/m}^2\cdot\text{day}} = 0.645$$

$$\frac{\overline{I}_{DH}}{\overline{I}_H} = 1.390 - 4.027K_T + 5.531{K_T}^2 - 3.108{K_T}^3$$
$$= 1.390 - 4.027\ (0.645) + 5.531\ (0.645)^2 - 3.108\ (0.645)^3 = 0.258$$

Solution- Details (Continued)

$$\overline{I}_{DH} = 0.258 \cdot 7.32 = 1.89 \text{ kWh/m}^2\text{-day}$$

$$\overline{I}_{DC} = \overline{I}_{DH}\left(\frac{1+\cos\Sigma}{2}\right) = 1.89\left(\frac{1+\cos 30^\circ}{2}\right) = 1.76 \text{ kWh/m}^2\text{-day}$$

$$\overline{I}_{RC} = \rho\,\overline{I}_{H}\left(\frac{1-\cos\Sigma}{2}\right) = 0.2 \cdot 7.32\left(\frac{1-\cos 30^\circ}{2}\right) = 0.10 \text{ kWh/m}^2\text{-day}$$

$$\overline{I}_{BH} = \overline{I}_{H} - \overline{I}_{DH} = 7.32 - 1.89 = 5.43 \text{ kWh/m}^2\text{-day}$$

$$\begin{aligned} H_{SRC} &= \min\{\cos^{-1}(-\tan L \tan\delta),\ \cos^{-1}[-\tan(L-\Sigma)\tan\delta]\} \\ &= \min\{\cos^{-1}(-\tan 37.73^\circ \tan 21.35^\circ),\ \cos^{-1}[-\tan(37.73-30)^\circ \tan 21.35^\circ]\} \\ &= \min\{107.6^\circ, 93.0^\circ\} = 93.0^\circ = 1.624 \text{ radians} \end{aligned}$$

$$\begin{aligned} \overline{R}_B &= \frac{\cos(L-\Sigma)\cos\delta\sin H_{SRC} + H_{SRC}\sin(L-\Sigma)\sin\delta}{\cos L\cos\delta\sin H_{SR} + H_{SR}\sin L\sin\delta} \\ &= \frac{\cos(37.73-30)^\circ\cos 21.35^\circ\sin 93^\circ + 1.624\sin(37.73-30)^\circ\sin 21.35^\circ}{\cos 37.73^\circ\cos 21.35^\circ\sin 107.6^\circ + 1.878\sin 37.73^\circ\sin 21.35^\circ} \\ &= 0.893 \end{aligned}$$

$$\overline{I}_{BC} = \overline{I}_{BH}\overline{R}_B = 5.43 \cdot 0.893 = 4.85 \text{ kWh/m}^2\text{-day}$$

$$\overline{I}_{C} = \overline{I}_{BC} + \overline{I}_{DC} + \overline{I}_{RC} = 4.85 + 1.76 + 0.10 = 6.7 \text{ kWh/m}^2\text{-day}$$

Spreadsheet

B82 Insolation on Tilted Collector.xlsx

I2 =ACOS(-TAN(F2*3.14/180)*TAN(G2*3.14/180))

Insolation on a tilted collector.xlsx

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1		n	SC	IH	Σ	L	δ	ρ	HSR	Io	KT	IDH/IH	IDH	IDC	IRC	IBH	HSRC1	HSRC2	HSRC	RBn	RBd	RB	IBC	IC
2	Oakland	197	1.37	7.32	30	37.73	21.4	0.2	1.878	11.35	0.645	0.2596	1.8999	1.773	0.098	5.42	1.8779	1.6239	1.624	1.001	1.12	0.89	4.842	6.71292
3		22	1.37	2.3	30	37.73	-20.1	0.2	1.284	4.784	0.481	0.387	0.89	0.83	0.031	1.41	1.2841	1.5212	1.284	0.833	0.44	1.88	2.653	3.5145
4	Seoul	197	1.37	7.32	30	37.5	21.4	0.2	1.875	11.35	0.645	0.2596	1.8999	1.773	0.098	5.42	1.8753	1.6223	1.622	0.999	1.12	0.89	4.833	6.70394
5		22	1.37	2.3	30	37.5	-20.1	0.2	1.287	4.823	0.477	0.3903	0.8978	0.838	0.031	1.402	1.2865	1.5227	1.287	0.836	0.45	1.87	2.626	3.49448
6																								
7																								
8																								
9																								
10																								

S2 =MIN(Q2,R2)

Insolation on a tilted collector.xlsx

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
1		n	SC	IH	Σ	L	δ	ρ	HSR	Io	KT	IDH/IH	IDH	IDC	IRC	IBH	HSRC1	HSRC2	HSRC	RBn	RBd	RB	IBC	IC
2	Oakland	197	1.37	7.32	30	37.73	21.4	0.2	1.878	11.35	0.645	0.2596	1.8999	1.773	0.098	5.42	1.8779	1.6239	1.624	1.001	1.12	0.89	4.842	6.71292
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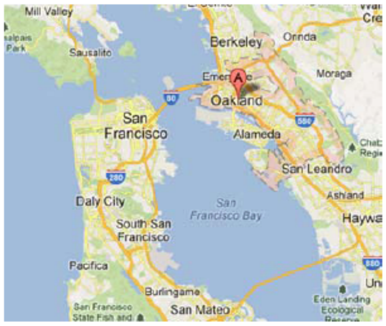
6																								
7																								
8																								

MathCad Solution

Insolation per day.xmcd   Charles Kim 2013

Q. Estimate the insolation on a south-facing collector at a tilt angle of 30 degress in Oakland, CA (latitude 37.73 degrres N). Assume that (1) Avrage horizontal Insolation (IH) in July is 7.32 kWh.m^2-day, and (2) ground reflectivity os 0.2



- Average horizontal insolation (I_H) in Oakland, California (latitude 37.73°N) in July is 7.32 kWh/m²-day. Assume ground reflectivity of 0.2.
- **Question**: Estimate the insolation on a south-facing collector at a tilt angle of 30° with respect to the horizontal.

2b27 Insolation per day.xmcd

$\rho := 0.2$
Ground Reflectivity

$L := 37.73$ Latitude   Try to change this angle for your city.

$IH := 7.32$

$S := 30$ tilted angle in degree

SOLUTION

0. Target $\quad \bar{I}_C = \bar{I}_H\left(1 - \frac{\bar{I}_{DH}}{\bar{I}_H}\right)\cdot \bar{R}_B + \bar{I}_{DH}\left(\frac{1+\cos\Sigma}{2}\right) + \rho \bar{I}_H\left(\frac{1-\cos\Sigma}{2}\right)$ Sunrise Hour Angle

1. Sun declination angle

$n := 197$ n for July 16

$\delta(x) := 23.45 \times \sin\left[\frac{360}{365}\times(x-81)\times deg\right]$

MathCad

$\delta(n) = 21.3537$

$$\boxed{H_{SR} = \cos^{-1}(-\tan L \tan\delta)}$$

2. Sunrise Hour Angle(HSR)

$HSR := acos(-\tan(L\times deg)\times \tan(\delta(n)\times deg)) = 1.8781$ radians

Solar Constant (SC)

3. Extraterrestrial Insolation (Io) with SC=1.37 kW/m^2 $SC := 1.37$ kW / m^2

$$\bar{I}_0 = \left(\frac{24}{\pi}\right)\mathrm{SC}\left[1 + 0.034\cos\left(\frac{360n}{365}\right)\right](\cos L\cos\delta\sin H_{SR} + H_{SR}\sin L\sin\delta)$$

$Io := \left(\frac{24}{\pi}\right)\times SC\times\left(1+0.034\times\cos\left(\frac{360\times n}{365}\times deg\right)\right)\times(\cos(L\times deg)\times\cos(\delta(n)\times deg)\times\sin(HSR) + HSR\times\sin(L\times deg)\times\sin(\delta(n)\times deg)) = 11.34$

4. Clearness Index (KT): KT = IH/Io

$KT := \frac{IH}{Io} = 0.6454$

5. Horizontal Diffuse Radiation (IDH):

$$\frac{\bar{I}_{DH}}{\bar{I}_H} = 1.390 - 4.027K_T + 5.531K_T^2 - 3.108K_T^3$$

$IDH := (1.390 - 4.027\times KT + 5.531\times KT^2 - 3.108\times KT^3)\times IH = 1.8982$

6. Diffuse Ration on the Collector (IDC):

$$\bar{I}_{DC} = \bar{I}_{DH}\left(\frac{1+\cos\Sigma}{2}\right)$$

$IDC := IDH\times\frac{(1+\cos(S\times deg))}{2} = 1.7710$ kWh/m^2-day

MathCad

7. Reflected Radiation on the collector (IRC)

$$\overline{I}_{RC} = \rho \overline{I}_H \left(\frac{1-\cos\Sigma}{2}\right)$$

$IRC := \rho \times IH \times \frac{(1-\cos(S \times deg))}{2} = 0.0981$ kWh/m^2-day

8. Horizontal Beam Radiation (IBH):

$$\overline{I}_H = \overline{I}_{DH} + \overline{I}_{BH} \rightarrow \overline{I}_{BH} = \overline{I}_H - \overline{I}_{DH}$$

$IBH := IH - IDH = 5.4218$ kWh/m^2-day

9. Sunrise Hour Angle on the Collector (HSRC)

$H_{SR} = \cos^{-1}(-\tan L \tan \delta)$

sunrise hour angle (in radians)

$H_{SRC} = \min\{\cos^{-1}(-\tan L \tan\delta), \cos^{-1}[-\tan(L-\Sigma)\tan\delta]\}$

sunrise hour angle for the collector

$HSRC1 := acos(-\tan(L \times deg) \times \tan(\delta(n) \times deg)) = 1.8781$

$HSRC2 := acos(-\tan(L \times deg - S \times deg) \times \tan(\delta(n) \times deg)) = 1.6239$

$HSRC := \min(HSRC1, HSRC2) = 1.6239$ radians

MathCad

10. Beam Tilt Factor (RB):

$$\overline{R}_B = \frac{\cos(L-\Sigma)\cos\delta\sin H_{SRC} + H_{SRC}\sin(L-\Sigma)\sin\delta}{\cos L\cos\delta\sin H_{SR} + H_{SR}\sin L\sin\delta}$$

$$RB := \frac{\cos(L \times deg - S \times deg) \times \cos(\delta(n) \times deg) \times \sin(HSRC) + HSRC \times \sin(L \times deg - S \times deg) \times \sin(\delta(n) \times deg)}{\cos(L \times deg) \times \cos(\delta(n) \times deg) \times \sin(HSR) + HSR \times \sin(L \times deg) \times \sin(\delta(n) \times deg)} = 0.8934$$

11. Beam Radiation on the Collector (IBC):

$$I_{BC} = I_{BH}\left(\frac{\cos\theta}{\sin\beta}\right) = I_{BH}R_B$$

$IBC := IBH \times RB = 4.8438$ kWh / m^2-day

12. Total Insolation on the Collector (IC): IC=IBC+IDC+IRC

$IC := IBC + IDC + IRC = 6.7129$ kWh/m^2-day

2b27 Insolation per day.xmcd

Calculation is complex, so we need

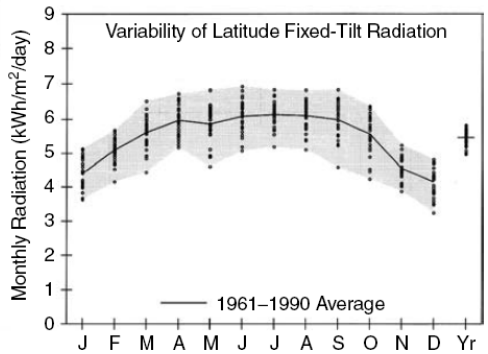
- Spreadsheet or Computer Analysis
- Pre-computed Data such as Solar Radiation Data Manual for Flat-Place and Concentrating Collectors (NREL, 1994)

Boulder, CO

WBAN NO. 94018

LATITUDE: 40.02° N
LONGITUDE: 105.25° W
ELEVATION: 1634 meters
MEAN PRESSURE: 836 millibars

STATION TYPE: Primary

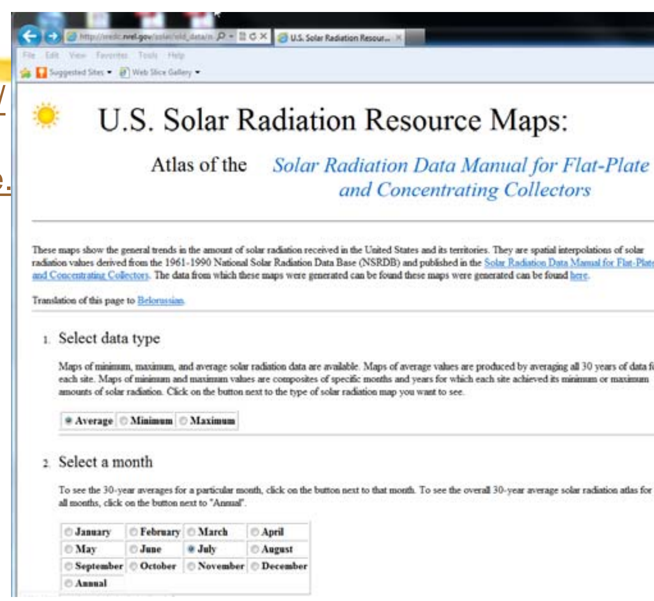


Solar Radiation for Flat-Plate Collectors Facing South at a Fixed Tilt (kWh/m² /day), Uncertainty ±9%

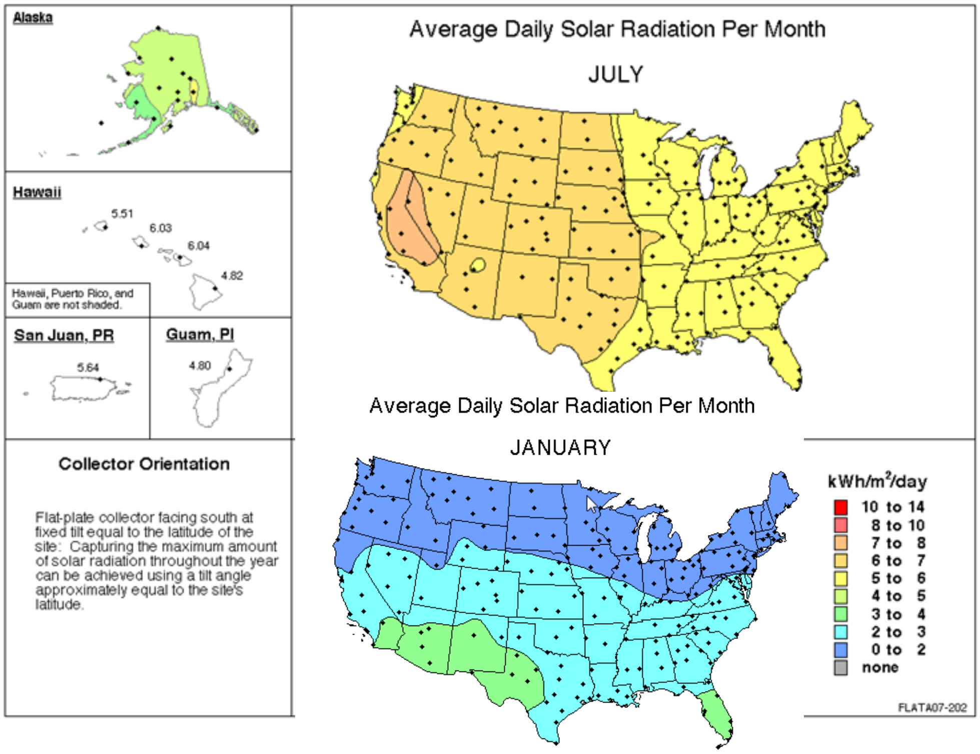
Tilt (°)		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Year
0	Average	2.4	3.3	4.4	5.6	6.2	6.9	6.7	6.0	5.0	3.8	2.6	2.1	4.6
	Min/Max	2.1/2.7	2.8/3.5	3.7/5.0	4.8/6.1	5.1/7.2	5.7/7.8	5.6/7.4	5.2/6.6	4.0/5.5	3.1/4.2	2.3/2.8	1.9/2.3	4.3/4.8
Latitude −15	Average	3.8	4.6	5.4	6.1	6.2	6.6	6.6	6.3	5.9	5.1	4.0	3.5	5.4
	Min/Max	3.2/4.4	3.8/5.1	4.3/6.2	5.3/6.8	4.9/7.3	5.5/7.6	5.6/7.4	5.3/7.1	4.6/6.7	4.0/5.8	3.4/4.6	2.8/4.1	4.9/5.7
Latitude	Average	4.4	5.1	5.6	6.0	5.9	6.1	6.1	6.1	6.0	5.6	4.6	4.2	5.5
	Min/Max	3.6/5.1	4.2/5.7	4.4/6.5	5.2/6.7	4.6/6.8	5.1/6.9	5.2/6.8	5.1/6.8	4.6/6.8	4.2/6.4	3.9/5.2	3.2/4.8	5.0/5.8
Latitude +15	Average	4.8	5.3	5.6	5.6	5.2	5.2	5.3	5.5	5.8	5.7	4.8	4.5	5.3
	Min/Max	3.9/5.6	4.3/5.9	4.4/6.5	4.8/6.2	4.1/6.0	4.4/5.9	4.5/5.9	4.6/6.2	4.4/6.6	4.2/6.5	4.1/5.6	3.5/5.3	4.8/5.6
90	Average	4.5	4.6	4.3	3.6	2.8	2.6	2.7	3.2	4.0	4.6	4.4	4.3	3.8
	Min/Max	3.6/5.4	3.7/5.2	3.5/5.0	3.0/4.0	2.3/3.1	2.2/2.8	2.3/2.9	2.7/3.6	3.1/4.6	3.4/5.3	3.7/5.1	3.4/5.2	3.4/4.1

