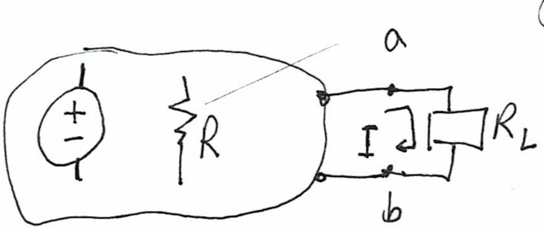
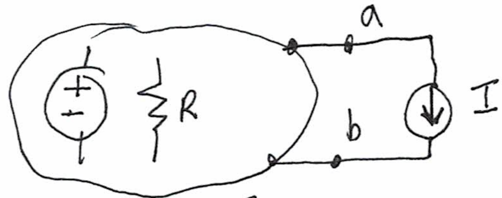
Thevenin Theorem (Principle) ← by Superposition



① Connect a Load

② Then current flows I

③ We replace the Load as a current source with I



④ Apply Superposition,

(i) Voltage source only:

then (↓) ⇒ 0 ⇒ open circuit.

→ $V_{oc} = V_1$

(ii) current source only

Voltage source shorted,

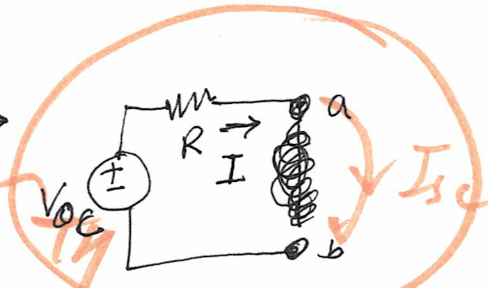
So,

$V_2 = -I \cdot R$

⑤ Total Voltage

$V_{ab} = V_1 + V_2$

$= V_{oc} - I \cdot R$ →



$V_{oc} = V_{Th}$ with series R

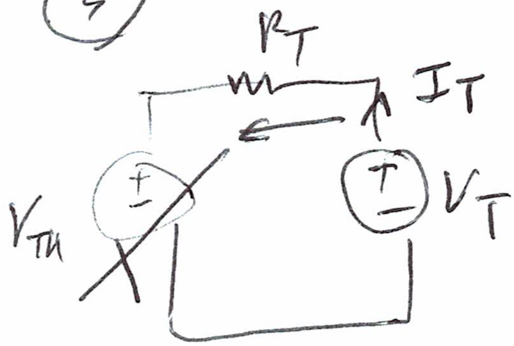
R : Equivalent Resistance after deactivation of All internal Sources

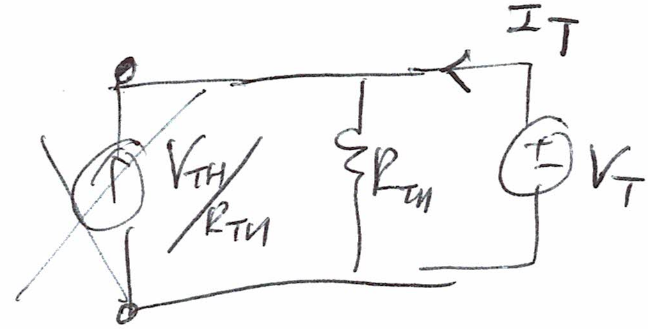
$V_{Th} = V_{oc}$

$R_{Th} = \frac{V_{oc}}{I_{sc}}$

(2) R_{TH} - conti

③





"Test voltage method"

$$R_{TH} = \frac{V_T}{I_T}$$

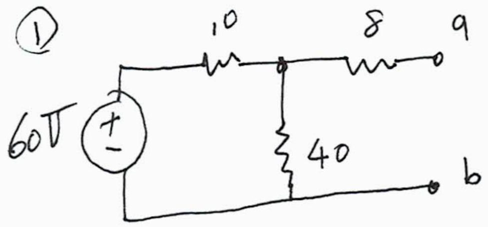
$$R_{TH} = \frac{V_T}{I_T}$$

"works for all <u>cases</u> source types"

	Applicable circuits	Src deactivation?	R_{TH}?
Input Resistance	1) Indepnd src only	yes	$R_{TH} = R_{ab}$
Short circuit	1) Indep src 2) Indep + dep src	NO	$R_{TH} = \frac{V_{TH}}{I_{SC}}$
Test voltage	1) Indep src 2) Indep + dep 3) dep src only	yes	$R_{TH} = \frac{V_T}{I_T}$

