EECE202 NETWORK ANALYSIS I Dr. Charles Kim

Class Note 19: Operational Amplifier (OP Amp)

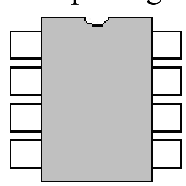
CHAPTER 5. The Operational Amplifier[1]

A. INTRODUCTION

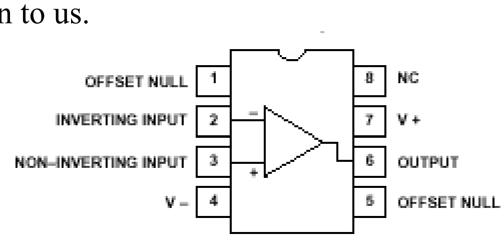
1. The *operational amplifier* or *op amp* for short, is a versatile circuit building block.
2. The *op amp* is an electronic unit that behaves like a voltage-controlled voltage source.
3. The *op amp* may also be regarded as a voltage amplifier with very high gain.
4. An op amp can sum signals, amplify a signal, integrate it, or differentiate it. The ability of the op amp to perform these mathematical operations is the main reason it is called an operational amplifier.
5. The term *operational amplifier* was introduced by John Regazzini and his colleagues, in their work on analog computers for the National Defense Research Council during World War II. The first op amps used vacuum tubes rather transistors.

B. OP AMP PACKAGE

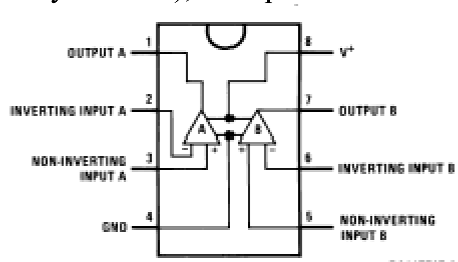
1. The op amp is an electronic device consisting of a complex arrangement of resistors, transistors, capacitors, and diodes.
2. Op amps are commercially available in integrated circuit (IC) packages in several forms. A typical one is the 8-pin single or dual in-line package (DIP) shown below.



3. For a single DIP op amp (typically **uA741)**, pin or terminal 8 is unused (NC), and terminals 1 and 5 are of little concern to us.

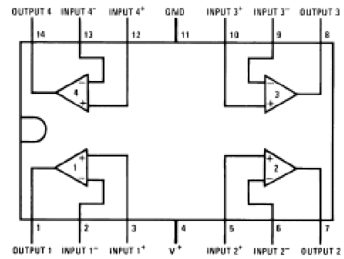


4. For a dual DIP op amp (typically **LM348**), all 8 pins are used for two op amps.



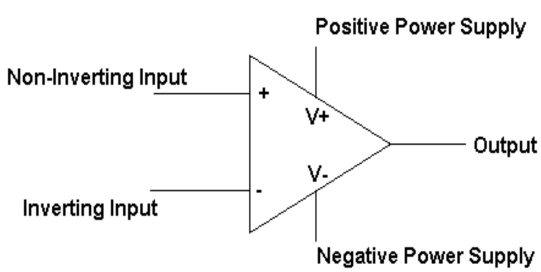
[1] We pass through quickly this subject because other courses, specifically Electronics I and II, cover this subject rather extensively in its semiconductor fabrication and design application. The main goal of this chapter is to get student to be familiar with the node voltage method application when active elements are present in a circuit.

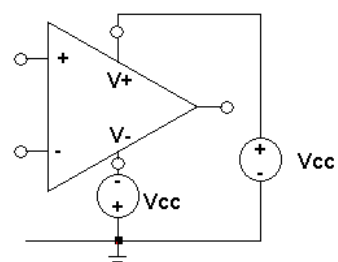
5. A typical quad op amp (LM324) is displayed below.