Daily Total Radiation incident on a tilted surface [kWh/m²/day]

- U. S. Solar Radiation Resource Maps:
- http://rredc.nrel.gov/solar/old_data/nsrdb/1961-1990/redbook/atlas/Table.html
- Data Types
  - Average
  - Minimum
  - Maximum
- Month Selection
- Orientation
  - Flat latitude
  - Flat latitude-15
  - Flat latitude+15
  - Etc
- View Map

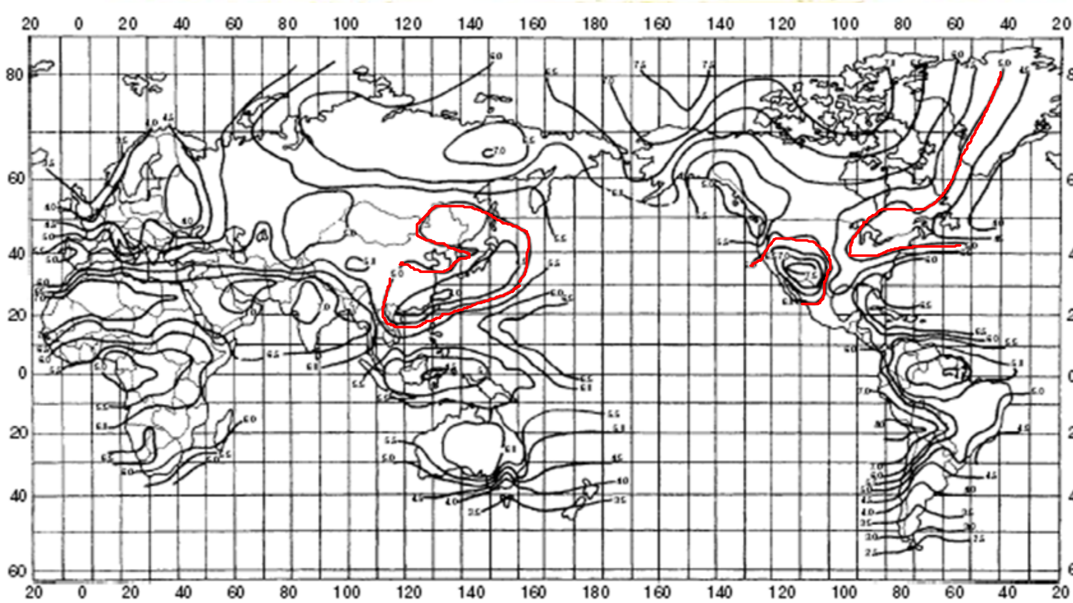


Average Solar Radiation, Jan/July, Flat, South Facing, Tilted Latitude



Daily Total Radiation incident on a tilted surface [kWh/m^2/day]

- Spring

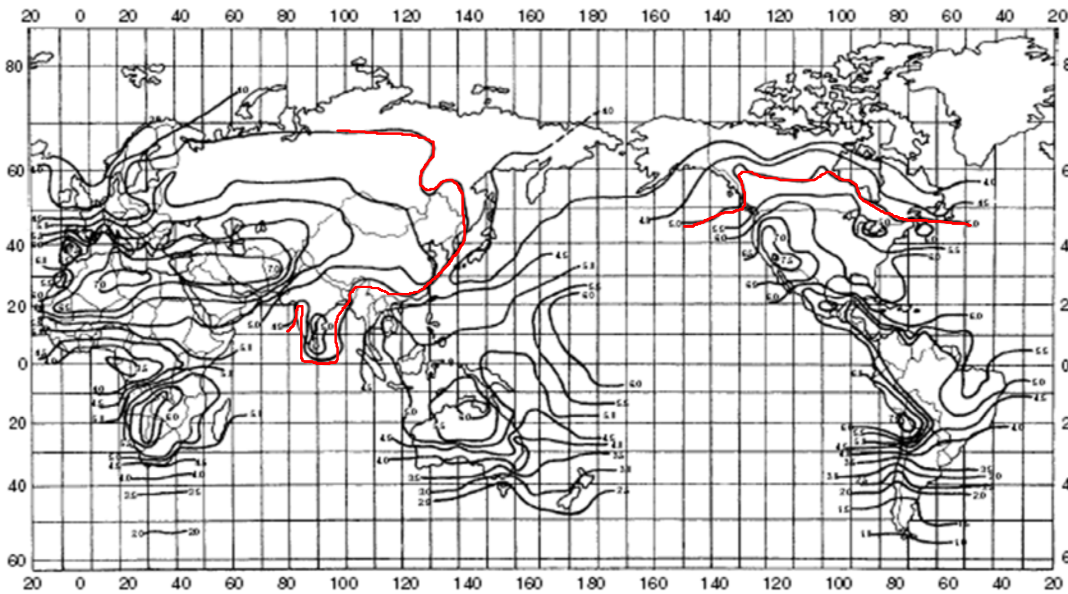


SPRING - Tilt angle equals the latitude angle

Daily total solar radiation incident on a tilted surface in kWH/m^2/day

Daily Total Radiation incident on a tilted surface [kWh/m^2/day]

- Summer

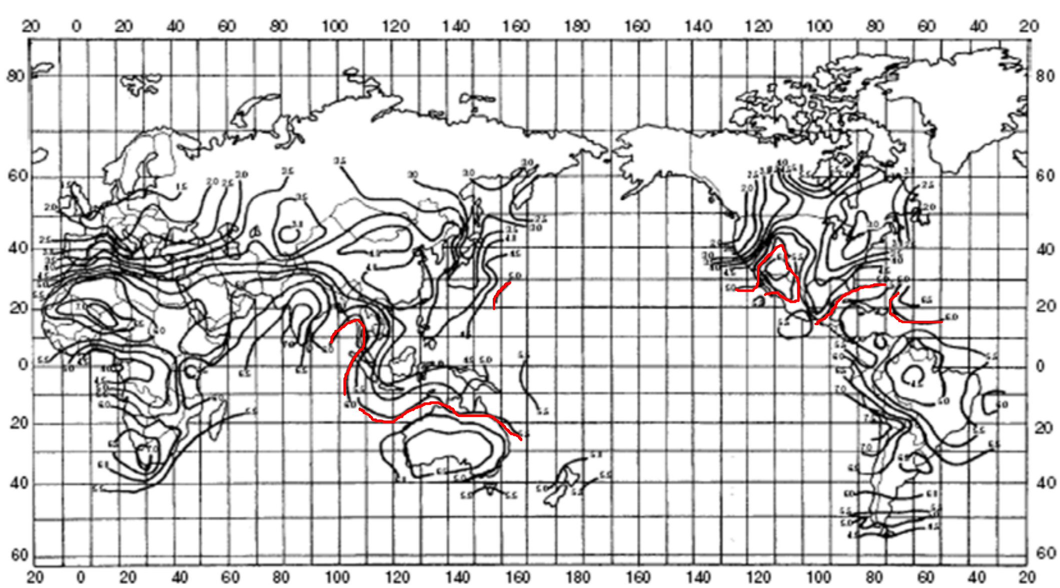


SUMMER - Tilt angle equals the latitude angle

Daily total solar radiation incident on a tilted surface in kWH/m^2/day

Daily Total Radiation incident on a tilted surface [kWh/m^2/day]

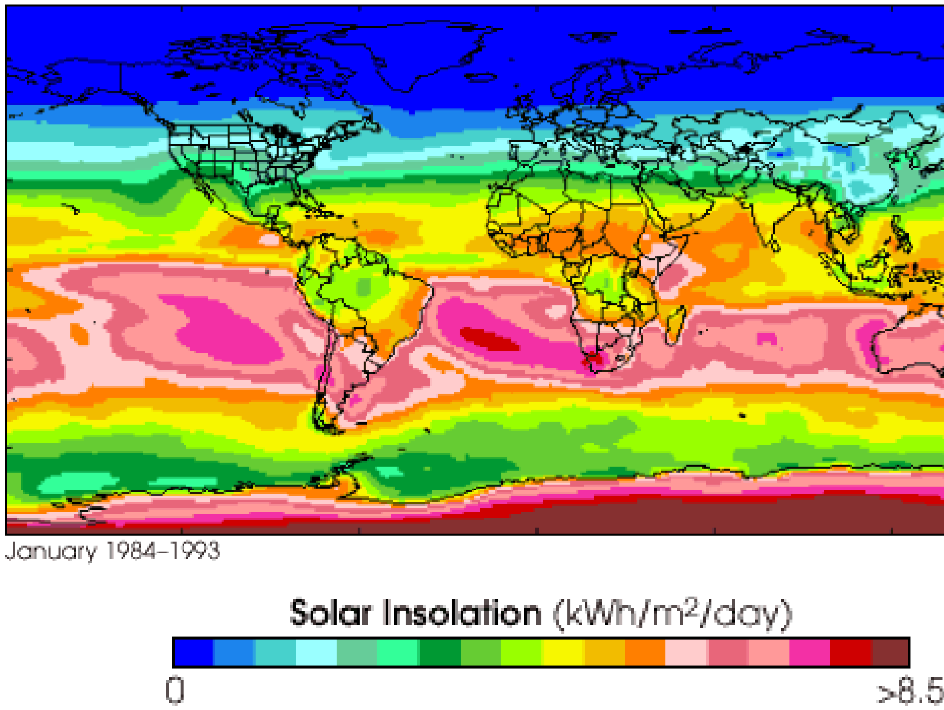
- Winter



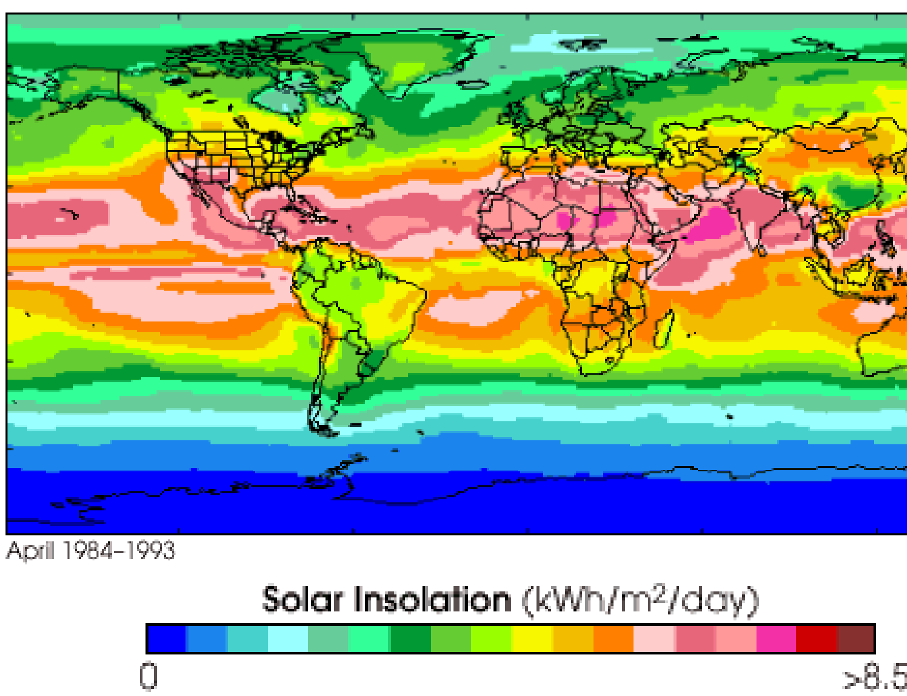
WINTER - Tilt angle equals the latitude angle

Daily total solar radiation incident on a tilted surface in kWH/m^2/day

Solar Insolation Map - January

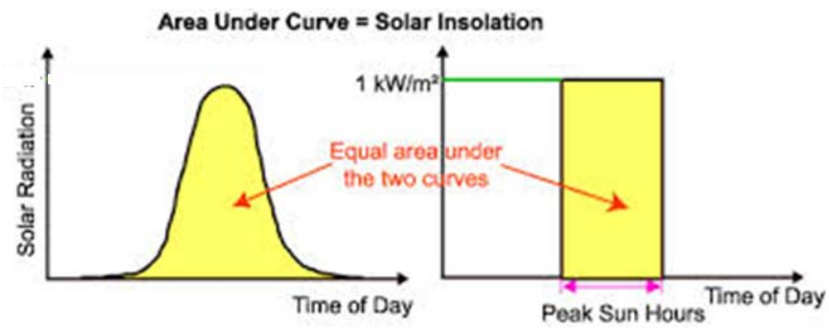


Solar Insolation Map - April



Peak Sun Hours

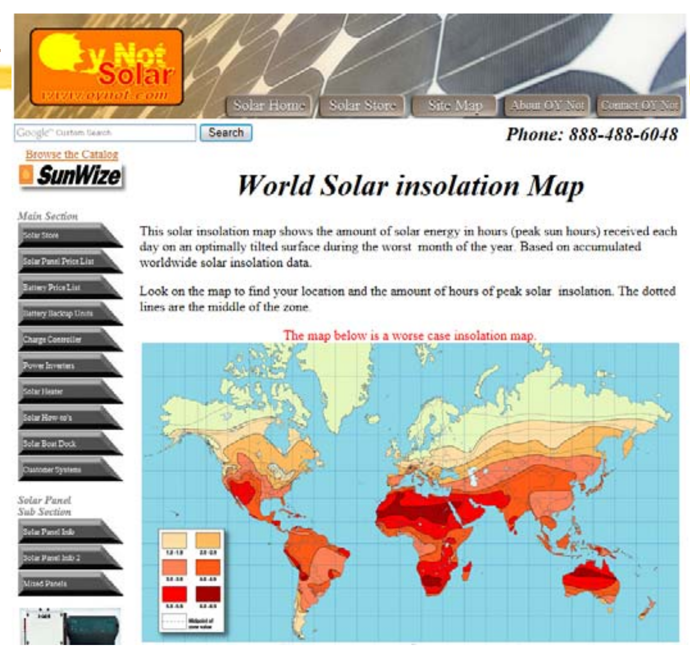
- Much simpler approach for PV
- "Peak Sun Hours": Total number of hours per day in which solar irradiance averages 1 kW/m^2 in worst month.



ENERGY = Rated_Power * Conversion_Efficiency * Peak_Sun_Hour/Day * 365 Day/Year

Peak Sun Hour Map

- http://www.oynot.com/solar-insolation-map.html
- The amount of solar energy in hours ("**peak sun**" hours) received each day on an optimally tilted surface during the **worst month** ("design month") of the year at 1 kW/m^2.