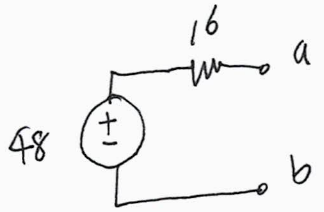
Thevenin Circuit Exercise problems

①

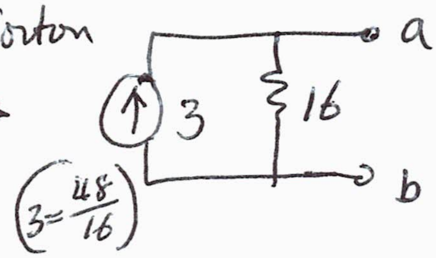


$$V_{oc} = 60 \cdot \frac{40}{10+40} = \frac{60 \cdot 40}{50} = 48\,V$$

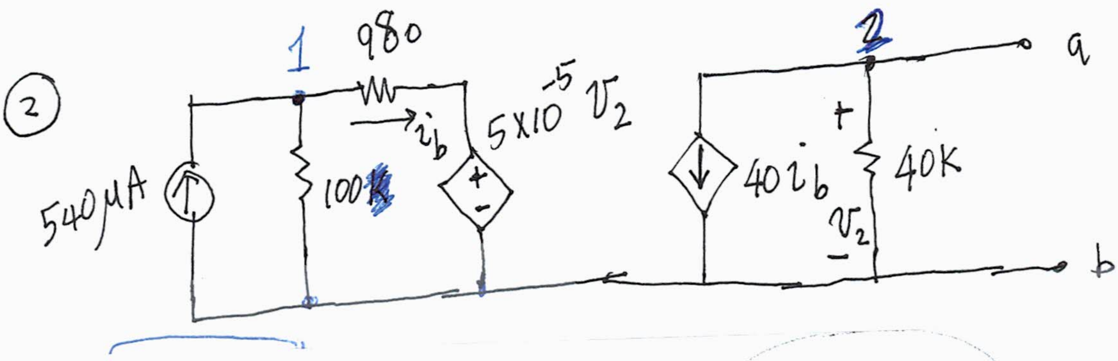
$$R_{Th} = 8 + 10 // 40 = 8 + \frac{10 \cdot 40}{50} = 16$$



Norton



②



@ 2; $V_2 = -40 \cdot i_b \cdot 40 \cdot 10^3 = -16 \times 10^5 \cdot i_b$ ①

$\boxed{V_{oc} = V_2}$

@ 1) $-540 \times 10^{-6} + \frac{V_1}{100} + \underbrace{\frac{V_1 - 5\times 10^{-5} V_2}{980}}_{i_b} = 0$ ②

$$i_b = \frac{V_1 - 5\times10^{-5} V_2}{980}$$

$$= \frac{V_1 - 5\times 10^{-5} \times (-16 \times 10^5) i_b}{980} = \frac{V_1 + 80 \cdot i_b}{980}$$

$980 i_b = V_1 + 80 i_b \rightarrow V_1 = 900 i_b$ ③

③ → ②: $-540 \times 10^{-6} + \frac{900 i_b}{100} + i_b = 0 \rightarrow -540 \times 10^{-6} + 10 \cdot i_b = 0$

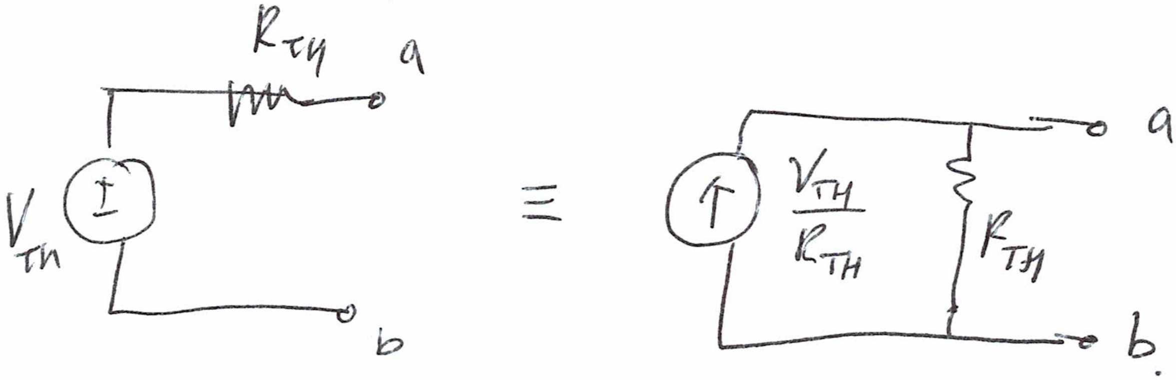
$\boxed{i_b = 54 \times 10^{-6}}$

② → $V_{oc} = V_2 = -16 \times 10^5 \cdot i_b = -16 \times 10^5 \cdot (54) \times 10^{-6}$

