EECE208 Intro to Electrical Engineering Lab   Dr. Charles Kim

5. Circuit Theorems
- Thevenin Theorem, Maximum Power Transfer, and Superposition

Objectives: This experiment emphasizes the following three circuit theorems: Thevenin Equivalent Circuit, Maximum Power Transfer, and Superposition. Theoretical calculations and experimental measurements will be performed to augment learning on the theorems.

Thévenin's theorem: Thévenin's theorem says that any two-terminal circuit with linear elements can be represented with an equivalent circuit containing a single voltage source ("Thevenin Voltage", V_{th}) in series with a single resistor ("Thevenin Resistor", R_{th}) as illustrated below.
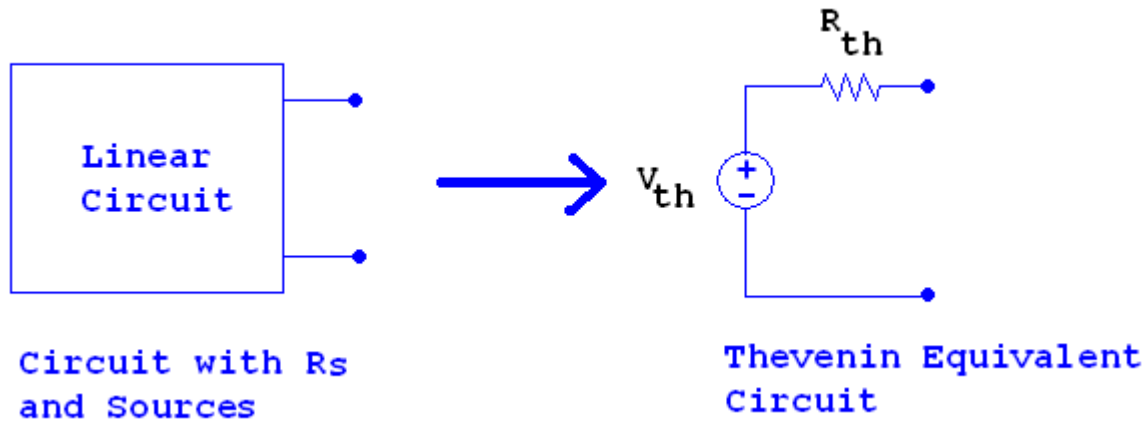

The Thevenin voltage is determined as the open circuit voltage Voc at the terminals. In other words, V_{th} is the same voltage as the terminal voltage, with two terminals remain open. In other words, $V_{th} = V_{oc}$. See illustration below.
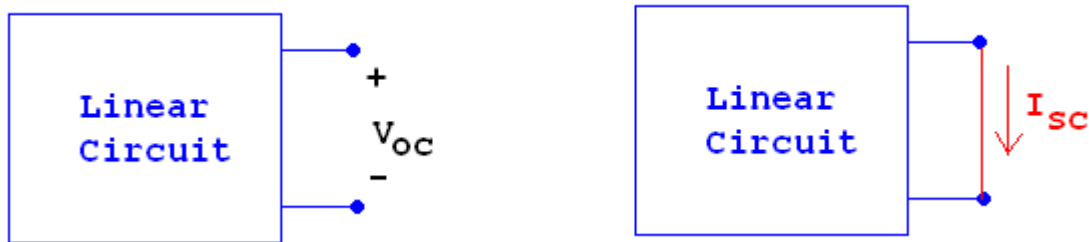
The Thevenin resistance R_{th} is determined by the open circuit voltage V_{oc} and the short circuit current I_{sc}(the current measured in the terminal after two terminals are shorted.) In other words,