ENERGY = Rated_Power * Conversion_Efficiency * Peak_Sun_Hour/Day * 365 Day/Year

Effect of adjusting the tilt angle

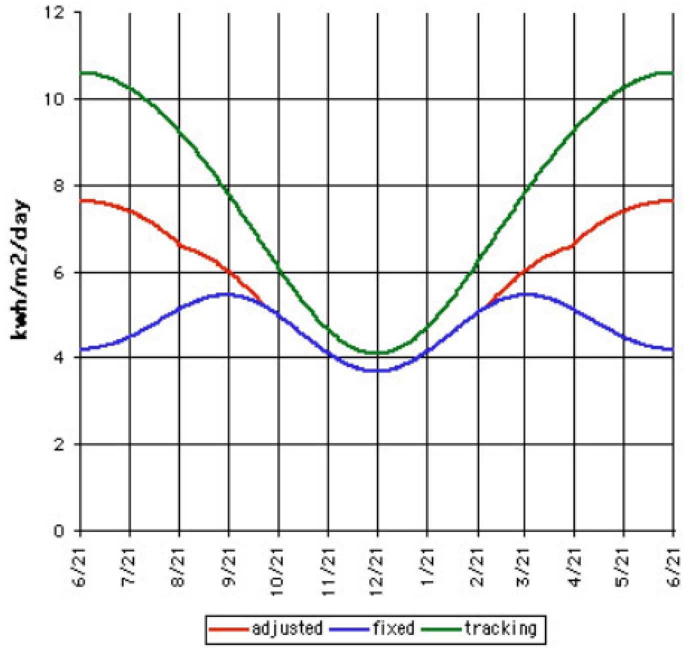
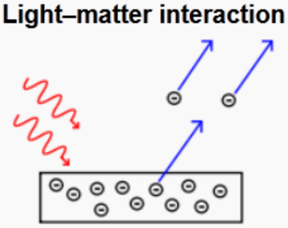


Photo-Electricity

Light–matter interaction



Low-energy phenomena:
Photoelectric effect

Mid-energy phenomena:
Thomson scattering
Compton scattering

High-energy phenomena:
Pair production

V·T·E

In the **photoelectric effect**, electrons are emitted from solids, liquids or gases when they absorb energy from light. Electrons emitted in this manner may be called *photoelectrons*.[1][2]

In 1887, Heinrich Hertz[2][3] discovered that electrodes illuminated with ultraviolet light create electric sparks more easily. In 1905 Albert Einstein published a paper that explained experimental data from the photoelectric effect as being the result of light energy being carried in discrete quantized packets. This discovery led to the quantum revolution. Einstein was awarded the Nobel Prize in 1921 for "his discovery of the law of the photoelectric effect".[4]

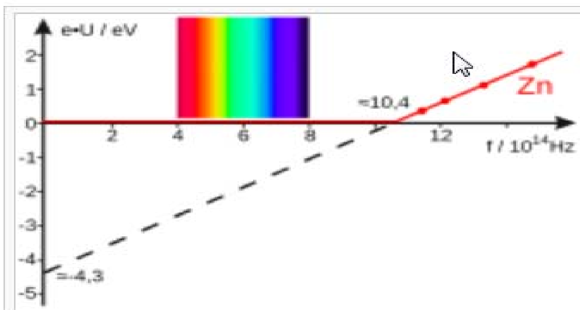
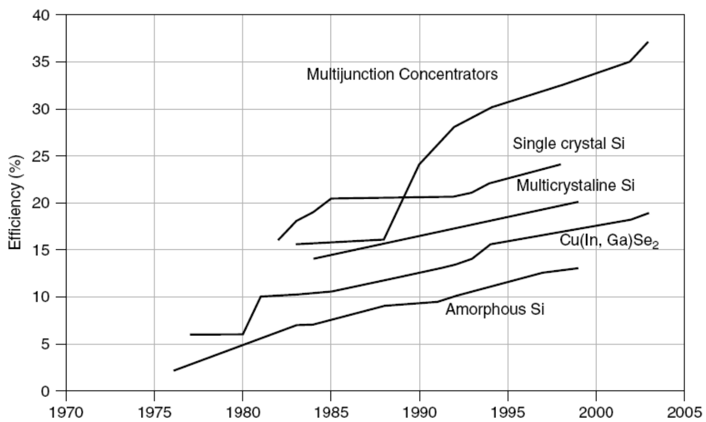


Diagram of the maximum kinetic energy as a function of the frequency of light on zinc

- ⌘ Silicon wafer

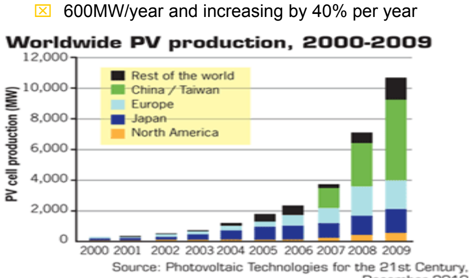
Photovoltaic Material and Electrical Characteristics

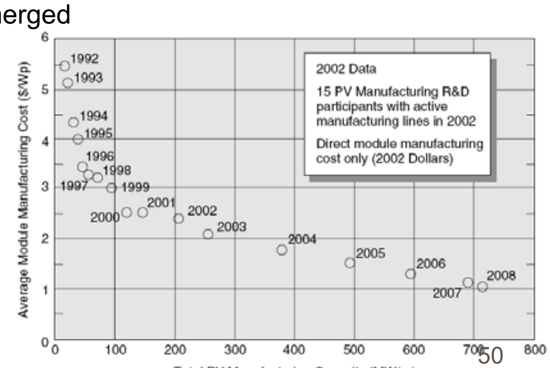
- Photovoltaic (PV): a device that is capable of converting the energy contained in photons of light into an electrical voltage or current
- A photon (short wavelength and high energy) break free electrons from the atoms in the photovoltaic material.
- **"The surface of the earth receives 6000 times as much solar energy as our total energy demand"**
- PV Cell Efficiency



PV History

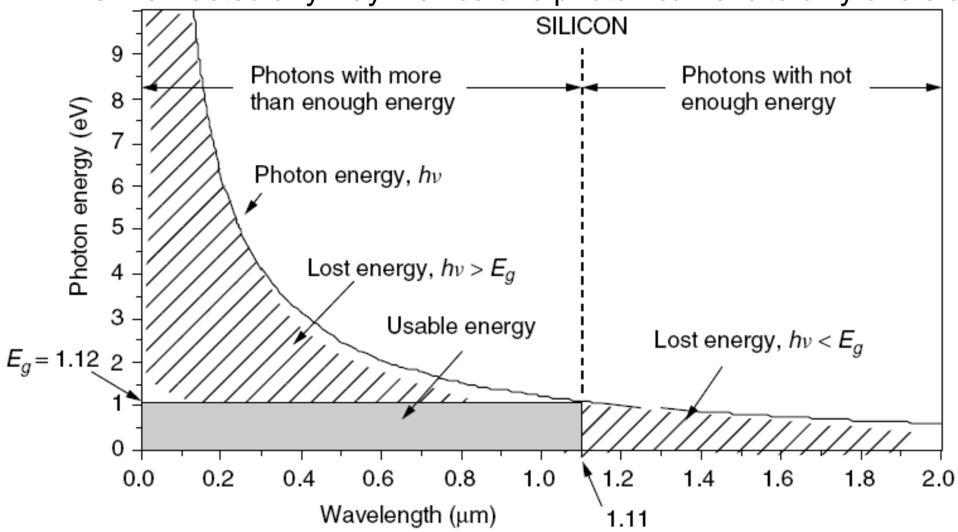
- 1829: Edmund Becquerel – voltage development on an metal electrode under illumination
- 1876: Adams and Day - PV effect on solid – built a cell made of Selenium with 1- 2 % efficiency
- 1904: Albert Einstein – Theoretical explanation of PV effect
- 1904: Czochralski (Polish Scientist) developed a method to grow perfect crystals of silicon →which later in 1940s and 1950s were adopted to make the first generation of single-crystal silicon PV cells, which continues to dominate the PV industry today
- Before 1958: Cost prohibitive
- 1958: Practical PV, used is space for Vanguard I satellite
- 1970s: Oil shock spurred the commercial PV development
- 1980s: High efficiency and low cost PV emerged
- 2002: Worldwide PV production
  - 600MW/year and increasing by 40% per year





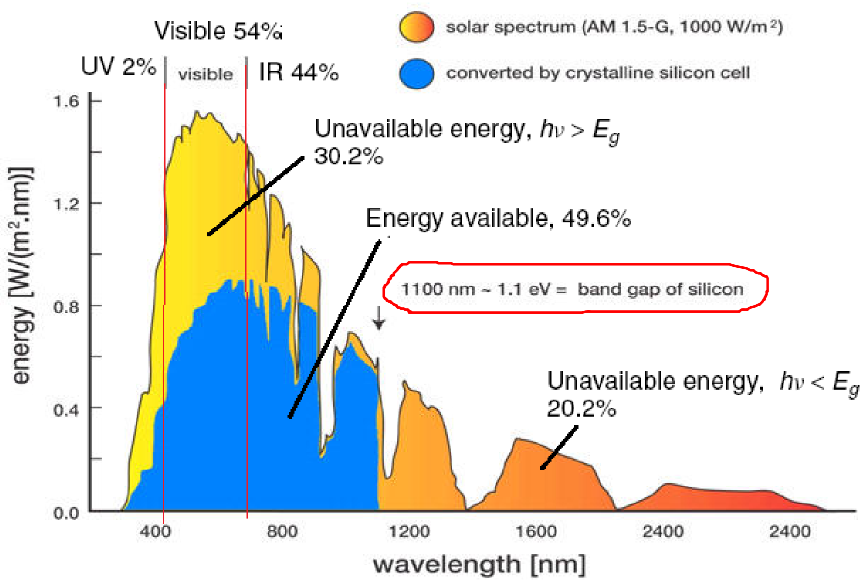
PV Semiconductor Physics

- For Si PV cells, photons with wavelength above **1.11 um** don't have the **1.12 eV** needed to excite an electron, and this energy is lost. Photons with shorter wavelengths have more than enough energy, but any energy above 1.12 eV is wasted any way – since one photon can excite only one electron.



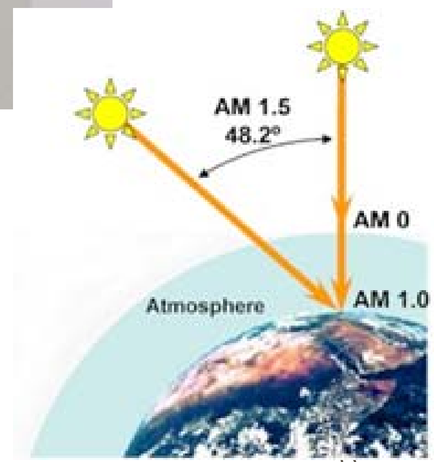
- h – Planck's constant = 6.626^(-34) [J-s]
- v – frequency [Hertz[

Solar Spectrum



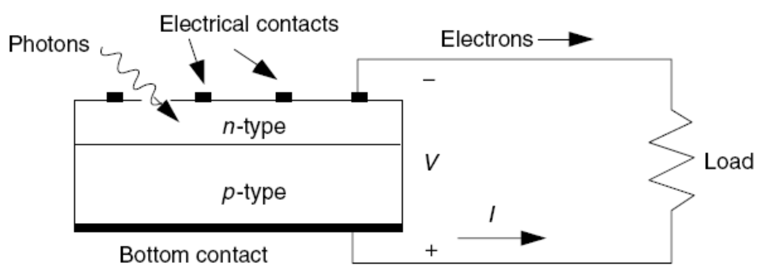
- AM (Air Mass) Ratio
  - AM0: Sun in space (no atmosphere0
  - AM1: Sun is directly overhead
  - AM1.5: Sun is 42 degrees above the horizon (standard condition)

AM Ratio and PV plate

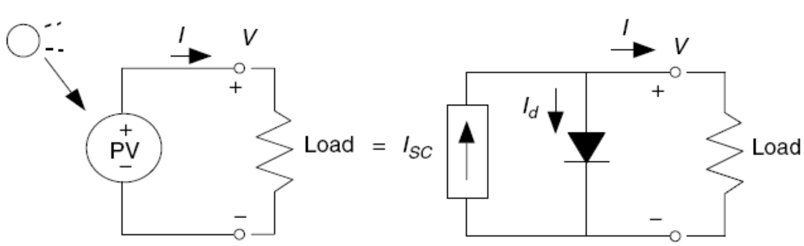


PV Cell Circuit

- Electron flow

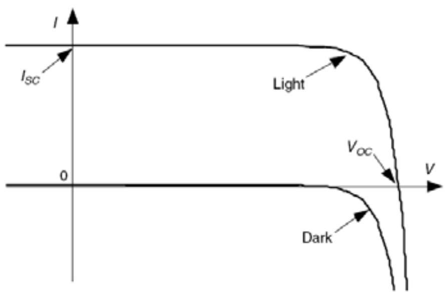


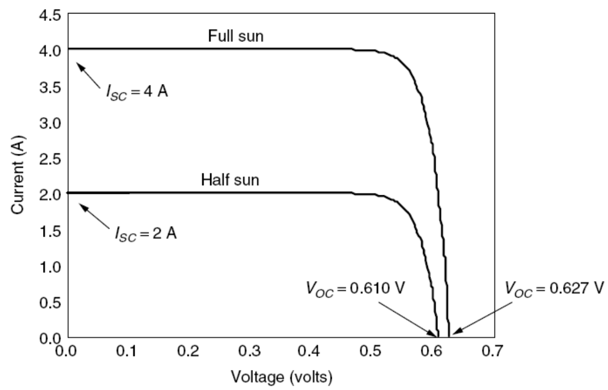
- Equivalent Circuit
  - Current source driven by sunlight in parallel with a real diode



I-V Curve

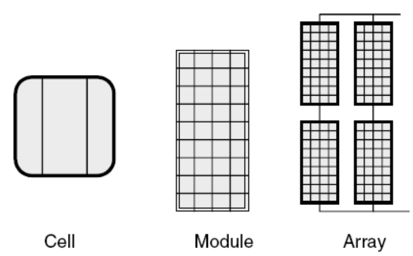
- Isc: Short Circuit Current
- Voc: Open Circuit Voltage
- "Full Sun": Peak Sun ---- solar intensity equal to 1 kW/m².