$= -86.4 \rightarrow V_{TH}$

(F) 2/18/22

Start from Thévenin circuit /or Norton Circ



(1) $V_{Th} = \underline{V_{ab}}(\text{open}) = \underline{\underline{V_{oc}}}$ $\Rightarrow V_{TH} = \frac{V_{TH}}{R_{TH}} \cdot R_{TH} = V_{TH}$

$= V_{ab}(\text{open}) = \underline{\underline{V_{oc}}}$

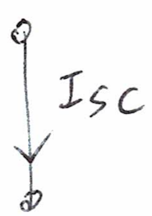
(2) $\underline{\underline{R_{TH}}}$

① $\underline{\underline{R_{ab}}}$ w/ $\underline{V_{TH}}$ $\underline{\text{deactivated}}$ ≡ $\underline{R_{ab}}$ w/ $\frac{V_{TH}}{R_{TH}}$ deactivated

(Short) (open)

"Input Resistance" → $\boxed{\text{Indp Source only}}$

②



$R_{TH} = \frac{V_{TH}}{V_{TH}/R_{TH}} = R_{TH}$

$R_{TH} = \frac{V_{TH}}{I_{SC}}$ with $\underline{\underline{\text{ab short}}}$

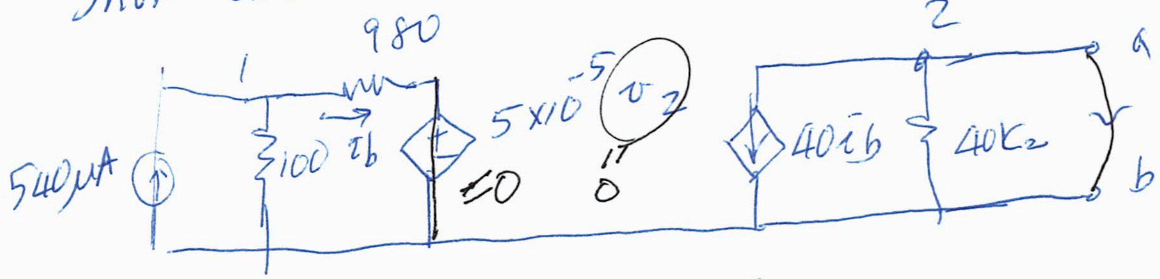
"Short Circuit Method" → $\boxed{\text{Indp + dep}}$

$\underline{\underline{\text{Not}}}$ for $\underline{\text{Dep only}}$

"No source" deactivation

② Conti-

Short circuit



$\leftarrow V_2 = 0$

$$\boxed{i_{sc} = -40 \cdot i_b}$$

S

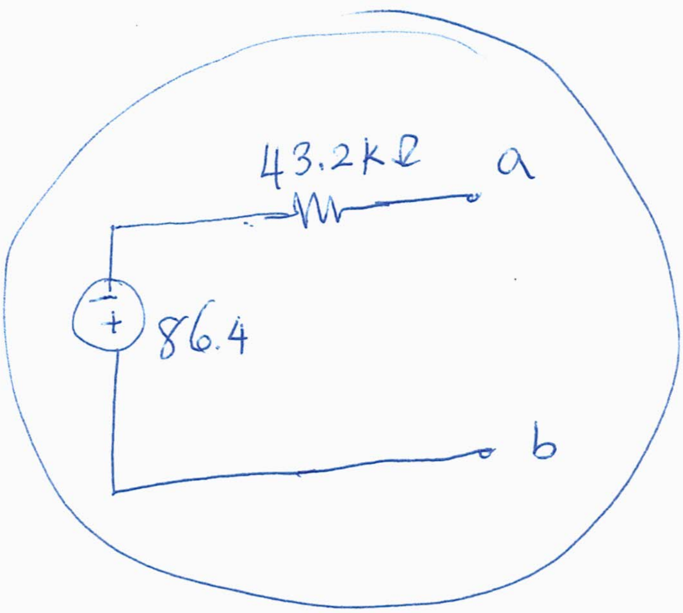
$$i_{b_{sc}} = \underline{540} \times 10^{-6} \cdot \frac{100}{100+980} = \frac{54}{1.08} \times 10^{-6} = \underline{50 \mu A}$$