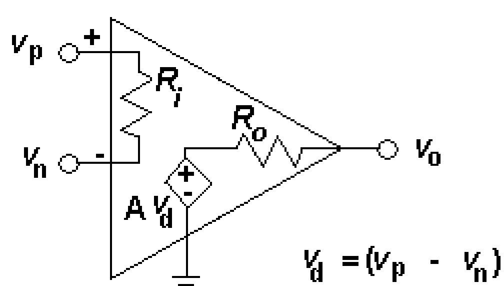
C. CIRCUIT SYMBOLS AND TERMINAL BEHAVIORS

1. The five important terminals in an op amp are:
   (a) Inverting input (-)
   (b) Noninverting input (+)
   (c) Output
   (d) Positive power supply, V+
   (e) Negative power supply, V-
2. The circuit symbol for the op amp is the triangle.
3. An input applied to the noninverting terminal appears with the same polarity at the output.
4. An input applied to the inverting terminal appears inverted at the output.
5. The op amp must be powered by a voltage supply.





3. An input applied to the noninverting terminal appears with the same polarity at the output.
4. An input applied to the inverting terminal appears inverted at the output.
5. The op amp must be powered by a voltage supply.
6. The equivalent circuit model of an op amp is shown below. The output section consists of a voltage-controlled dependent voltage source in series with the output resistance R_o. The output resistance R_o is the Thevenin equivalent resistance seen at the output terminal. The input resistance R_i is the Thevenin equivalent resistance seen at the input terminals.



7. Therefore, according to above equivalent circuit of op amp, it can be said that the op amp senses the difference between the two inputs, multiplies it by the gain ***A***, and causes the resulting voltage to appear at the output.
8. The output voltage of the equivalent circuit then is given by, $v_o = A(v_p - v_n)$ where v_p is the voltage between the non inverting terminal and ground, and v_n is the voltage between the inverting terminals and ground. ***A*** is called the open-loop voltage gain since it is the gain of the op amp without any external feedback from output to input.
9. Typical values of voltage gain A, input resistance, output resistance, and supply voltage:

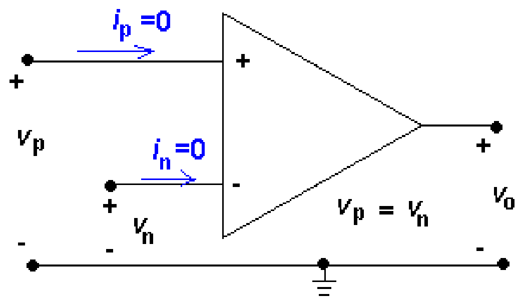
Parameters	**Typical Range**	**Ideal Values**
Open-loop gain, ***A***	10^5 - 10^8	∞
Input Resistance, R_i	10^6 - 10^{13} Ω	∞ Ω
Output Resistance, R_o	10 - 100 Ω	0 Ω
Supply Voltage, V_{cc}	5 – 24 V	

9. Output voltage limitation: The magnitude of the output voltage cannot exceed $|V_{cc}|$. Depending on the power supply voltage and the differential input voltage $v_d = v_p - v_n$, op amp can operate in three modes:
   (a) Positive saturation: $v_o = V_{cc}$
   (b) Linear region: $-V_{cc} \le v_o \le V_{cc}$
   (c) Negative saturation: $v_o = -V_{cc}$
10. The voltage transfer characteristics combine the three regions of mode.

$$v_o = \begin{cases} -V_{cc} & if \quad A(v_p - v_n) < -V_{cc} \\ A(v_p - v_n) & if \quad -V_{cc} \le A(v_p - v_n) \le +V_{cc} \\ +V_{cc} & if \quad A(v_p - v_n) > +V_{cc} \end{cases}$$