$$R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{V_{th}}{I_{sc}}.$$

Maximum Power Transfer: What is says is that the value of the load resistance that absorbs the maximum power from a two-terminal circuit is equal to the Thévenin resistance.   In other words, to deliver the maximum power from a circuit (like an audio amplifier) to a load (like a speaker), the load resistance (i.e., the speaker resistance) must be the same as the circuit's resistance (i.e., the Thevenin resistance of the circuit.). See the illustration shown in the next page.
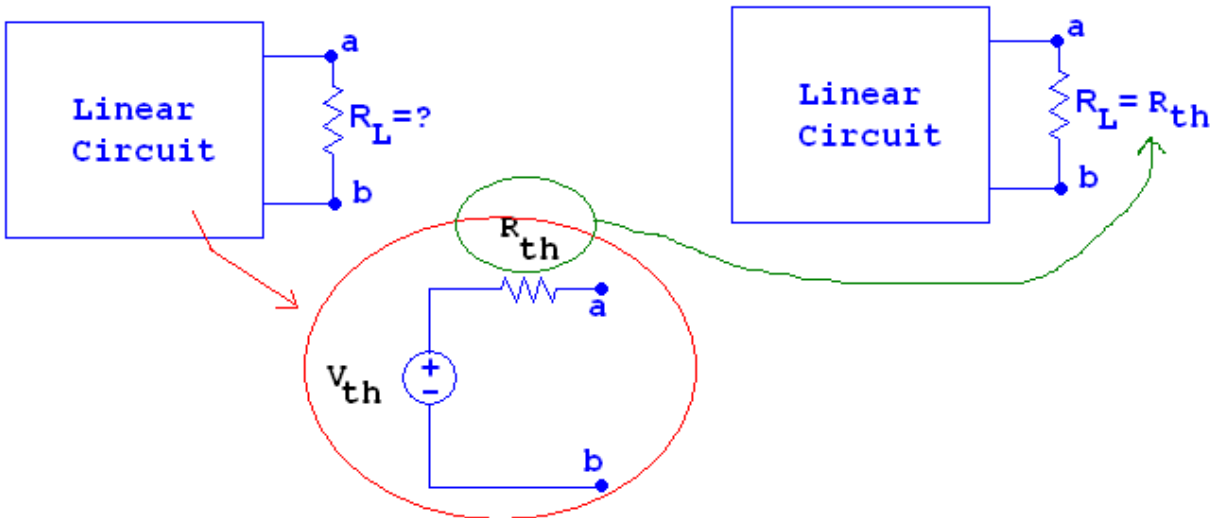
When the maximum transfer condition is met (i.e. $R_L = R_{th}$), the total resistance of the circuit is $2R_{th}$. Since the current through the load is $I_L = \dfrac{V_{th}}{2R_{th}}$, the power delivered to the load will

be: $P_L = I_L^2 \cdot R_{th} = \dfrac{V_{th}^2}{4R_{th}}$.

Superposition: If a circuit has two or more independent sources, one way to analyze the circuit is to use nodal or mesh method. Another way of analysis is to determine the contribution of each independent source and then add them up. This approach is called the **superposition**. The superposition principle says that "the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone."

Steps of Applying Superposition Principle:
(a) Deactivate all independent sources except one source. Find the output (V or I) due to that active source.
(b) Repeat step (a) for each of the other independent sources.
(c) Find the total contribution by adding algebraically all the contributions due to the independent sources.

Let's have an example. Find V in the circuit using the superposition theorem.
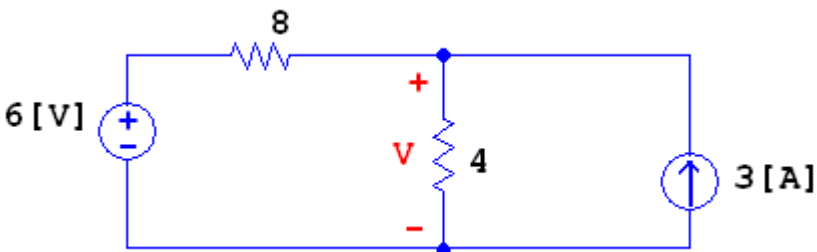
Now, we consider one source at a time, therefore, we deactivate the current source. The deactivation of current source means (since there will be no current) that we replace the current source by an open circuit.
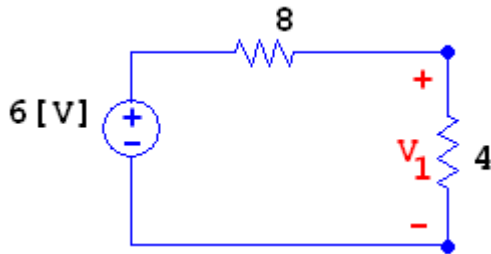
Then the voltage with only the voltage source is, by voltage divider: $V_1 = 6 \cdot \dfrac{4}{4+8} = 2[V]$.