$$R_{Th} = \frac{V_{oc}}{i_{sc}} = \frac{-86.4}{-40 \cdot i_b} = \frac{-86.4}{-40 \cdot 50 \mu A}$$

$$= \frac{(-86.4)\, 10^6}{(-2000)}$$

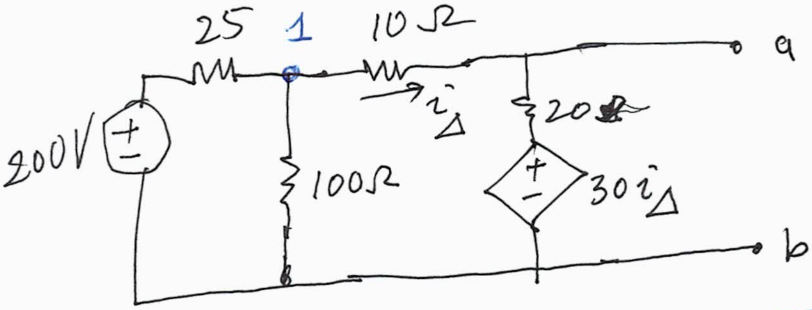
$$= \left(\frac{-86.4}{-2}\right) \cdot 10^3$$

$$= (43.2)\, 10^3 = 43.2 k\Omega$$

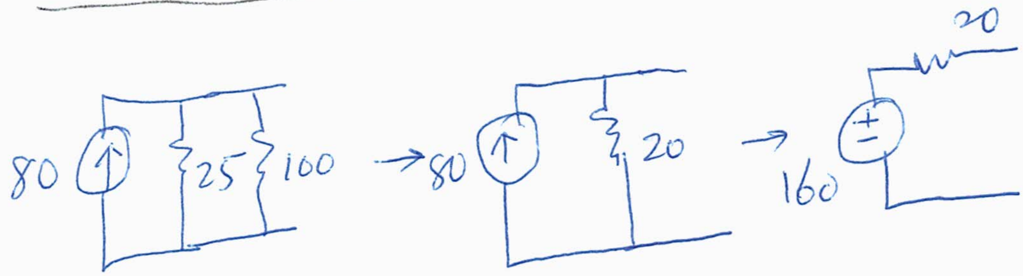


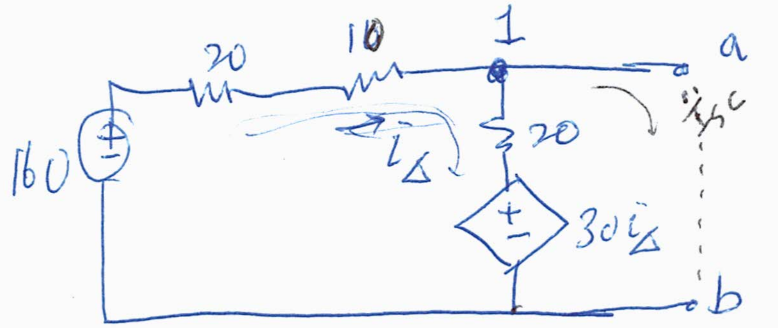
Thevenin Theorem Exercise Problem (Continue)

(3) Thevenin Circuit at Terminal a & b



Simplify the left side by source transformation.





$V_{oc} = V_1 = (20)\, i_\Delta + 30\, i_\Delta$

$V_1 = 50\, i_\Delta$

@ 1: $\dfrac{V_1 - 160}{30} + \dfrac{V_1 - 30\, i_\Delta}{20} = 0$

$2(50\, i_\Delta - 160) + 3(20\, i_\Delta) = 0 \rightarrow 160\, i_\Delta = 2 \cdot 160$