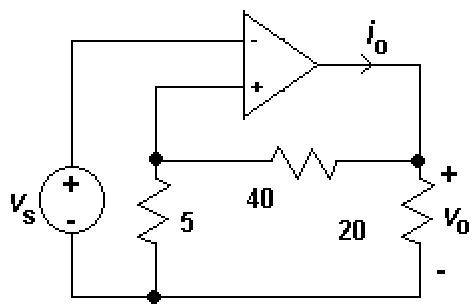
D. IDEAL OP AMP MODEL

1. An op amp is ideal if it has the following characteristics:
   (a) Infinite open-loop gain, i.e., A = ∞
   (b) Infinite input resistance, i.e., R_i=∞ Ω
   (c) Zero output resistance, i.e., R_o= 0 Ω
2. Two important characteristics of the ideal op amp for circuit analysis:
   (a) The current into both input terminals are zero, i.e., $\boldsymbol{i_p = i_n = 0}$. This is due to infinite input resistance: an open circuit exists between two terminals and current cannot flow through.
   (b) The voltage across the input terminals is negligibly small, i.e., $\boldsymbol{v_p = v_n}$.. This is due to infinite open-loop gain. In the linear region, the magnitude of the output voltage must be less than the supply power voltage, i.e., $-V_{cc} \le A(v_p - v_n) \le V_{cc}$. Even for a practical op amp, the gain A is about the 10^5, and the V_{cc} is just about 24V. Therefore, to satisfy the inequality for the linear region mode, $(v_p - v_n) \le \frac{24}{10^5} = 0.24 \times 10^{-3} = 0.24$ [mV].



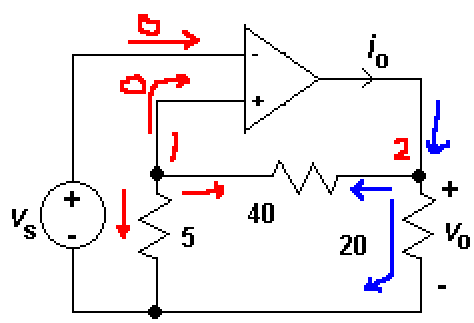
4. Example using the ideal op amp model and the non-ideal op amp equivalent circuit model:

Q: Calculate the closed-loop gain (i.e., there *is* a feedback) $\frac{v_o}{v_s}$, and find i_o when $v_s = 1$ [V].



Solution A: using the ideal op amp model

(a) By the constraints of ideal op amp, $v_p=v_n$ and $i_p=i_n=0$, and node voltage method application:



(b) Constraints and hidden values: by the ideal op amp model, $V_1=V_s$ and $V_o=V_2$.

(c) @ node 1: $0+\frac{V_1-V_o}{40}+\frac{V_1}{5}=0$ ---> $V_o=9V_1=9V_s$

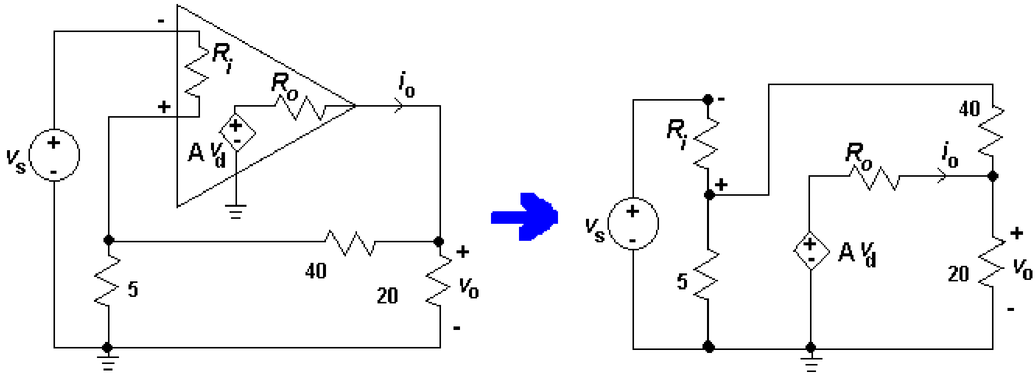
Therefore the closed-loop gain is: $\frac{v_o}{v_s}=9$