We then activate the current source, and deactivate the voltage source. The deactivation of voltage source means (since no voltage means no resistance) that we replace it by a short circuit.
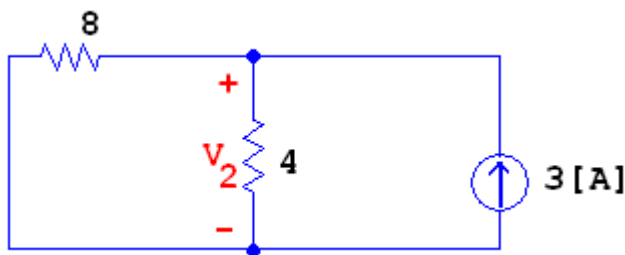
Then the voltage contribution by the current source only, current divider,

is: $V_2 = 4 \cdot (3 \cdot \dfrac{8}{4+8}) = 8[V]$.

Finally, the voltage by both sources is: $V = V_1 + V_2 = 2 + 8 = 12\,[V]$.

Caveat: Some weakness and caution
(a) Applying the superposition may very likely involve more works.
(b) Since the principle is based on linearity, you <u>cannot</u> directly apply it to the <u>power</u> due to each source: power absorbed by a resistor depends on the square of the voltage or current.[1] Therefore, if power value is needed, the current through (or voltage across) the element must be calculated first using superposition.
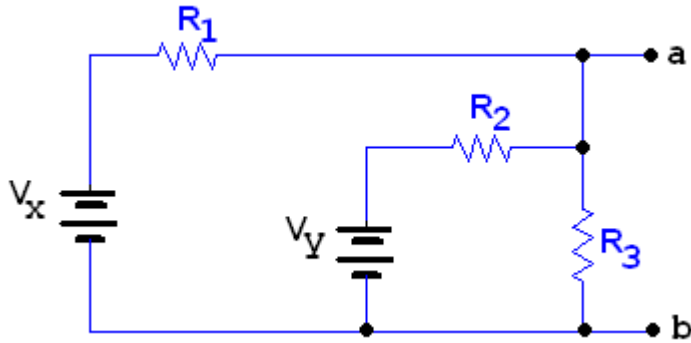
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[1] For example, when current i_1 flows through resistor R, the power is $P_1 = R*i_1^2$, and when current i_2 flows through R, the power is $P_2 = R*i_2^2$. If current (i_1+i_2) flows through R, the power is $P_3 = R*(i_1+i_2)^2$, and $P_3 \neq (P_1+P_2)$. Power relation is nonlinear.

1. Find the Thevenin equivalent circuit at the terminals **a** and **b** for the network shown below.

2. Draw the Thevenin equivalent Circuit with values found from 1.

3. Using the Superposition Theorem, find the voltage V_{ab} at the terminals **a** and **b**.

LAB PROCEDURE

<u>**Equipment:**</u>

1. Breadboard.
2. Resistors
3. DMM for voltage and current measurements.
4. Power supply

<u>**Procedures:**</u>

A. Thevenin Theorem and Maximum Power Transfer
1. Construct the circuit on the breadboard as shown below. Randomly select three different resistors (for R1, R2, and R3) in the range of [2K - 10K].   Apply using Dual Power Supply V_x and V_y with any value in the range of [5V - 10V].
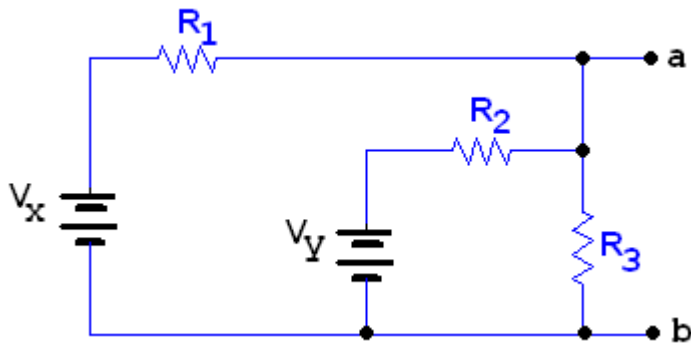
2. Now, by calculation, find the Thevenin Voltage, V_{th}, and Thevenin resistance R_{th} of the circuit. (Hint: An *alternative* way to the Thevenin resistance (other than the short circuit current method): first deactivate two voltage sources, and find the equivalent resistance of the circuit at the terminals **a** and **b**. See step 8 below for more on this.)