$\therefore i_\Delta = 2$

$V_1 = 50\, i_\Delta = 50 \cdot 2 = 100 = V_{oc} = V_{th}$

or

$160 = 50\, i_\Delta + 30 \cdot i_\Delta$

$= 80\, i_\Delta$

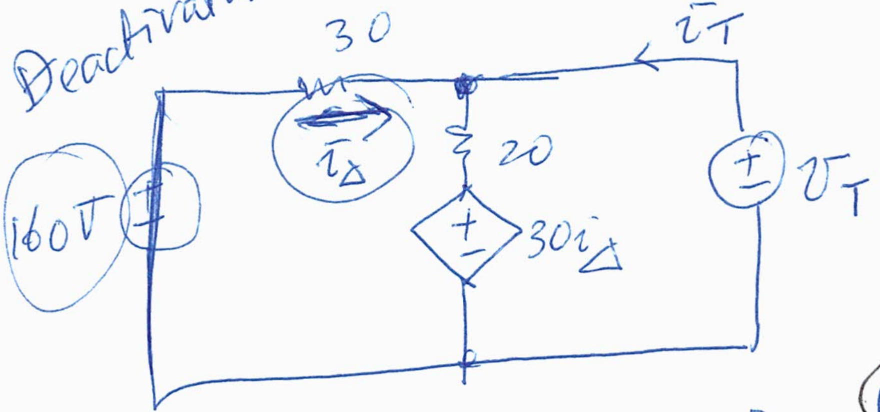
$\therefore i_\Delta = 2$

$V_{th} = 100$

Now ~~use~~ ~~to~~ practice

Apply Test Voltge Method $R = \frac{V_{th}}{I_T}$

Deactivation



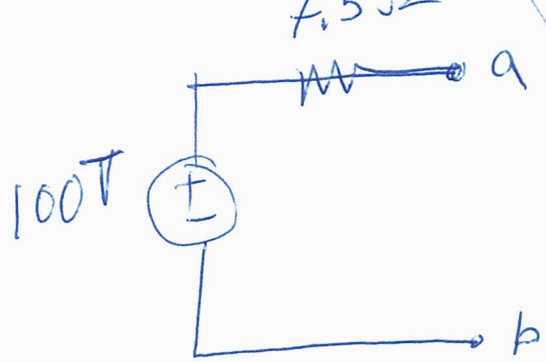
$$i_\Delta = -\frac{v_T}{30}$$

$$\frac{v_T}{30} + \frac{v_T - 30 i_\Delta}{20} - i_T = 0$$

$$2 \cdot v_T + 3 v_T - 90\left(-\frac{v_T}{30}\right) - 60 i_T = 0$$

$$v_T(2 + 3 + 3) = 60 i_T$$

$$\therefore \frac{v_T}{I_T} = \frac{60}{8} = \frac{15}{2} = 7.5 \rightarrow R_{th}$$



Short ckt case

$$\frac{v_1 - 160}{30} + \frac{v_1 - 30 i_\Delta}{20} + i_{sc} = 0$$

$v_1 = 0$

$$\frac{-160}{30} + \frac{-30 i_\Delta}{20} + i_{sc} = 0$$

$$i_\Delta = \frac{160}{30}$$

$$\frac{-160}{30} + \frac{-160}{20} + i_{sc} = 0$$

$$-\frac{16}{3} - \frac{16}{2} = -i_{sc}$$

$$\frac{-32 - 48}{6} = -i_{sc}$$

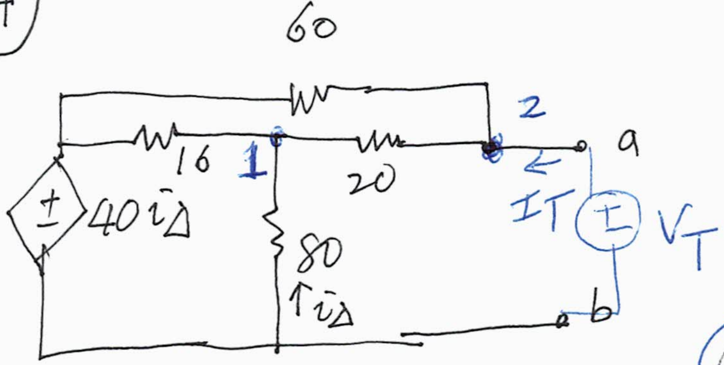
$$\frac{80}{6} = i_{sc}$$

$$R_{th} = \frac{100}{\frac{80}{6}} = \frac{60}{8}$$

$$= 7.5$$

Thevenin Theorem Practice Prob (Conti-)

(4)



ⓐ $V_{th} = 0$, there is no independent source

$V_2 = V_T$

$i_\Delta = -\frac{V_1}{80}$

ⓑ $R_{th} = \frac{V_T}{I_T}$

@ 2 : $-I_T + \frac{V_T - V_1}{20} + \frac{V_T - 40 i_\Delta}{60} = 0$ (1)