(d) @ node 2: $i_0=\frac{V_o-V_1}{40}+\frac{V_o}{20}=\frac{2V_o-V_s}{40}$ ---(with V_s=1)----

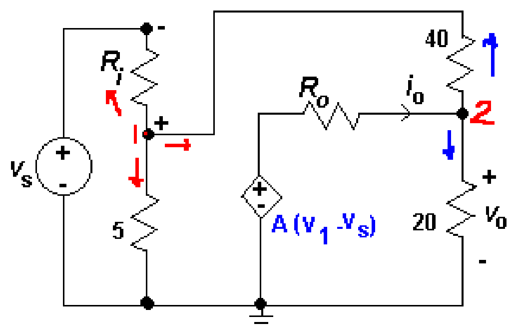
$>i_0=\frac{3(9V_s)-V_s}{40}=\frac{26}{40}=0.65$ [A]

Solution B: using the non-ideal op amp model (with R_i=2MΩ, R_0=50 Ω, and open-loop gain A=$2x10^5$.)

(a) The circuit diagram and its redrawn circuit are shown below:



(b) Let's apply the node-voltage method to the right circuit.



(c) @ node 1: $\frac{V_1 - V_s}{R_i} + \frac{V_1}{5} + \frac{V_1 - V_o}{40} = 0$ (Since V_o=V_2).

The first term is much smaller than the other two terms (because R_i is too big), therefore, the equation is approximated to: $\frac{V_1}{5} + \frac{V_1 - V_o}{40} = 0$ ---->$V_o = 7V_1$

(d) @ node 2: $\frac{V_o - V_1}{40} + \frac{V_2}{20} + \frac{V_0 - A(V_1 - V_s)}{R_o} = \frac{V_o - V_1}{40} + \frac{V_2}{20} + \frac{V_0 - A(V_1 - V_s)}{50} = 0$

It simplifies to: $5V_0 - 5V_1 + 10V_2 + 4V_0 - 4AV_1 + 4AV_s = 0$

Again, $19V_0 - V_1(5 + 4A) + 4AV_s = 0$

Since V_o and number 5 are much smaller compare to A: $\underline{V_1 = V_s}$

Therefore, $V_1 = V_s = \frac{V_o}{7}$ ----->$\underline{\frac{V_o}{V_s} = 7}$

(e) For the current i_o: applying KCL at node 2 yields: $i_o = \frac{V_o - V_1}{40} + \frac{V_o}{20} = \frac{6V_s + 14V_s}{40} = \frac{V_s}{2}$

Therefore, when V_s=1 [V] , i_o=0.5 [A]

(f) NOTE: This result is reached by gross approximations in the calculation.

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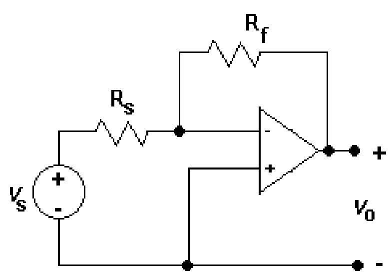
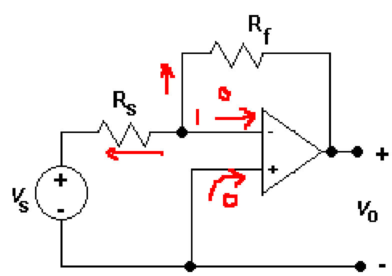
Class Note 20: OP Amp Circuits --SOLUTION

1. Let's consider some useful op amp circuits that often serve as modules for more complex circuits. The op amp circuits we will discuss are:
   - Inverting Amplifier
   - Noninverting Amplifier and Voltage Follower
   - Summing Amplifier
   - Difference Amplifier
2. Example problems are included for op amp circuit analysis practices.

A. Inverting Amplifier

1. An inverting amplifier reverses the polarity of the input signal while amplifying it.
2. A voltage V_s is connected to the inverting input.
3. The noninverting terminal is grounded.