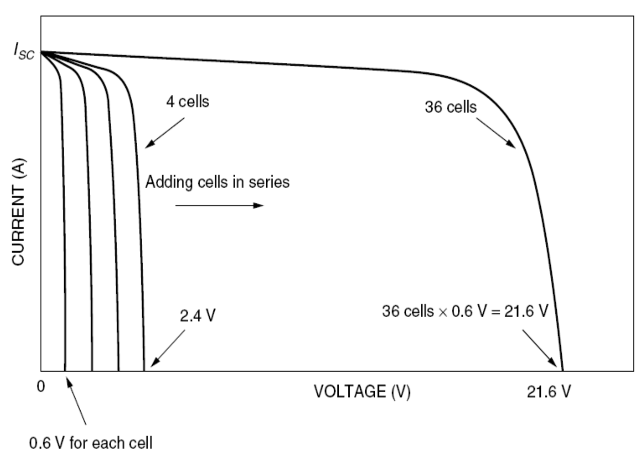




PV Cells, Modules, and Arrays

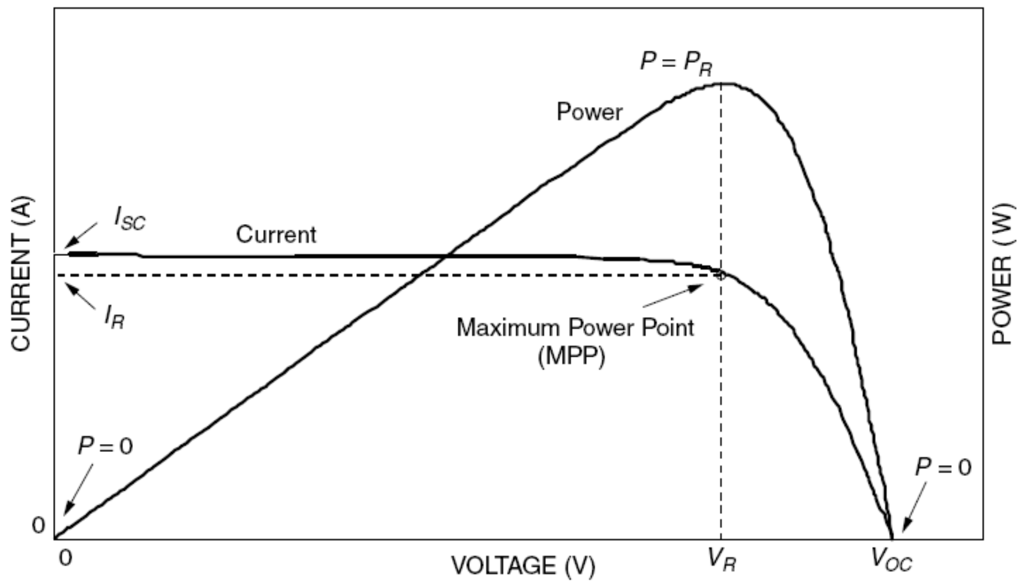
- Individual cell: 0.5V
- Module:
  - 36 cells in series – 12V module
  - 72 cells in series – 24V module
- Parallel – increase current
- Series- increase voltage





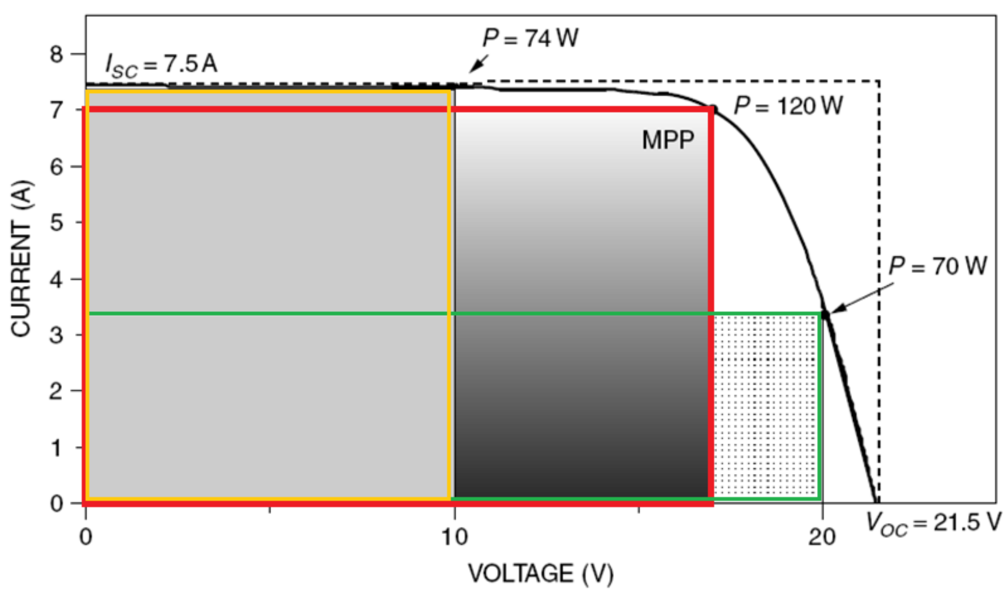
I-V Curve and Power Output

- Maximum Power Point (MPP)
- I_R: Rated Current
- V_R: Rated Voltage



MPP and FF

- The biggest possible rectangle – the area is power
- Fill Factor (FF): performance measure: ratio of the power at MPP to the product of V_{oc} and I_{sc}. (solid_rectangle/dotted_rectangle)

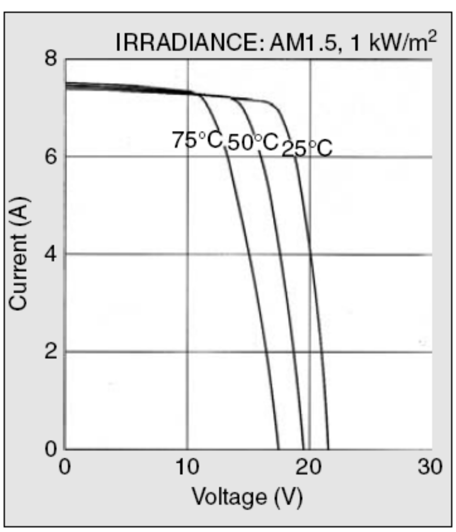


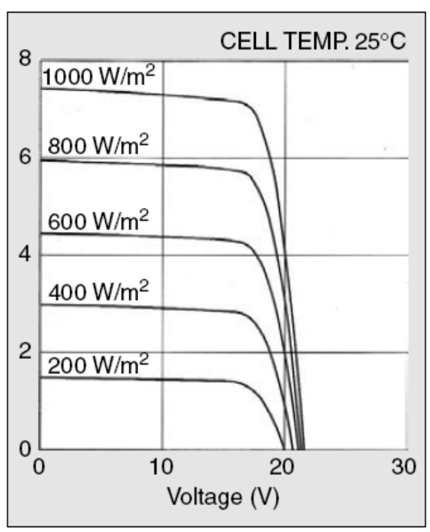
PV Module Performance Examples

Manufacturer	Kyocera	Sharp	BP	Uni-Solar	Shell
Model	KC-120-1	NE-Q5E2U	2150S	US-64	ST40
Material	Multicrystal	Polycrystal	Monocrystal	Triple junction a-Si	CIS-thin film
Number of cells n	36	72	72		42
Rated Power $P_{DC,STC}$ (W)	120	165	150	64	40
Voltage at max power (V)	16.9	34.6	34	16.5	16.6
Current at rated power (A)	7.1	4.77	4.45	3.88	2.41
Open-circuit voltage V_{OC} (V)	21.5	43.1	42.8	23.8	23.3
Short-circuit current I_{SC} (A)	7.45	5.46	4.75	4.80	2.68
Length (mm/in.)	1425/56.1	1575/62.05	1587/62.5	1366/53.78	1293/50.9
Width (mm/in.)	652/25.7	826/32.44	790/31.1	741/29.18	329/12.9
Depth (mm/in.)	52/2.0	46/1.81	50/1.97	31.8/1.25	54/2.1
Weight (kg/lb)	11.9/26.3	17/37.5	15.4/34	9.2/20.2	14.8/32.6
Module efficiency	12.9%	12.7%	12.0%	6.3%	9.4%

Insolation and Temperature Effect

- Decrease in insolation, decrease in short-circuit current
- Increase in cell temperature, substantial decrease in open-circuit voltage, and slight decrease in short-circuit current
- Kyocera 120-W multicrystal-Si module example



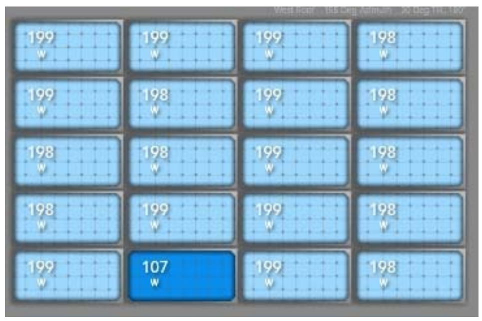


Solar Cell Cooling

Floating Solar Cells

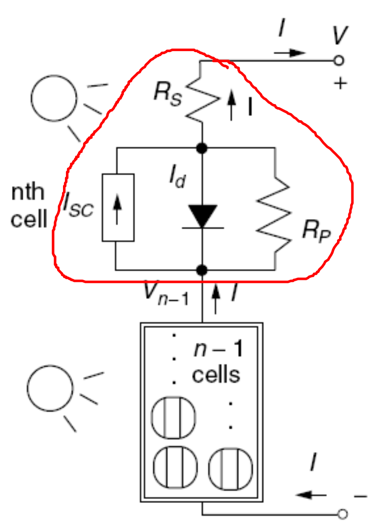
Shading Effect and Bypass Diode

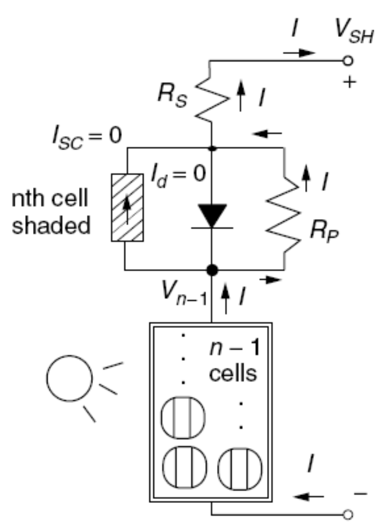
- Output of a PV module can be reduced dramatically when even a small portion of it is shaded.
- Even a single cell under shade in a long string of cells can easily cut output power by more than half.
- External diodes mitigate the impacts of shading



Physics of Shading

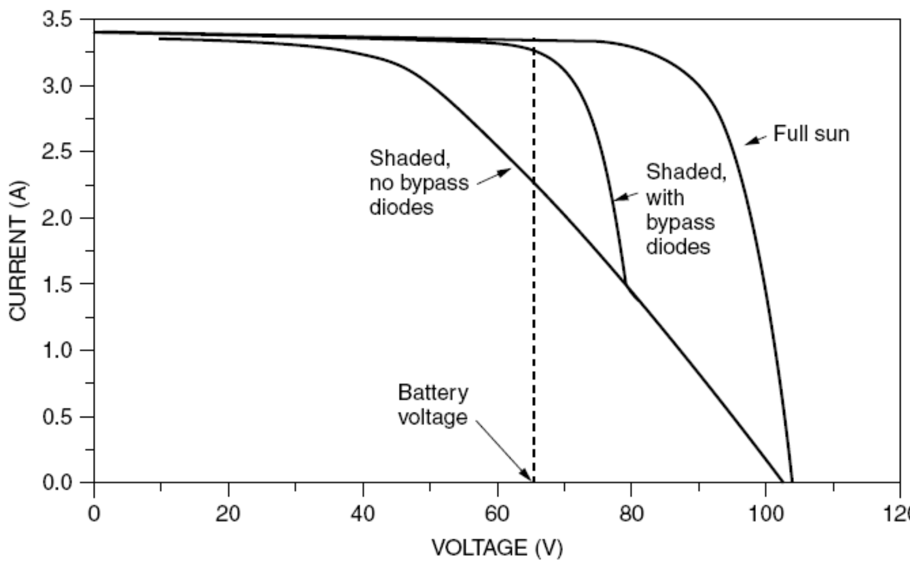
- All cells under sun
  - The same current flows through each cell
- Top cell under shade
  - The current source is reduced to zero for the cell
  - Now the current from other cells must flow through Rp, which drop the voltage, instead of adding voltage.



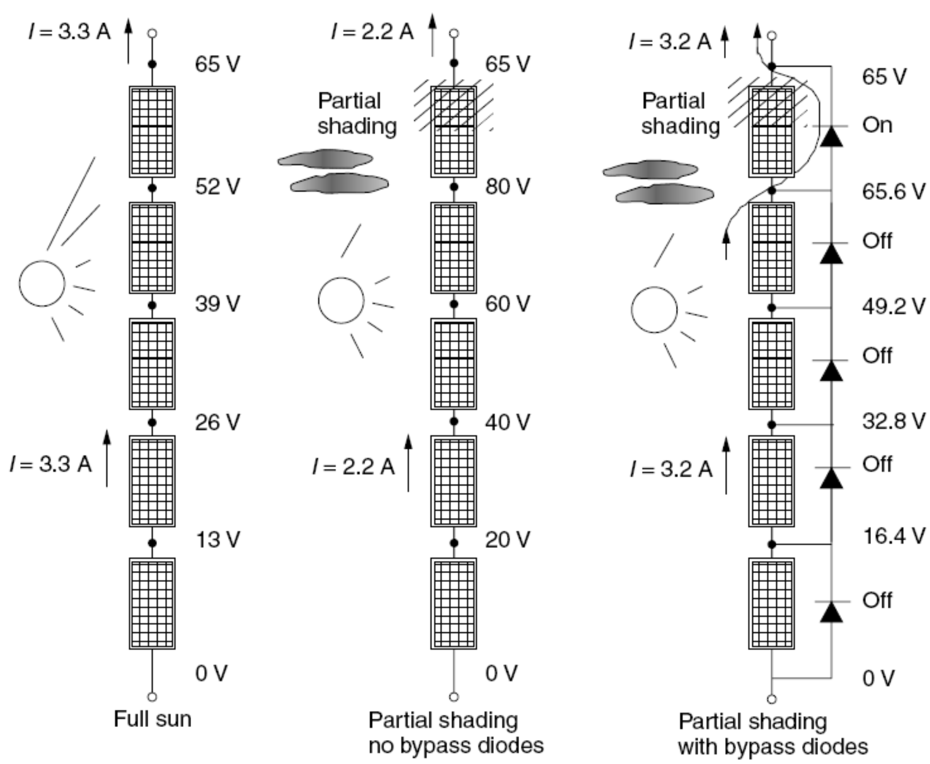


Impact of Bypass Diode

- For a 5 PV modules in series delivering 65V to a battery bank – one module has 2 shaded cells.
- Charging current drops to 2.2A from 3.3A
- With a bypass diode, the current is recovered to 3,2 A

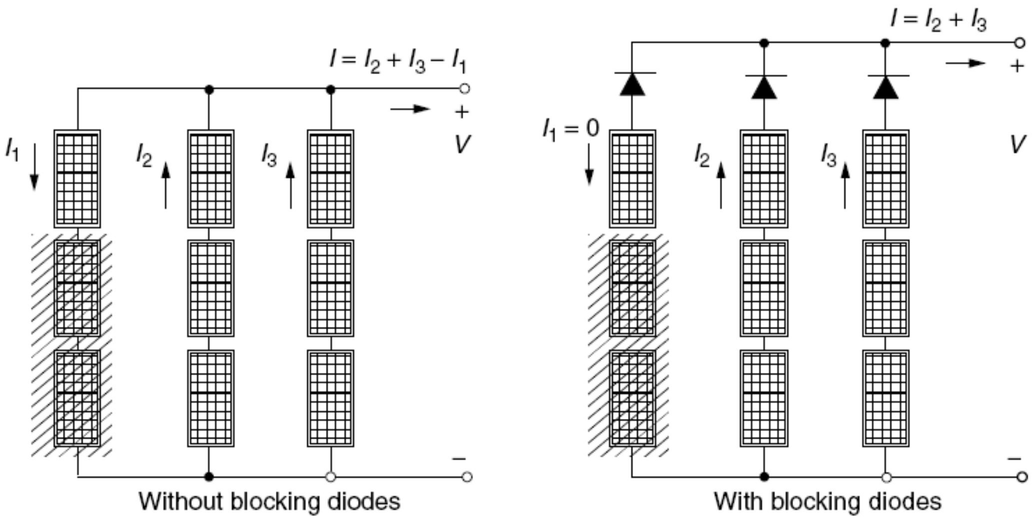


Mitigation by Bypass Diode



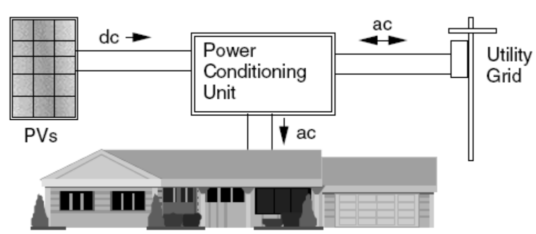
Partial Cell under Performance – Blocking Diode

- In Parallel Combination of strings of cells: Separate the malfunctioning or shaded string of cells by blocking (or "Isolation") diode at the top of each string

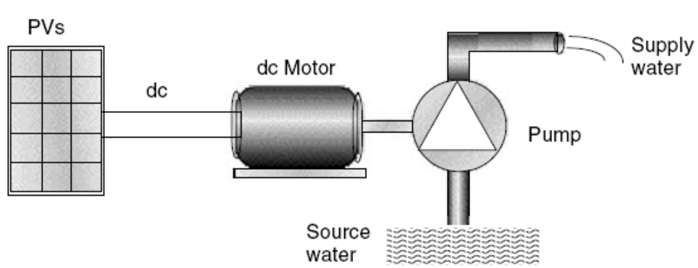


PV System Configurations

- Utility connected PV System: Feed/get power directly from/to the utility grid and PV