@ 1 : $\frac{V_1 - 40 i_\Delta}{16} + \frac{V_1}{80} + \frac{V_1 - V_T}{20} = 0$ (2)

(1) → $-60(I_T) + 3V_T - 3V_1 + V_T - 40\left(-\frac{V_1}{80}\right) = 0$

$4V_T - 2.5V_1 = 60 I_T$ (1)'

(2) → $5V_1 - 5 \cdot 40\left(-\frac{V_1}{80}\right) + V_1 + 4V_1 - 4V_T = 0$

$5V_1 + 2.5V_1 + V_1 + 4V_1 - 4V_T = 0$

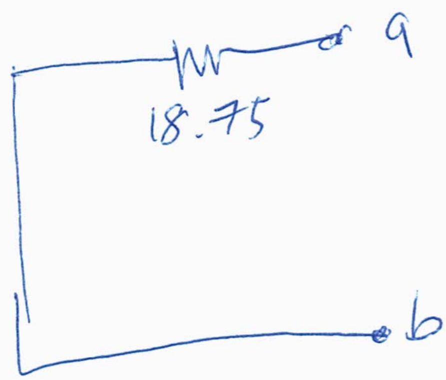
$12.5V_1 = 4V_T \rightarrow V_1 = \frac{4}{12.5} V_T$ (2)'

(2)' → (1)' $4V_T - \cancel{2.5}\frac{4}{\cancel{12.5}_{5}} V_T = 60 I_T$ $\quad \frac{20-4}{5} V_T = 60 I_T$ $\quad \frac{V_T}{I_T}$

$$4V_T - \frac{4}{5}V_T = 60\, I_T$$

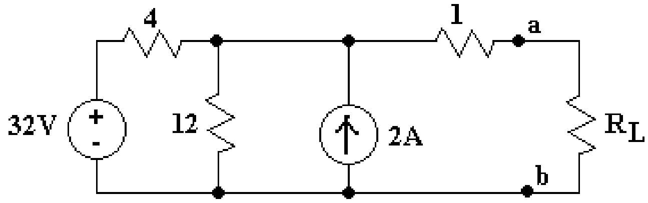
$$\frac{20-4}{5} V_T = 60\, I_T \longrightarrow \frac{16}{5} V_T = 60\, I_T$$

$$\frac{V_T}{I_T} = \frac{60}{16/5} = \frac{60(5)}{16} = 18.75$$



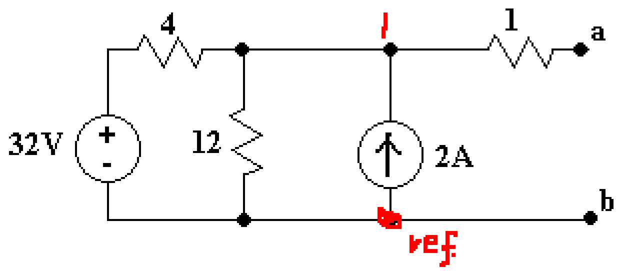
8. An example of **Input Resistance Method**

Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals ***a*** and ***b***. Then, find the current through the load resistor $R_L = 6\ \Omega$.



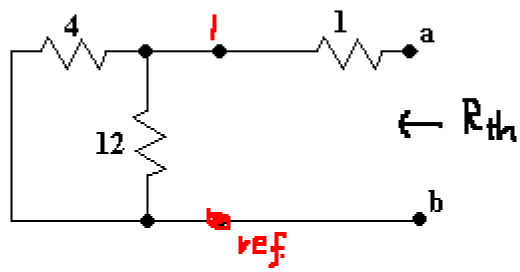
Solution:

(a) Finding V_{th} : Open-circuit voltage. Since two terminals a and b are open, there is no current flowing through 1 Ω resistor. If we apply the node-voltage method, the open circuit voltage is the same as the node voltage V_1.



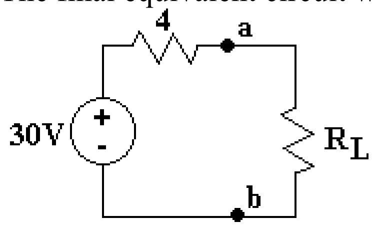
Therefore, @node 1: $\frac{V_1 - 32}{4} + \frac{V_1}{12} - 2 = 0$ ---> V_1=30 V ----->V_{th}=30 V