3. Now draw the Thevenin equivalent circuit of the above network.

4. Measure the open-circuit voltage using the DMM and Voltmeter: $V_{oc} = (V_{ab} = V_{th}) = $ _____

5. Measure the short-circuit current using the DMM and Ammeter: $I_{sc} = $ _____

6. Calculate $R_{TH} = \dfrac{V_{oc}}{I_{sc}} = $ _____

7. Compare the calculation results (from 2) and the measurement results (from 4 - 6).

8. Disconnect V_x and V_y and replace them by short circuits, then measure the resistance between the terminals **a** and **b** using DMM as Ohmmeter. $R_{meas} = $ _____

9. Explain why (or why not) the two values obtained from 6 and 8, respectively, are the same.

10. Obtain a variable resistor of 0 - 10K (or any value or any shape available. If question, ask Mr. Bowman) illustrated below. Using the DMM as Ohmmeter, measure the maximum resistance of the variable resistor between pins 2 and 3.

  Now at the terminals, connect the variable resistor as illustrated below.
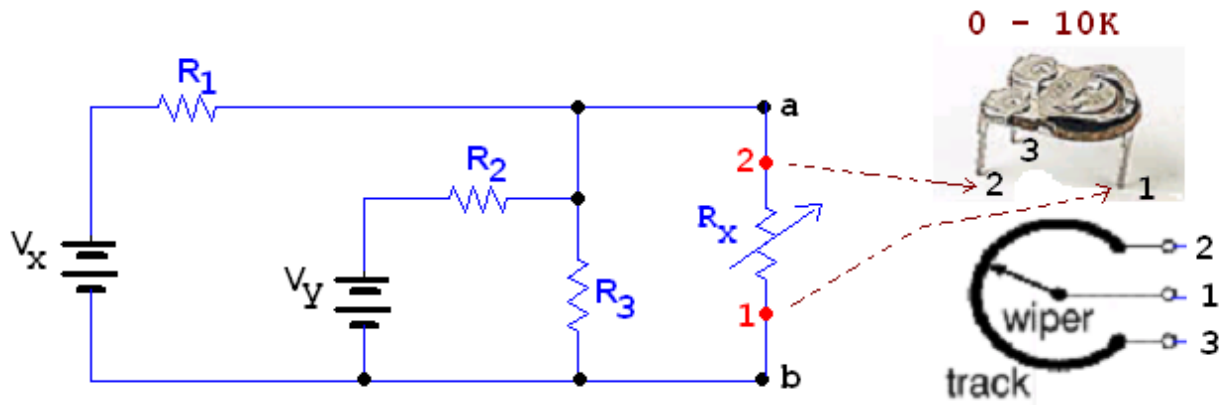
11. By the way, this is another way of finding the Thevenin resistance.   Turn the wiper of the variable resistor to the left-most position.   By slowly turning the wiper from left most position to the right most position by the quarter, (a) measure the voltage across **a** and **b**, (b) measure current flowing through the variable resistor, and (c) Calculate the power consumed by the variable resistor by the voltage and the current measures.

Wiper Position	Left Most	1/4 position	Center	3/4 position	Right Most
Voltage [V] measured					
Current [A] measured					
Power [W] calculated					