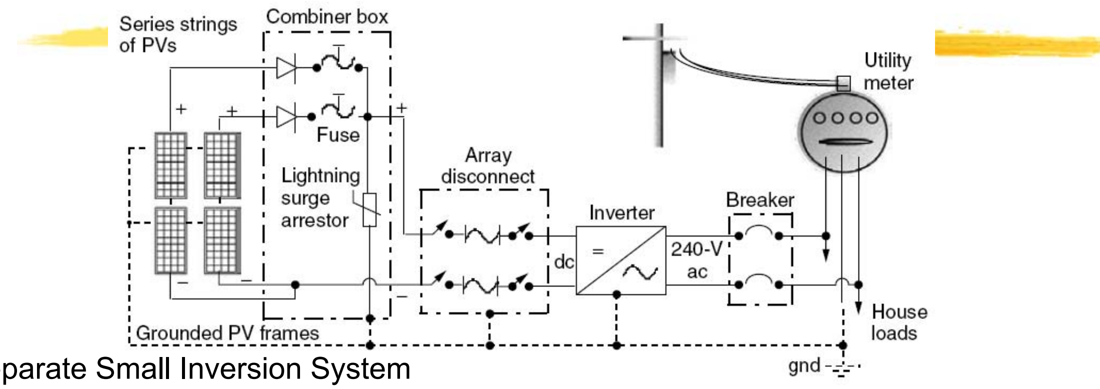


- Stand-alone system: Charge batteries (with or without Generator backup) and serves load

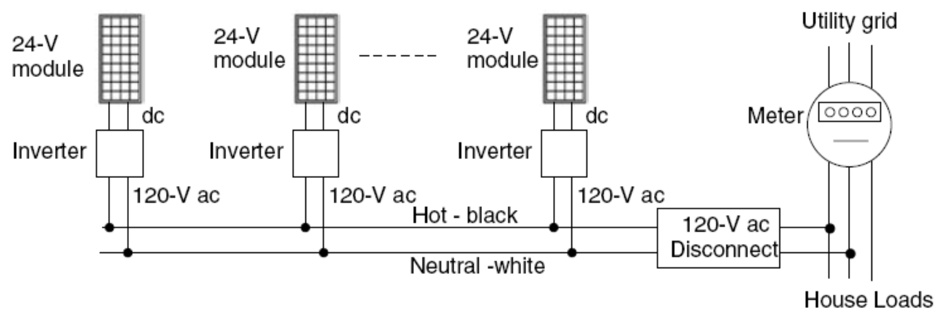


Grid-Connected PV System

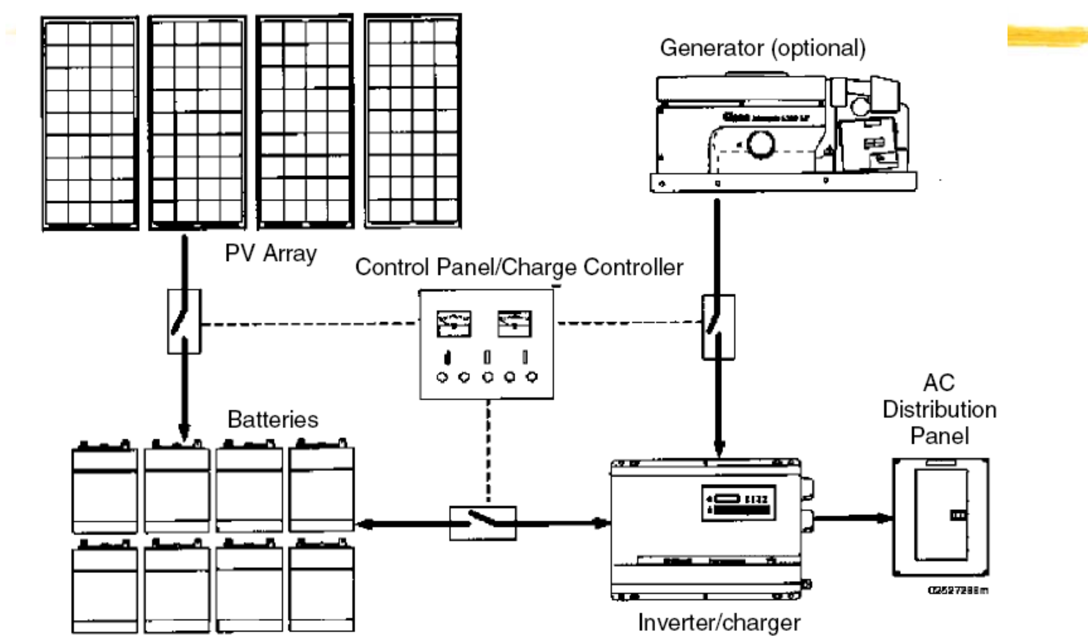
- Combined Inversion system



- Separate Small Inversion System

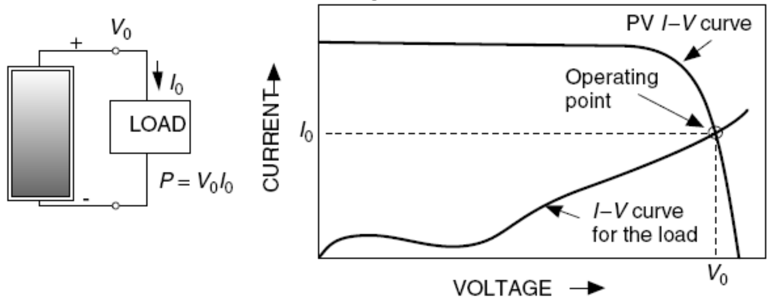


Example Stand-Alone PV System

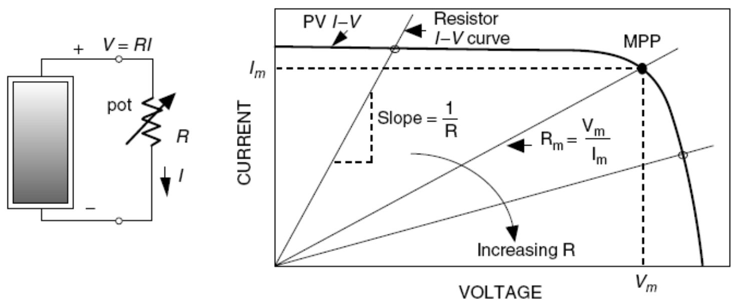


Operating Point

- PV Cell's I-V Curve
- Load's I-V Curve
- The intersection point is the operating point.

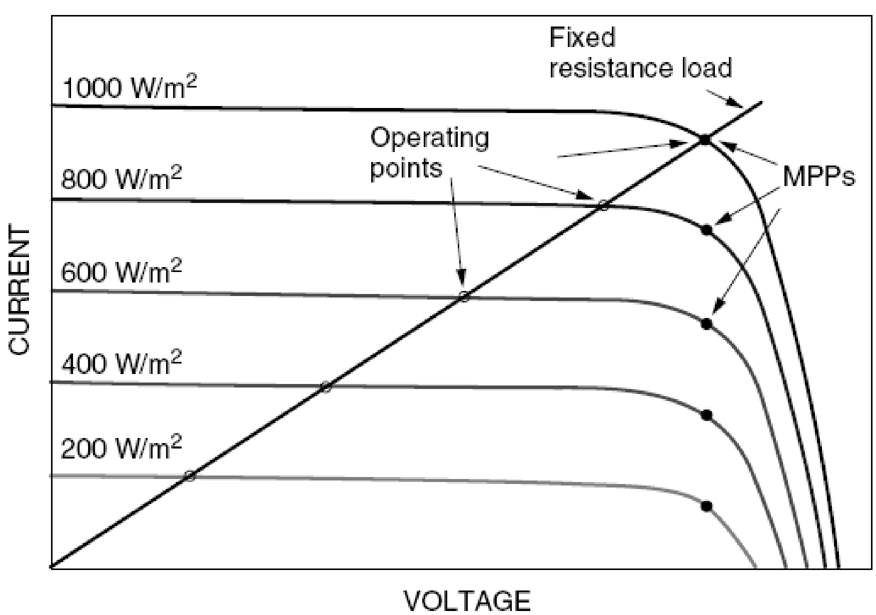


- Changes in Operating Points by the changes in resistance



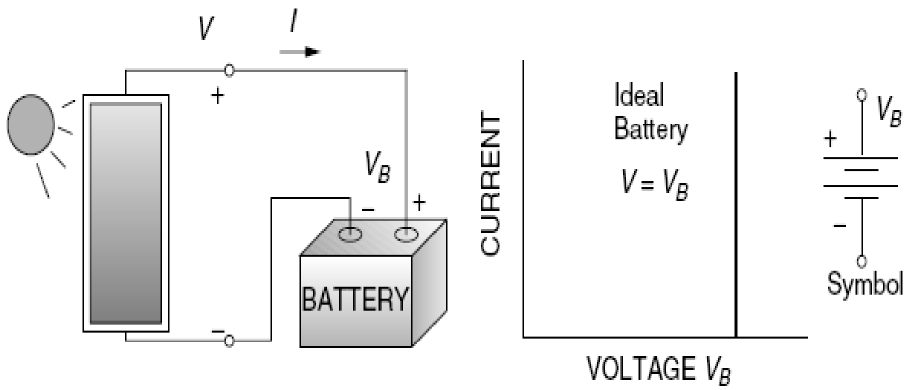
Operating Point Change over Insolation

- With fixed resistance, the operating point moves down off the MPP as the Insolation condition changes and the PV is less efficient



Battery I-V Curve

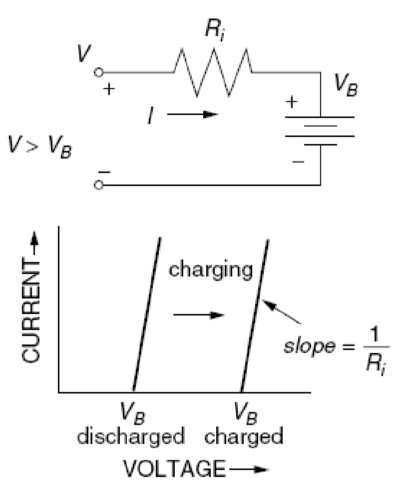
- Ideal: Voltage remains constant no matter how much current is drawn
- I-V Curve: Straight up-and-down line



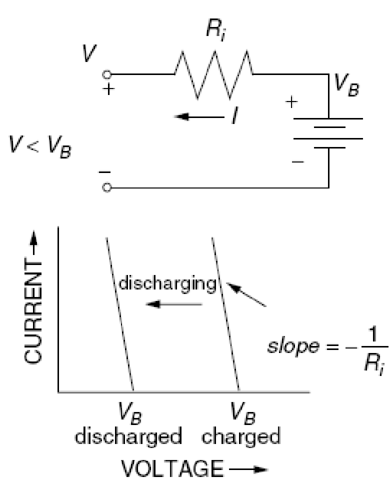
Battery I-V Curve

Real Battery

- Real battery has internal resistance: $V = V_B + R_i * I$
  - Charging: Applied voltage must be bigger than V_B
  - Discharging: Output Voltage is less than V_B.



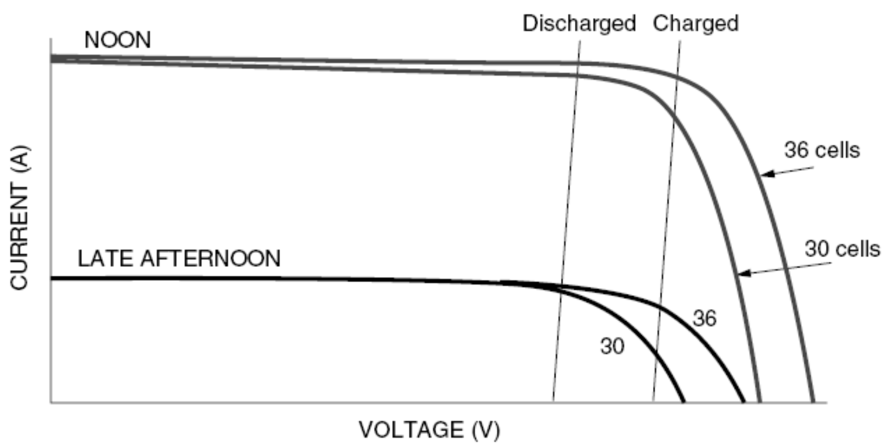
(a) Charging



(b) Discharging

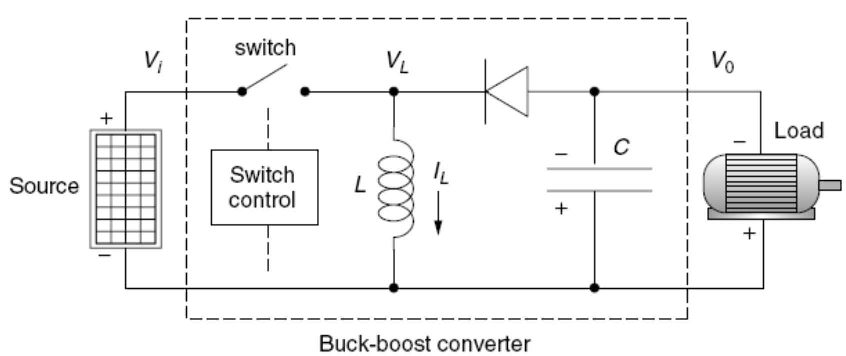
Charging and Discharging

- Charging moves I-V curve toward the right during the day (from PV) → So current lowers and prevents overcharging
- Discharging moves I-V curve toward left during late afternoon (from PV)



Voltage Control

- Benefit of operating PV near the knee (MPP) of the I-V Curve throughout the ever-changing daily conditions
- Conversion of DC voltages → Switched mode dc-to-dc converter {on-off switch to allow current to pass or block}
- Boost Converter: Step-up
- Buck Converter: Step-Down
- Buck-Booster Converter: Combination

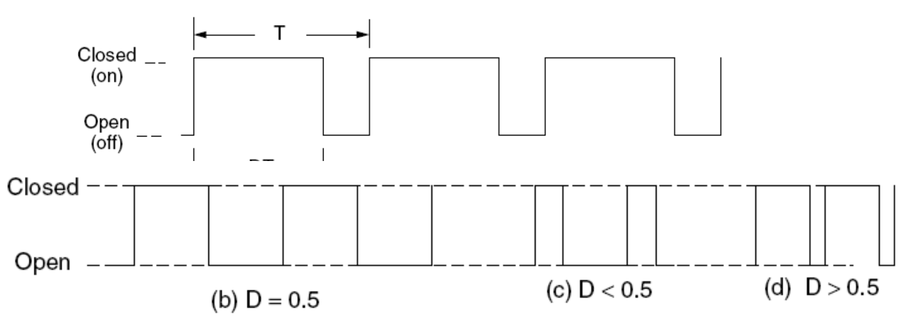


Circuit Operational Principle

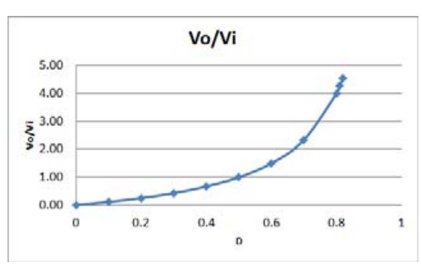
- When the switch is closed, the input voltage V_i is applied across the inductor, driving current I_L through the inductor. All of the source current goes through the inductor since the diode blocks any flow to the rest of the circuit. During this portion of the cycle, energy is being added to the magnetic field in the inductor as current builds up. If the switch stayed closed, the inductor would eventually act like a short-circuit and the PVs would deliver short-circuit current at zero volts.
- When the switch is opened, current in the inductor continues to flow as the magnetic field begins to collapse (remember that current through an inductor cannot be changed instantaneously—to do so would require infinite power). Inductor current now flows through the capacitor, the load, and the diode. Inductor current charging the capacitor provides a voltage (with a polarity reversal) across the load that will help keep the load powered after the switch closes again.
- If the switch is cycled quickly enough, the current through the inductor doesn't have a chance to drop much while the switch is open before the next jolt of current from the source. With a fast enough switch and a large enough inductor, the circuit can be designed to have nearly constant inductor current. That's our first important insight into how this circuit works: Inductor current is essentially constant.
- If the switch is cycled quickly enough, the voltage across the capacitor doesn't have a chance to drop much while the switch is closed before the next jolt of current from the inductor charges it back up again. Capacitors, recall, can't have their voltage change instantaneously so if the switch is cycling fast enough and the capacitor is sized large enough, the output voltage across the capacitor and load is nearly constant. We now have our second insight into this circuit: Output voltage *Vo* is essentially constant (and opposite in sign to *Vi*).