(b) Finding R_{th}: After deactivating independent sources, we have,



Therefore, R_{th}=R_{ab}=1 + (4//12) = 1+3=4 Ω

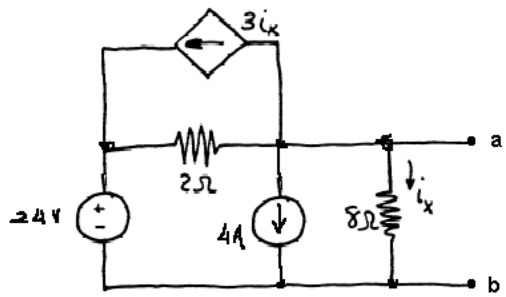
(c) Finding the load current: The final equivalent circuit with the load is reduced to:



Therefore, $I_L = \frac{30}{4+6} = 3\,[\mathrm{A}]$

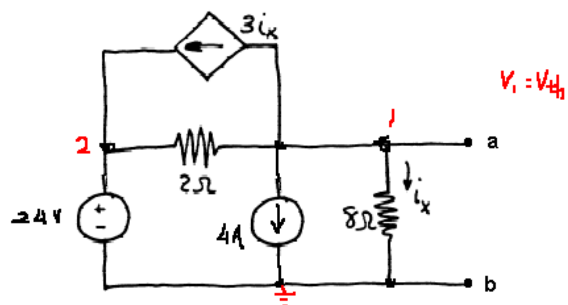
9. An example of **Short Current Method** and **Test Voltage Method**

Find the Thevenin equivalent circuit of the following circuit.



Solution:

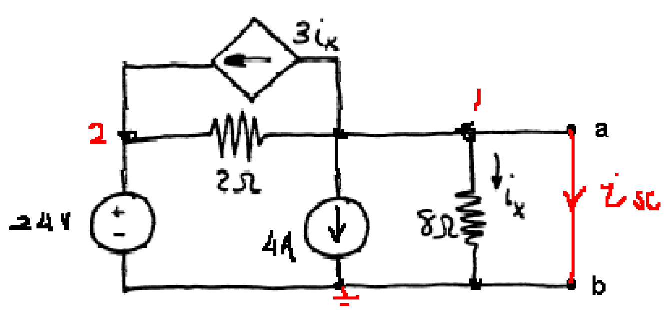
(a) **Derivation of V_{th}.**



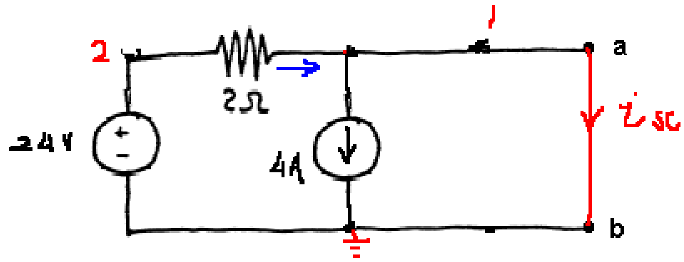
Constraints: $V_2 = 24$; $i_x = \dfrac{V_1}{8}$

@ node 1: $\dfrac{V_1 - 24}{2} + 4 + \dfrac{V_1}{8} + 3 \cdot \dfrac{V_1}{8} = 0$ ------------>V_1=8 [V]=V_{th}

(b) **Derivation of R_{th} by Short Current Method**: First, two terminals *a* and *b* are shorted to find the short current I_{sc}.



When and b are shorted out, there is no current through 8 resistor, therefore, i_x=0. Hence, the dependent source disappears from the circuit. Therefore, the circuit has changed to:

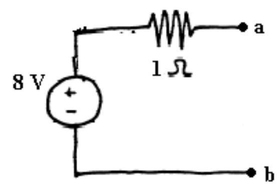


The circuit is very weird, but somehow we may apply node-voltage equation like:

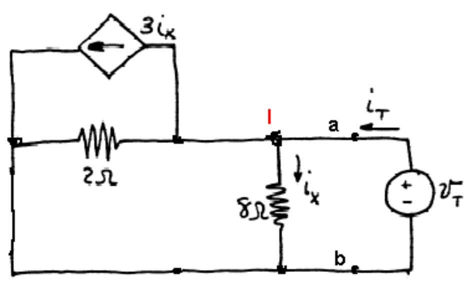
$$\frac{24}{2} = 4 + I_{sc}\text{, so } I_{sc}=8.$$

Therefore $R_{th}=8/8=1$ [Ω]

So, the Thevenin equivalent circuit is:



(c) **Derivation of R_{th} by Test Voltage Method**: After deactivation of the independent sources, we have the following circuit.