12. Plot **V**, **I**, and **P** obtained from 11 for the variable resistor.
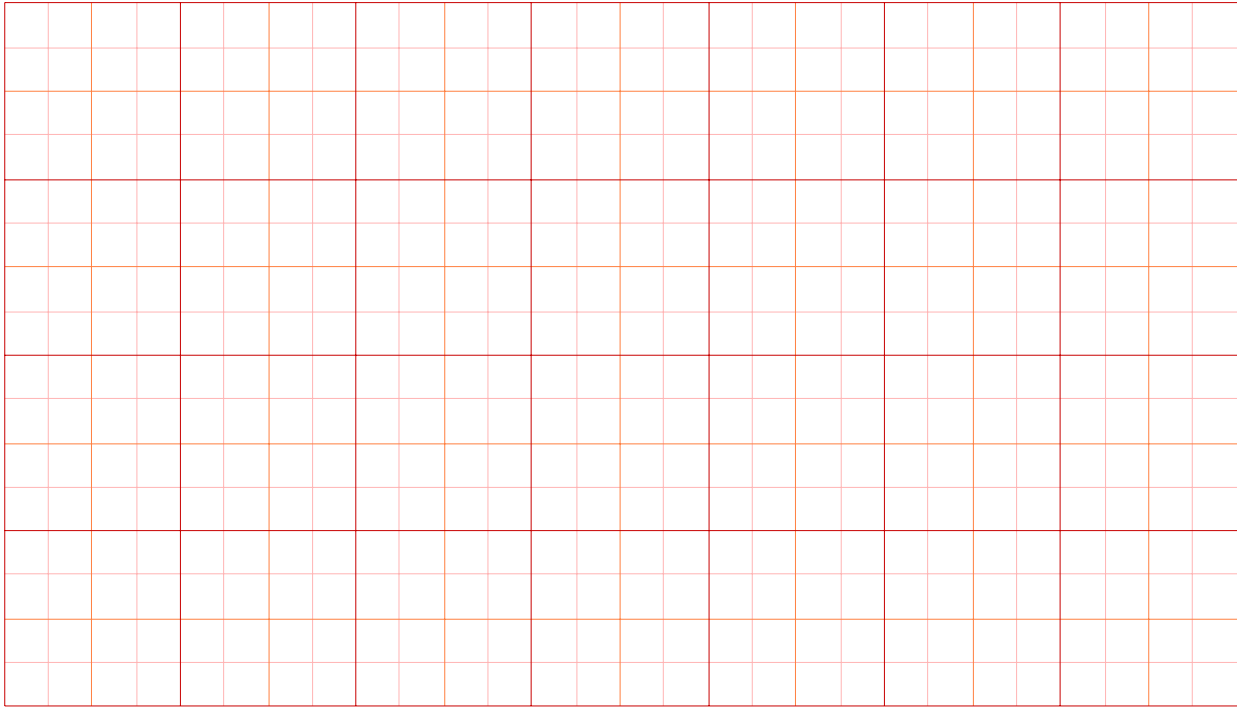
13. From the plot, explain (and devise) how do we get the accurate Thevenin resistance of the circuit.

14. From the experimental design you found from 14, perform the measurement, and find the Thevenin resistance of the circuit.

B. Superposition

1. Now take off the variable resistor from the circuit, so that we have the original circuit as illustrated below.   Keep the values of resistance and voltage as we they were.
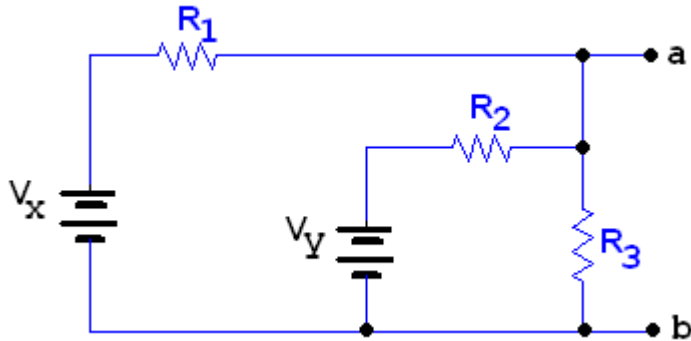
2. By calculation, using superposition principle, calculate the voltage across R3. Record your calculation results below.

Source	V_{ab}	Show the calculation works here.
V_x only		
V_y only		
Sum		

3. From the circuit, keep V_x and replace V_y by short-circuit, and measure the voltage across R3.
4. Now, reconnect V_y and replace Vx by short-circuit, and measure the voltage across R3.
5. Fill the table below using the findings from 3 and 4.

Source	Measured V_{ab}
V_x only	
V_y only	
Sum	

6. Compare the results from 2 and 5.

END