Input – Output Voltage by Duty Cycle

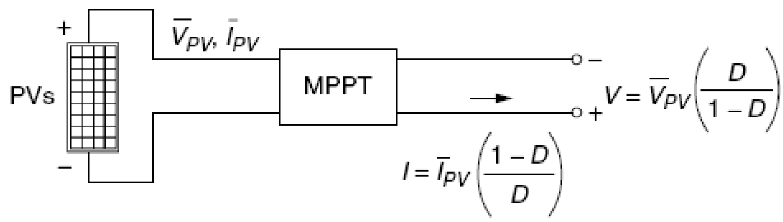
- The duty cycle of the switch itself controls the relationship between the input and output voltages of the converter.
- The duty cycle *D* ($0 < D < 1$) is the fraction of the time that the switch is closed. This variation in the fraction of time the switch is in one state or the other is referred to as *pulse-width modulation* (PWM).

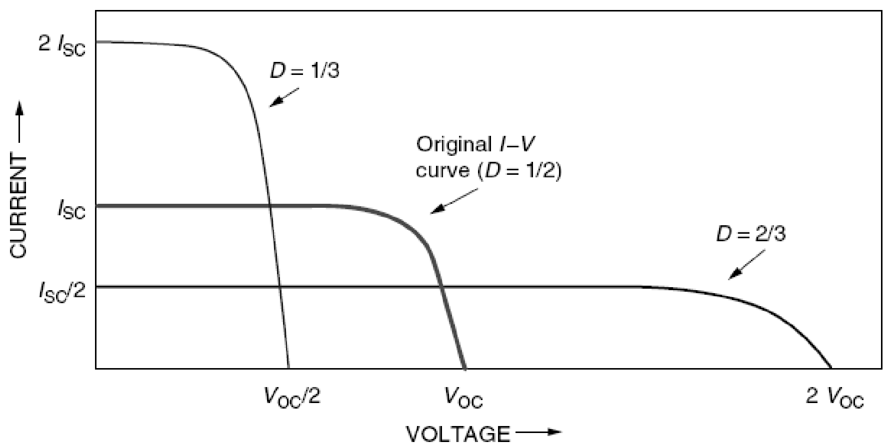


$$\frac{V_0}{V_i} = -\left(\frac{D}{1-D}\right)$$



MPPT and PV I-V with Duty Cycle





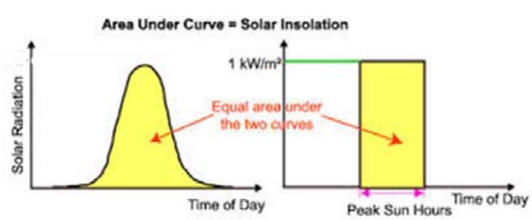
Estimation of PV Performance

- "1-sun" ("peak sun hour")of insolation is defined as 1 kW/m^2
- (EX)5.6 kWh/m^2-day = 5.6 h/day of 1-sun = 5.6 h of "peak sun"
- P_{ac} =AC power delivered by an array under 1-sun insolation.
- Daily kWh delivered = [rated AC power]*[number of hours of peak sun]

$$\text{Energy (kWh/day)} = \text{Insolation}\left(\frac{\text{kWh/m}^2}{\text{day}}\right)\cdot A\ (\text{m}^2)\cdot\bar{\eta}$$

A is the area of the PV array

$\bar{\eta}$ is the average system efficiency over the day.



DC power from the system 1-sun of insolation,

$$P_{dc}\,(\text{kW}) = \left(\frac{1\ \text{kW}}{\text{m}^2}\right)\cdot A\ (\text{m}^2)\cdot \eta_{1-\text{sun}}$$

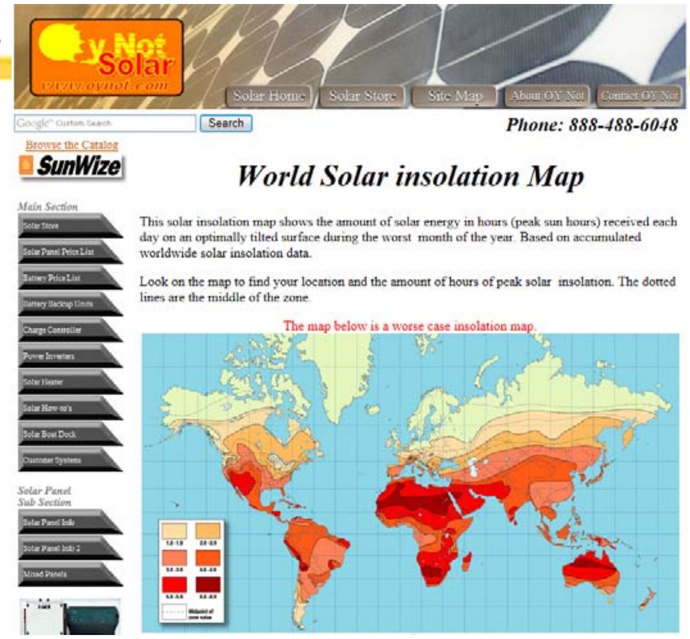
η_{1-sun} is the system efficiency at 1-sun.

$$P_{ac}(\text{kW}) = P_{dc}\,(\text{kW})\cdot \text{[Dc-AC Conversion Efficiency]}$$

$$\text{Energy (kWh/day)} = P_{ac}(\text{kW})\cdot(\text{h/day of "peak sun"})$$

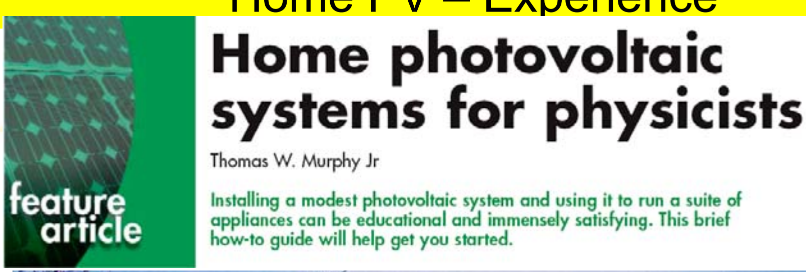
REMINDER ---- Peak Sun Map

- http://www.oynot.com/solar-insolation-map.html
- The amount of solar energy in hours ("**peak sun**" hours) received each day on an optimally tilted surface during the **worst month** ("design month") of the year.



ENERGY = Rated_Power * Conversion_Efficiency * Peak_Sun_Hour/Day * 365 Day/Year

Home PV – Experience



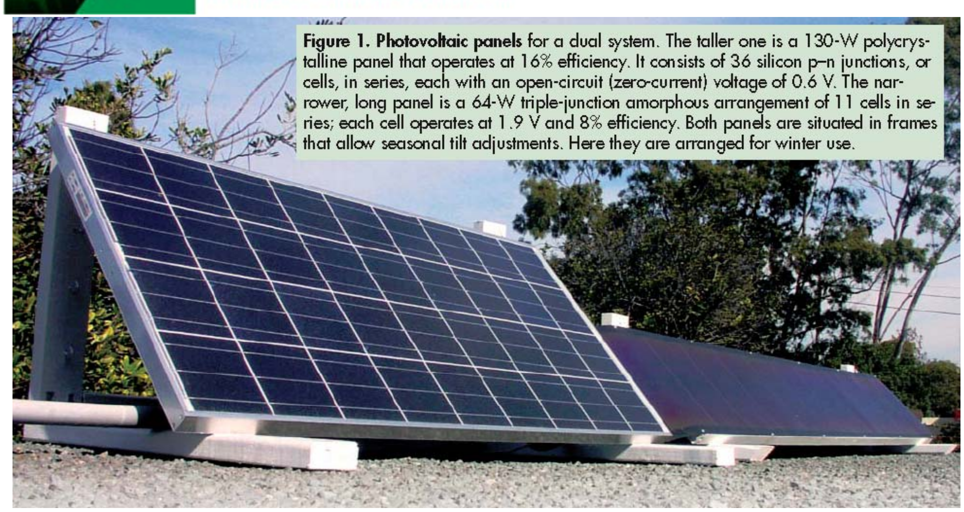


Figure 1. Photovoltaic panels for a dual system. The taller one is a 130-W polycrystalline panel that operates at 16% efficiency. It consists of 36 silicon p–n junctions, or cells, in series, each with an open-circuit (zero-current) voltage of 0.6 V. The narrower, long panel is a 64-W triple-junction amorphous arrangement of 11 cells in series; each cell operates at 1.9 V and 8% efficiency. Both panels are situated in frames that allow seasonal tilt adjustments. Here they are arranged for winter use.

- Tom Murphy, "Home Photovoltaic systems for physicists" Physics Today, July 2008.

HOME PV

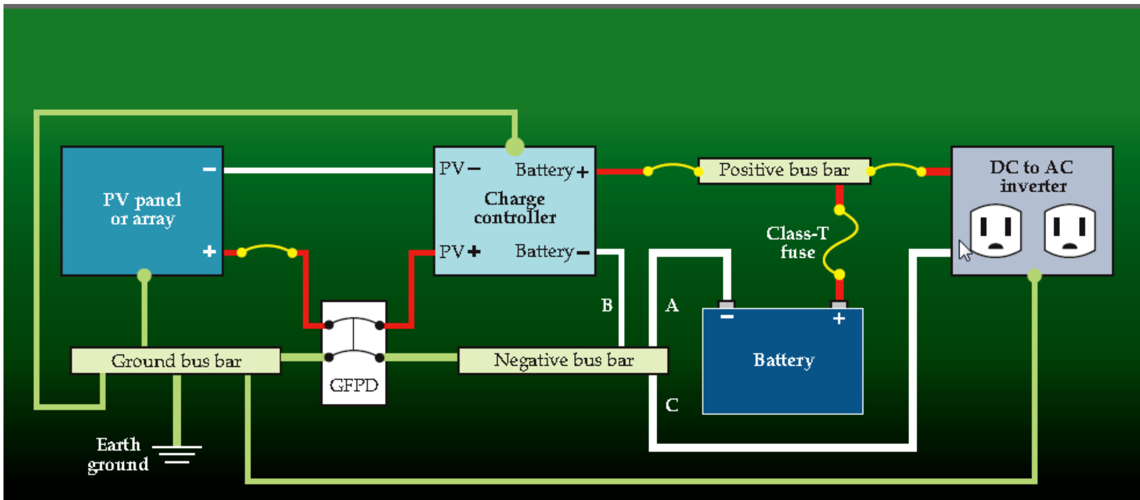


Figure 2. Circuit diagram for a standalone photovoltaic system. The breakers (yellow arcs) serve as overcurrent protection and disconnect simultaneously, but they can be implemented as fuses and disconnect switches. If the PV panels are not attached to a dwelling, the ground-fault protection device (GFPD) can be eliminated and the same bus bar, a metal strip that allows branch connections, can be used for negative and ground. All modules have grounded frames. In-line shunts can be placed at locations A, B, and C for measuring net battery current, solar input current, and load current, respectively—although, since those currents obey a sum rule, any two will suffice. Ground wires are green, negative white, and positive red.

A HOME PV

Figure 3. Power center for my dual PV system. Only half the parts are necessary for a more typical single system. Cut off at the bottom edge of the photo are two 12-V, 150-Ah batteries (one for each system), and immediately above them are 110-A class-T fuses. At left are two charge controllers. The black one is a maximum power-point tracking charge controller, as described in the text; it is certainly overkill for this small system. In the upper right are two 400-W inverters. Below them is an MPPT charge controller not currently used. To the left of the inverters is the monitoring system, capable of measuring two voltages and three currents. In the center is the exposed breaker box, showing three breakers per system, four shunts for current measurement (I use only three at a time), and for each panel, connected grounded bus bars, metal strips that allow branch connections. Ground wires are green, negative white, and positive red. The green extension cords on the right deliver AC power to appliances in the house. This particular system implementation does not require a ground-fault protection device.

A Home PV

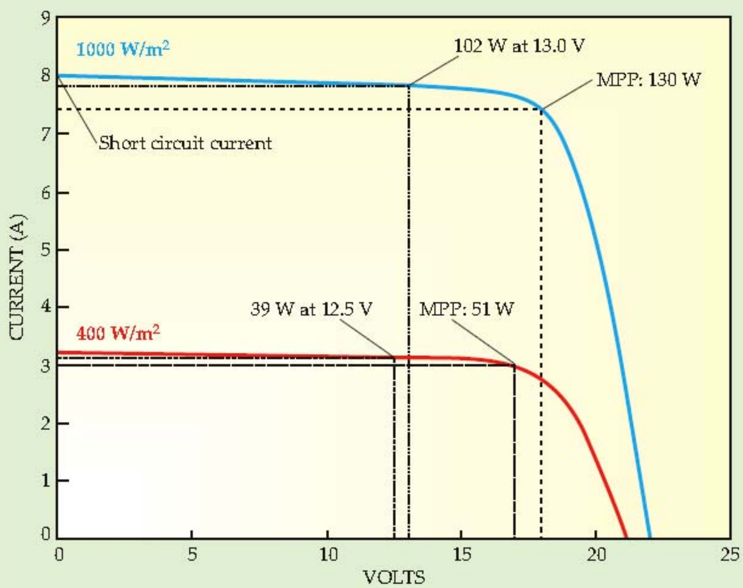


Figure 4. Current–voltage curves for a photovoltaic panel rated at 130 W at 25 °C. The blue curve shows full illumination; the red curve 40% illumination. Typical PV panels, with 36 cells in series, have an open-circuit voltage of around 22 V. Maximum power is typically delivered at around 15–18 V. At lower voltages, the PV current saturates; in full sun the saturation current is called the short-circuit current. The maximum power point (MPP) is indicated for both curves, as is the reduced power achieved by extracting current at typical battery voltages.

A Home PV

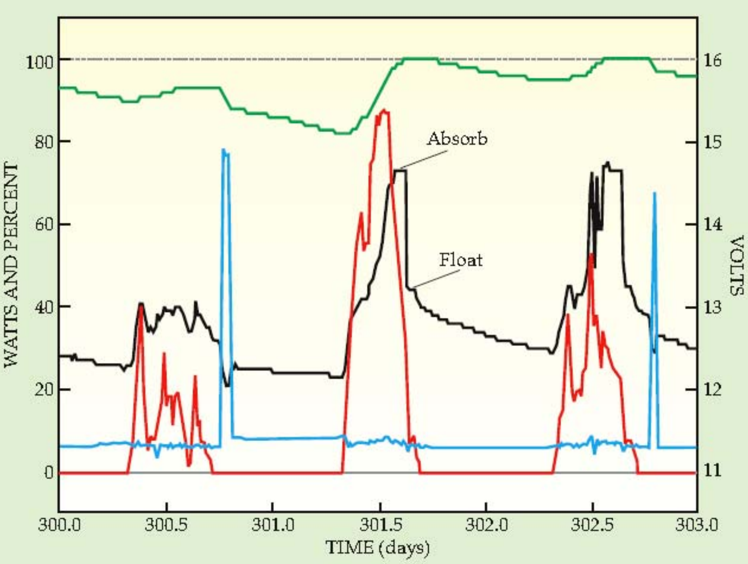


Figure 5. Sample data for a 130-W solar panel powering an entertainment system. Day 300 had a heavy but variable overcast; day 301 was mostly sunny; day 302 was cloudy. The red curve traces the solar power delivered by the panel to the charge controller, and the blue curve traces the load. Note the constant "off" load plus brief intervals of television usage. The black curve is battery voltage, as indicated on the right-hand scale. The battery reached absorption stage on days 301 and 302 and also showed intervals of float stage—indicating a fully charged battery—at the end of both days. The green curve indicates battery charge as a percent of capacity. The first day made only a small positive contribution to the battery charge, but the sunny day that followed made up for the deficit. Note that the falling edge of the solar power on the sunny day is not due to a decline in illumination, but reflects the diminishing current demand in maintaining absorption-stage voltage. Additional data samples are available at http://physics.ucsd.edu/~tmurphy/pv_for_pt.html.

PV Energy Delivery Calculation

- Estimate the annual energy delivered by the 1-kW (dc, STC) array in Madison, WI, which south-facing, and has a tilt angle equal to its latitude minus 15◦. Assume the dc-to-ac conversion efficiency at 72%.