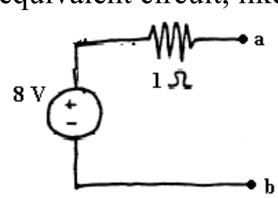


Constraint: $i_x = \frac{V_1}{8}$, $V_1=V_T$.

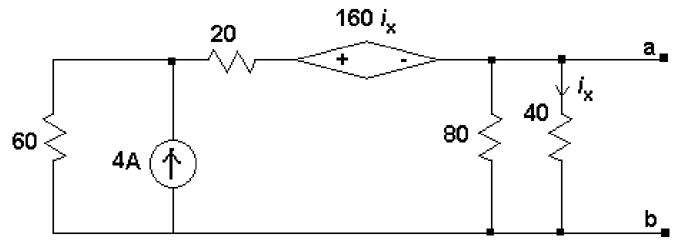
Applying **KCL** at node 1: $I_T = i_x + \frac{V_T}{2} + 3i_x = \frac{8V_T}{8} = V_T$

Therefore, $R_{th} = \frac{V_T}{I_T} = 1$

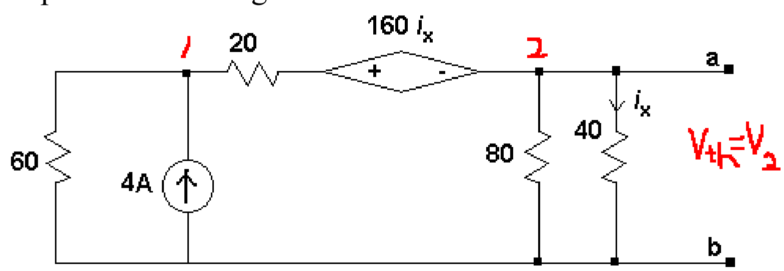
So we have the same Thevenin equivalent circuit, like this.



10. Another Thevenin equivalent circuit problem.



(a) V_{th} derivation: Open-circuit voltage



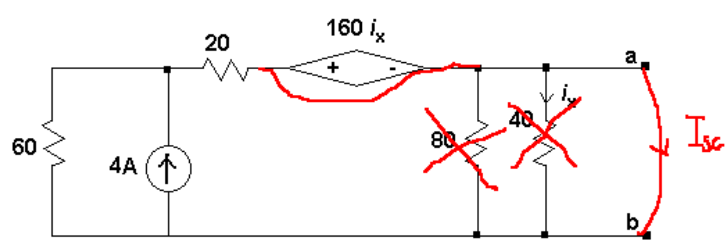
Constraint: $i_x = \frac{V_2}{40}$, therefore $160i_x=4V_2$

@ node 1: $\frac{V_1}{60} - 4 + \frac{V_1 - V_2 - 4V_2}{20} = 0$ ------(1)

@ node 2: $\frac{V_2}{80} + \frac{V_2}{40} + \frac{V_2 - V_1 + 4V_2}{20} = 0$ -----(2)

From (1) and (2): $V_2=V_{th}=30$ [V]

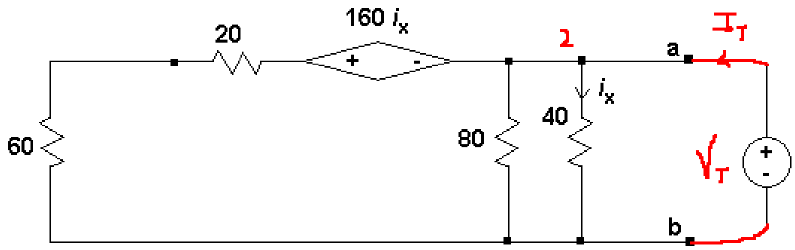
(b) Derivation of R_{th} by Short Current Method: If you short the terminal, then the circuit becomes like below: (Remember $i_x=0$)



By current-division, we have: $I_{sc} = 4 \cdot \frac{60}{60+20} = 3$

Therefore, $R_{th} = 30/3 = 10$ [Ω]

(c) Derivation of R_{th} by Test Voltage Method: After deactivation of the independent source and applying a test voltage, we have the following circuit.



Constraint: $i_x = \frac{V_T}{40}$, therefore $160i_x=4V_T$

[Applying KCL] @ node 2: $I_T = \frac{V_T}{80} + \frac{V_T}{40} + \frac{V_T + 4V_T}{80}$,

from this $\frac{V_T}{I_T} = 10 = R_{th}$

(d) Final Thevenin equivalent circuit?