- Insolation Table for Madison

Tilt	Madison, WI Jan	Feb	Mar	Apr	May	Jun	Latitude 43.13°N July	Aug	Sept	Oct	Nov	Dec	Year
Lat − 15	3.0	3.9	4.5	5.1	5.8	6.2	6.2	5.7	4.8	3.8	2.5	2.3	4.5
Lat	3.4	4.3	4.7	5.0	5.5	5.7	5.8	5.5	4.8	4.0	2.8	2.6	4.5
Lat + 15	3.6	4.4	4.6	4.6	4.8	4.9	5.0	5.0	4.6	4.0	2.9	2.8	4.3
90	3.5	4.0	3.7	3.2	2.9	2.8	2.9	3.2	3.4	3.3	2.6	2.7	3.2
1-Axis (Lat)	3.9	5.0	5.8	6.4	7.3	7.8	7.7	7.1	6.0	4.8	3.2	3.0	5.7
Temp. (°C)	−4.0	−1.1	5.3	13.7	20.5	25.7	28.0	26.4	21.9	15.5	6.7	−1.2	13.1

Solution

- From 72% Conversion efficiency
  - P_{ac}=1.kW*0.72 = 0.72kW
- From the Insolation Table, the annual average insolation is 4.5 kWh/m^2-day
  - Same as 4.5 h “peak sun”/day
- Energy Calculation

$$\text{Energy} = 0.72\ \text{kW} \times 4.5\ \text{h/day} \times 365\ \text{day/yr} = 1183\ \text{kWh/yr}$$

MathCad Solution

2b85 PV Annual Energy Delivery.xmcd

PV Annual Energy Delivery.xmcd   Charles Kim 2013

Q. Estimate the annual energy delivered by the 1 kW (DC) arrary in Madison, WI, which south-facing, and has a tilt angle equal to its latutude minus 15 degrees. Assume the DC-to-AC conversion efficiency at 72%.

Insolation Table for Madison (i.e., Peak-Sun Hours)

	Madison, WI					Latitude 43.13°N							
Tilt	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sept	Oct	Nov	Dec	Year
Lat − 15	3.0	3.9	4.5	5.1	5.8	6.2	6.2	5.7	4.8	3.8	2.5	2.3	4.5
Lat	3.4	4.3	4.7	5.0	5.5	5.7	5.8	5.5	4.8	4.0	2.8	2.6	4.5
Lat + 15	3.6	4.4	4.6	4.6	4.8	4.9	5.0	5.0	4.6	4.0	2.9	2.8	4.3
90	3.5	4.0	3.7	3.2	2.9	2.8	2.9	3.2	3.4	3.3	2.6	2.7	3.2
1-Axis (Lat)	3.9	5.0	5.8	6.4	7.3	7.8	7.7	7.1	6.0	4.8	3.2	3.0	5.7
Temp. (°C)	−4.0	−1.1	5.3	13.7	20.5	25.7	28.0	26.4	21.9	15.5	6.7	−1.2	13.1

SOLUTION

$Pdc := 1$ kW

$\eta := 0.72$ DC to AC Conversion Efficiency

$Pac := Pdc \times \eta$ kW

$PSH := 4.5$ [4.5 h/day] From the table, the last column - Annual average peak-sun hours

$Epv := Pac \times PSH \times 365 = 1.1826 \times 10^{3}$ kWh/year

Detailed Monthly Analysis

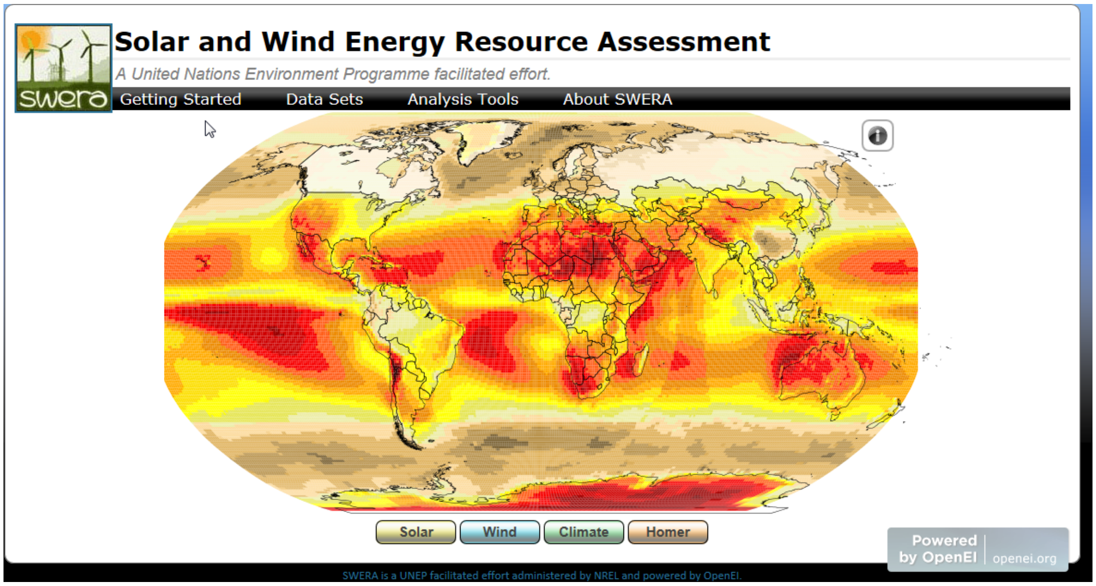
Madison, WI, South L-15

dc Power	1 kW at STC
Temp. coef.	0.5%/°C
Mismatch	0.03
Dirt	0.04
Inverter	0.90
NOCT	47°C

Month	Insolation (kWh/m^2-day)	Avg Max Temp. (°C)	Cell Temp. (°C)	Array dc Power (kW)	Array ac Power (kW)	Energy (kWh/mo)
Jan	3.0	−4.0	29.8	0.98	0.82	76
Feb	3.9	−1.1	32.7	0.96	0.81	88
Mar	4.5	5.3	39.1	0.93	0.78	109
Apr	5.1	13.7	47.5	0.89	0.74	114
May	5.8	20.5	54.3	0.85	0.72	129
Jun	6.2	25.7	59.5	0.83	0.69	129
July	6.2	28.0	61.8	0.82	0.68	131
Aug	5.7	26.4	60.2	0.82	0.69	122
Sept	4.8	21.9	55.7	0.85	0.71	102
Oct	3.8	15.5	49.3	0.88	0.74	87
Nov	2.5	6.7	40.5	0.92	0.77	58
Dec	2.3	−1.2	32.6	0.96	0.81	57
Avg:	4.5	13.2			kWh/yr =	1202

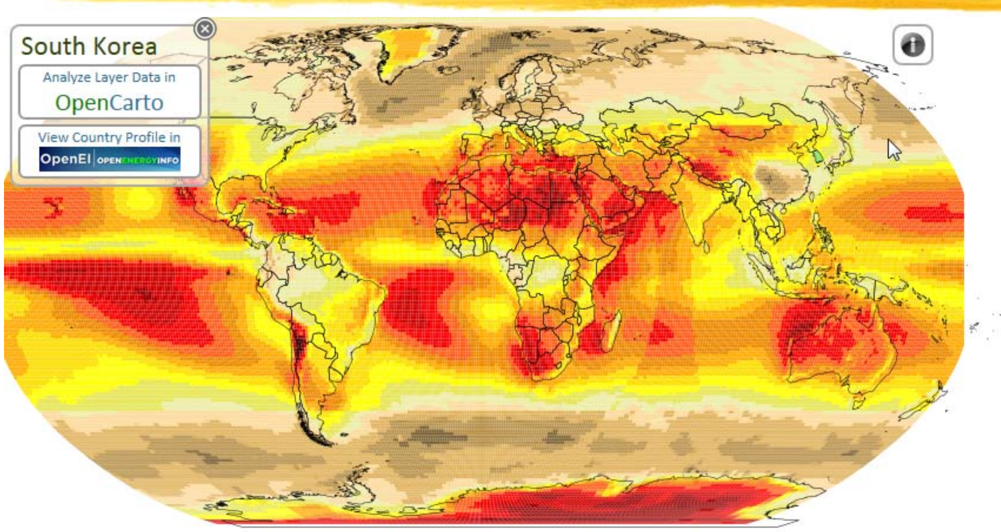
SWERA

- ⌘ Solar and Wind Energy Resource Assessment
- ⌘ http://en.openei.org/apps/SWERA/



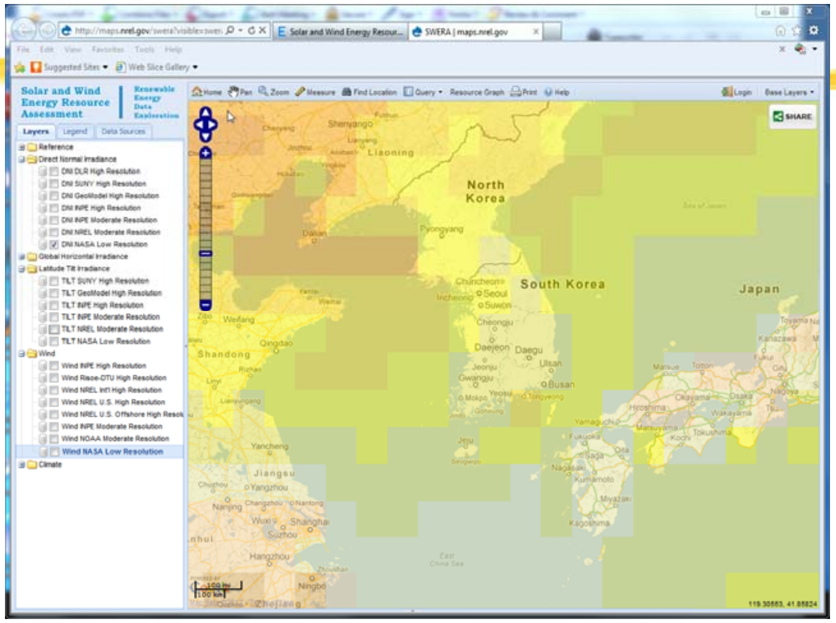
Getting data for Insolation and Wind speed

- ⌘Select Korea



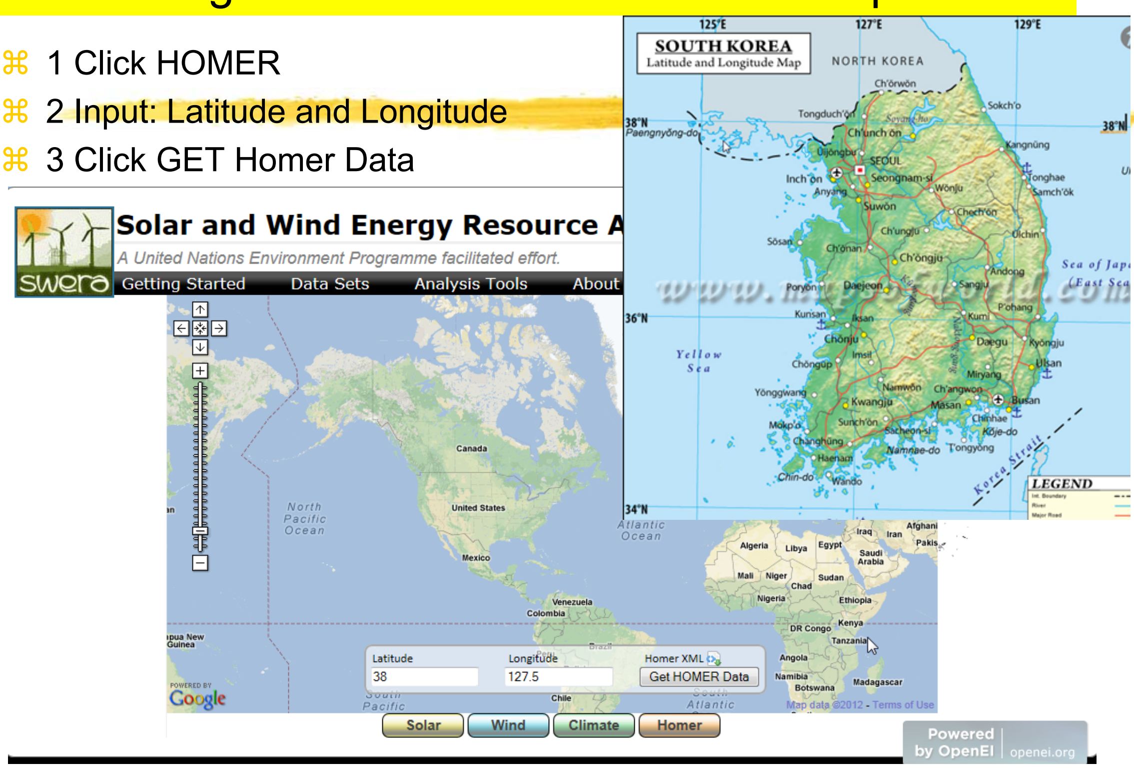
Layers of Data

- Solar
  - DNI: Direct Normal Irradiance
  - GHI: Global Horizontal Irradiance
  - TLT: Tilted Flat Plate Collector
- Wind
  - Hub height: 50m
  - Speed in m/s



Getting data for Insolation and Wind speed

- 1 Click HOMER
- 2 Input: Latitude and Longitude
- 3 Click GET Homer Data



Data for Homer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Import XLM File from SWERA

- SWERA
  - Lat & Longs → Get Homer
  - From the XLM data screen
    - CTRL+S (save to a xlm file)
- Now with HOMER
  - File>"Import XLM"
  - Wind Resources are automatically filled
  - Solar Resources are automatically filled
    - Lat N, Long E → marking error
    - But kWh/m^2 is kept the same.

Capacity Factor

- Capacitor Factor (CF): Ratio with Rated Power
- CF of 0.4 means:
  - the system delivers full-rated power 40% of the time and no power at all the rest of the time, or
  - the system deliver 40% of rated power all of the time.

$$\text{Energy (kWh/yr)} = P_{ac}\text{(kW)} \cdot \text{CF} \cdot 8760\text{(h/yr)}$$

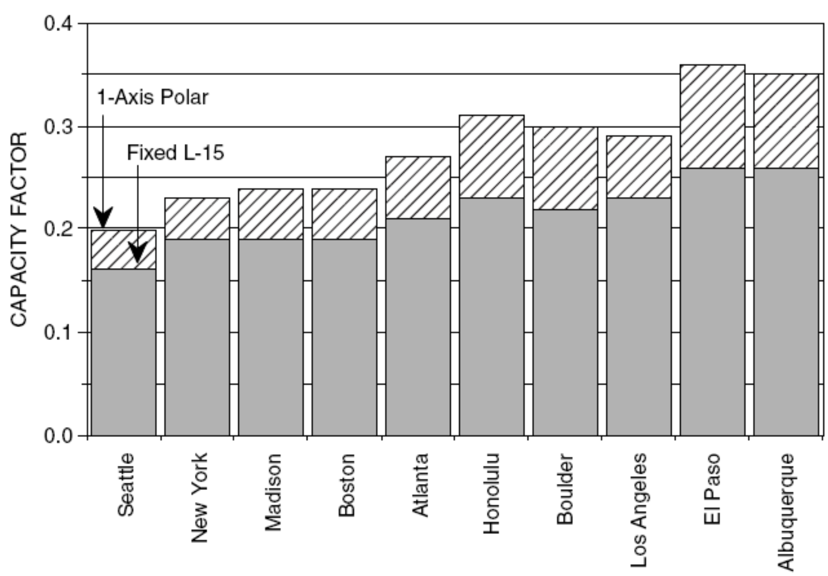
$$\text{Energy (kWh/day)} = P_{ac}\text{(kW)} \cdot \text{(h/day of "peak sun")}$$

- CF for Grid-Connected PV:

$$\text{Capacity factor (CF)} = \frac{\text{(h/day of "peak sun")}}{24\ \text{h/day}}$$

CFs for a number of U.S. cities

- CF: 0.16 – 0.26 for fixed south-facing panel at tilt L-15
- CF: 0.20 – 0.36 for single axis polar mount tracker



PV System Sizing

- Questions
  - How many kWh/yr are required?
  - How many peak watts of dc PV power are needed to provide that amount?
  - How much area will that system require?
  - What real components are available ?
- Example
  - An energy efficient house in Fresno (Latitude at 22°) is to be fitted with a rooftop PV array that will annually displace all of the 3600 kWh/yr of electricity that the home uses.
  - **Question: How many kW (dc, STC) of panels will be required and what area will be needed?**
  - Assumptions:
    - Roof is south-facing with a moderate tilt angle
    - Annual insolation for L-15 is 5.7kWh/m^2-day
    - Dc-to-ac conversion efficiency at 75%
    - Solar system average 1-sun efficiency at 12.5%

Sizing Solution

- Roof is south-facing with a moderate tilt angle
- Annual insolation for L-15 is 5.7kWh/m^2-day
- DC-to-AC conversion efficiency at 75%
- Solar Cell efficiency at 12.5%
- 1. Annual Energy Equation

$$\text{Energy (kWh/yr)} = P_{ac}(\text{kW}) \cdot (\text{h/day @1-sun}) \cdot 365 \text{ days/yr}$$

- 2. AC Power

$$P_{ac} = \frac{3600 \text{ kWh/yr}}{5.7 \text{ h/day} \times 365 \text{ days/yr}} = 1.73 \text{ kW}$$

- 3. DC Power

$$P_{dc,STC} = \frac{P_{ac}}{\text{Conversion efficiency}} = \frac{1.73 \text{ kW}}{0.75} = 2.3 \text{ kW}$$

- 4. Area Calculation

$$P_{dc,STC} = 1 \text{ kW/m}^2 \text{ insolation} \cdot A \text{ (m}^2) \cdot \eta$$

$$A = \frac{2.3 \text{ kW}}{1 \text{ kW/m}^2 \cdot 0.125} = 18.4 \text{ m}^2 \text{ (198 ft}^2)$$

PV and Inverter Modules

Module:	Sharp NE-K125U2	Kyocera KC158G	Shell SP150	Uni-Solar SSR256
Material:	Poly Crystal	Multicrystal	Monocrystal	Triple junction a-Si
Rated power $P_{dc,STC}$:	125 W	158 W	150 W	256 W
Voltage at max power:	26.0 V	23.2 V	34 V	66.0 V
Current at max power:	4.80 A	6.82 A	4.40 A	3.9
Open-circuit voltage V_{OC}:	32.3 V	28.9 V	43.4 V	95.2
Short-circuit current I_{SC}:	5.46 A	7.58 A	4.8 A	4.8
Length:	1.190 m	1.290 m	1.619 m	11.124 m
Width:	0.792 m	0.990 m	0.814 m	0.420 m
Efficiency:	13.3%	12.4%	11.4%	5.5%

Manufacturer:	Xantrex	Xantrex	Xantrex	Sunny Boy	Sunny Boy
Model:	STXR1500	STXR2500	PV 10	SB2000	SB2500
AC power:	1500 W	2500 W	10,000 W	2000 W	2500 W
AC voltage:	211–264 V	211–264 V	208 V, 3Φ	198–251 V	198–251 V
PV voltage range MPPT:	44–85 V	44–85 V	330–600 V	125–500 V	250–550 V
Max input voltage:	120 V	120 V	600 V	500 V	600 V
Max input current:	—	—	31.9 A	10 A	11 A
Maximum efficiency:	92%	94%	95%	96%	94%

Sizing Solution -- Continued

- PV Module selection
  - Kyocera KC158G 158-W module: 23.2V
  - Number of modules?
  - From DC Power = 2300W→ 2300/158=14.6
    - 2-string: 23.2x2=46.4V
    - 3-string: 23.2x3=69.6V --- Pick this. Open Circuit voltage (28.9x3=86.7V) is still below 120V max of the STXR2500 inverter
  - 3x5 (15 modules)
- Inverter Module
  - Xantrex STXR2500 Inverter:
    - MPPT Input voltage 44-85V
    - Max input voltage: 120V
- Check if the energy requirement is met

$$\text{Area} = 15 \text{ modules} \times 1.29 \text{ m} \times 0.99 \text{ m} = 19.1 \text{ m}^2 \ (206 \text{ ft}^2)$$

$$P_{dc,STC} = 158 \text{ W/module} \times 15 \text{ modules} = 2370 \text{ W}$$

$$\text{Energy} = 2.37 \text{ kW} \times 0.75 \times 5.7 \text{ h/day} \times 365 \text{ day/yr} = 3698 \text{ kWh/yr}$$

Module:	Kyocera KC158G
Material:	Multicrystal
Rated power $P_{dc,STC}$:	158 W
Voltage at max power:	23.2 V
Current at max power:	6.82 A
Open-circuit voltage V_{OC}:	28.9 V
Short-circuit current I_{SC}:	7.58 A
Length:	1.290 m
Width:	0.990 m
Efficiency:	12.4%

Manufacturer:	Xantrex
Model:	STXR2500
AC power:	2500 W
AC voltage:	211–264 V
PV voltage range MPPT:	44–85 V
Max input voltage:	120 V
Max input current:	—
Maximum efficiency:	94%

Excel Solution

B130 PV Sizing.xlsx - Microsoft Excel

F27

	A	B	C	D	E	F	G
1	peak sun [h/day]	Solar 1-sun effic	DC2AC effi	kWh/yr required to produce	Pac [kW]	Pdc [kW]	Area (m2)
2	**5.7**	**0.125**	**0.75**	**3600**	**1.73035328**	**2.307138**	**18.4571**
3	4.7	0.125	0.75	3600	2.09851355	2.798018	22.38414
4	3.7	0.125	0.75	3600	2.66567938	3.554239	28.43391

=D2/(A2*365)

C	D	E	F	G
DC2AC effi	kWh/yr required to produce	Pac [kW]	Pdc [kW]	Area (m2)
0.75	**3600**	**1.73035328**	**2.307138**	**18.4571**
0.75	3600	2.09851355	2.798018	22.38414
0.75	3600	2.66567938	3.554239	28.43391

B130 PV Sizing.xlsx

MathCad Solution

2b97 PV Sizing.xmcd

PV Sizing.xmcd Charles Kim 2013

(Background) An energy efficient house in Fresno, CA (Latitude at 22 degrees) is to ve fitted with a rooftop PV array that will annually replace all of the 3600 kWh/year of electricty that the home uses.

(Assumption):
(1) Roof is south facing with a moderate tilt angle.
(2) Annual Insolation for L-15 is 5.7 kWh/m²-day ----> (5.7 Peak Sun Hours)
(3) DC-to-AC conversion efficiency is 75%
(4) PV array average 1-sun efficiency (or Capacifty Factor, CF)is 12.5%

(Q): How many kW (DC rating) of PV panels will be required and what are will be needed?

Annual Energy Production Equation (for PV)

$$\text{Energy (kWh/yr)} = P_{ac}(\text{kW}) \cdot (\text{h/day @1-sun}) \cdot 365 \text{ days/yr}$$

MathCad

SOLUTION

1. Annual Energy (AC) Required is 3600 kWh (Eann): $Eann := 3600$ kWh/year

$PSH := 5.7$ Peak sun hour

2. Required AC Power Generation from PV system (Pac):

$$Pac := \frac{Eann}{PSH \times 365} = 1.7304 \quad \text{kW}$$

3. Required DC Power Generation from PV array (Pdc): $D2A := 0.75$ DC to AC Conversion Efficiency

$$Pdc := \frac{Pac}{D2A} = 2.3071 \quad \text{kW}$$

4. PV Array Area (A): $P_{dc,STC} = 1\ \text{kW/m}^2\ \text{insolation} \cdot A\ (\text{m}^2) \cdot \eta$

PV Efficiency (η): $\eta := 0.125$

$$A := \frac{Pdc}{1 \times \eta} = 18.4571 \quad \text{m}^2$$

5. Selection of PV Module

6. Selection of Inverter Module

MathCad

Module:	Kyocera KC158G
Material:	Multicrystal
Rated power $P_{dc,STC}$:	158 W
Voltage at max power:	23.2 V
Current at max power:	6.82 A
Open-circuit voltage V_{OC}:	28.9 V
Short-circuit current I_{SC}:	7.58 A
Length:	1.290 m
Width:	0.990 m
Efficiency:	12.4%

Manufacturer:	Xantrex
Model:	STXR2500
AC power:	2500 W
AC voltage:	211–264 V
PV voltage range MPPT:	44–85 V
Max input voltage:	120 V
Max input current:	—
Maximum efficiency:	94%

$Prated := 0.158$ $PVlen := 1.29$ $PVwid := 0.99$

MathCad

6. Number of Modules Needed (Npv): $Npv := \frac{Pdc}{Prated} = 14.6021$ $Npv := round(Npv) = 15.0000$

3 strings: 23.2 [V] *3 = 69.6 [V]
5 rows

DIY PV System Costs

Item	Cost
28 Evergreen PV modules (blems)	$18,514
PVPowered inverter	2,895
Misc. electrical	2,772
PV support, homemade	2,437
PV rack, homemade	1,554
Travel expense (to pick up parts, equipment)	861
Books	132
Total	$29,165
Less 30% Tax Credit	$8,750
Grand Total	$20,415

7. Verification

$PdcV := Prated \times Npv = 2.3700$ kW

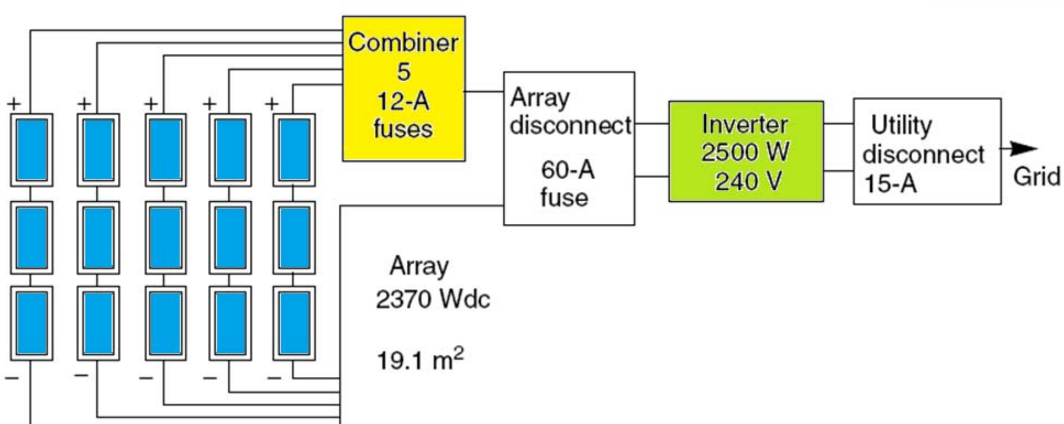
$EannV := PdcV \times D2A \times PSH \times 365 = 3.6981 \times 10^3$ kWh/year

$AV := PVlen \times PVwid \times Npv = 19.1565$ m^2

Final Design

NEC Article 690

- Other requirements
  - NEC – 600V max voltage limit
  - Fuse and disconnect switch: withstand 125% of expected dc voltage
  - Consider potential exceeded solar insolation: give 125%
  - Combiner fuse rating: (7.58 PV short circuit current)x(1.25)x(1.25) =11.8A
  - Array disconnect switch rating: 11.8Ax 5 = 59.2A
  - Inverter fuse rating (125%): 1.25x[2500W/240V]=13A



NEC Article 690

- Article 690, which consists of nine parts, provides electrical requirements for photovoltaic (PV) systems.
- PV source circuits [690.4(B)(1)].
- PV output and inverter circuits [690.4(B)(2)].
- Multiple systems. Conductor of each system where multiple systems are present [690.4(B)(3)].
- Module connection. Arrange module connections so the removal of a module doesn’t interrupt the grounded conductor to other PV source circuits [690.4(C)].

NEC Article 690

Grid-Connected PV System Economics

- Estimation of the cost of electricity generated by PV
  - Amortizing cost of **Principal** (**P** \$) over a **period** (***n*** year) with **interest rate of *i*** for Loan payment.
  - **Annual Payment** (**A** \$/yr) divided by **Annual kWh** → \$/kWh
- **CRF** (Capital Recovery Factor):

$$\mathrm{CRF}(i,n) = \frac{i(1+i)^n}{(1+i)^n - 1}$$

- Annual Loan Payment (A):

$$A = P \cdot \mathrm{CRF}(i,n)$$

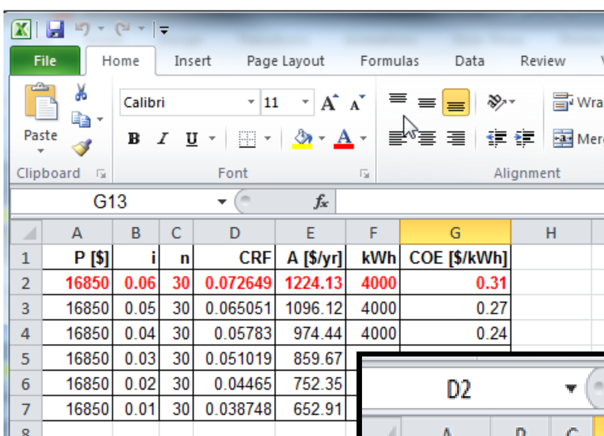
- **Example**: A PV system costs \$16,850 to deliver 4000 kWh/yr. If the system is paid for with a 6% 30-year loan, what would be the cost of electricity, ignoring income tax benefit, loan tax deduction, etc?

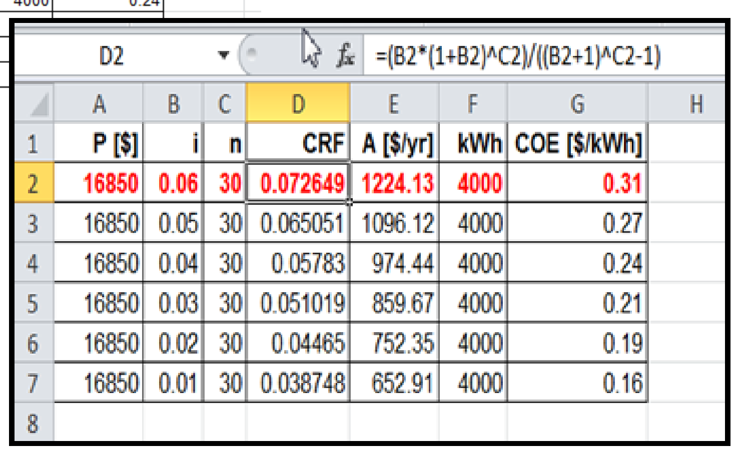
$$\mathrm{CRF}(i,n) = \frac{i(1+i)^n}{(1+i)^n - 1} = \frac{0.06(1.06)^{30}}{(1.06)^{30} - 1} = 0.07265/\mathrm{yr}$$

$$\mathrm{A} = \mathrm{P\ CRF}(i,n) = \$16{,}850 \times 0.07265/\mathrm{yr} = \$1224/\mathrm{yr}$$

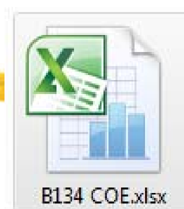
$$\text{Cost of electricity} = \frac{\$1224/\mathrm{yr}}{4000\ \mathrm{kWh/yr}} = \$0.306/\mathrm{kWh}$$

Excel Solution





	A	B	C	D	E	F	G
1	P [\$]	i	n	CRF	A [\$/yr]	kWh	COE [\$/kWh]
2	16850	0.06	30	0.072649	1224.13	4000	0.31
3	16850	0.05	30	0.065051	1096.12	4000	0.27
4	16850	0.04	30	0.05783	974.44	4000	0.24
5	16850	0.03	30	0.051019	859.67	4000	0.21
6	16850	0.02	30	0.04465	752.35	4000	0.19
7	16850	0.01	30	0.038748	652.91	4000	0.16



MathCad Solution

2b110 PV Economics.xmcd

PV Economics.xmcd Charles Kim 2013

(Background)A PV system costs $16,850 to deliver 4000 kWh per year. The system is paid fro with a 6% 30-year loan.

(Q): What would be the cost of electricity.
Ignore income tax benefit, loan tax deduction, etc.

SOLUTION

Annual Energy (Eann): $Eann := 4000$ kWh

Interest rate (i): $i := 0.06$

Project Period (n): $n := 30$ years

Capital Recovery Factor (CRF):

$$\mathrm{CRF}(i, n) = \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$CRF := \frac{i \times (1+i)^n}{(1+i)^n - 1} = 0.0726$$

Principal (P): $P := 16850$ $

Annual Loan Payment (A):

$$A = P \cdot \mathrm{CRF}(i, n)$$

$$A := P \times CRF = 1.2241 \times 10^3$$ $/year

Cost of Energy (COE): $COE := \frac{A}{Eann} = 0.3060$ $/kWh