4. Input-Output relationship (see the figure above right)

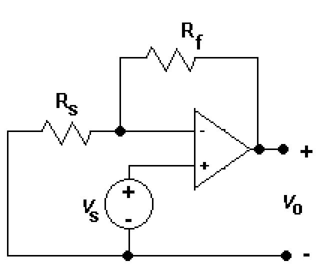
   (a) By ideal op amp model: V_1=0 (Why? V_p is grounded, so $V_p=0=V_n=V_1$)

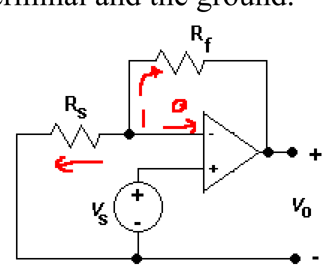
   (b) @ node 1: $\frac{0-V_s}{R_s}+\frac{0-V_o}{R_f}=0$ ---->$V_o=-\frac{R_f}{R_s}V_s$

   (c) Closed-loop gain: $A_v=-\frac{R_f}{R_s}$

B. Noninverting Amplifier and Voltage Follower

1. A noninverting amplifier provides a positive voltage gain.
2. An input voltage V_s is applied to the noninverting terminal.
3. A resistor Rs is connected to between the inverting terminal and the ground.





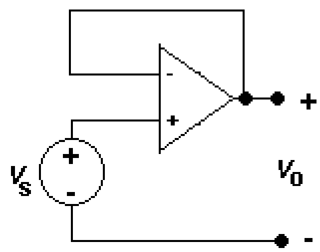
4. Input-Output relationship (see the figure above right)

   (a) By ideal op amp model: $V_1=V_s$ (Why? $V_p=V_s=V_n$)

(b) @ node 1: $\frac{V_s}{R_s}+\frac{V_s-V_o}{R_f}=0$ ----> $V_o=\left(1+\frac{R_f}{R_s}\right)V_s$

(c) Closed-loop gain: $A_v=1+\frac{R_f}{R_s}$

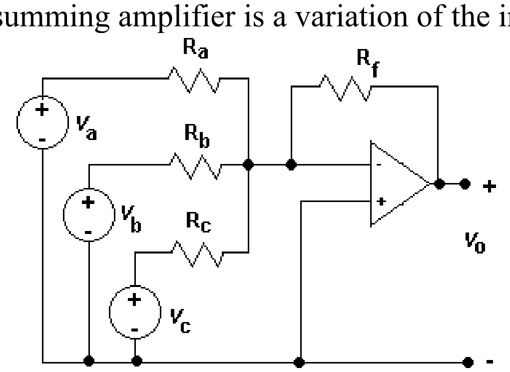
5. If the feedback resistor $R_f=0$ (short circuit) and/or input resistor $R_s=\infty$, then the closed-loop gain $A_v=1$. This unity gain amplifier is called a **voltage follower** because the output follows the input. Thus, for a voltage follower, $V_o=V_s$.

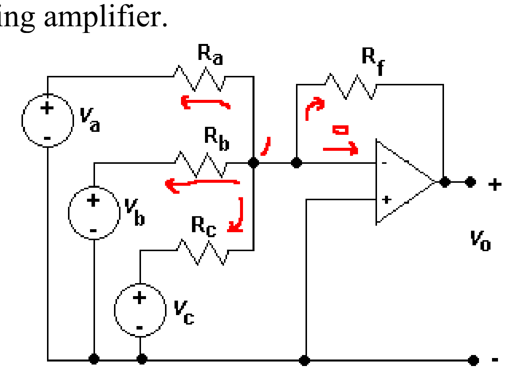


6. A voltage follower is useful as an intermediate stage amplifier (or a buffer amplifier) to isolate once circuit module from another.

C. Summing Amplifier

1. A summing amplifier combines several inputs and produces an output that is the weighted sum of the inputs.
2. A summing amplifier is also called a summer.
3. A summing amplifier is a variation of the inverting amplifier.

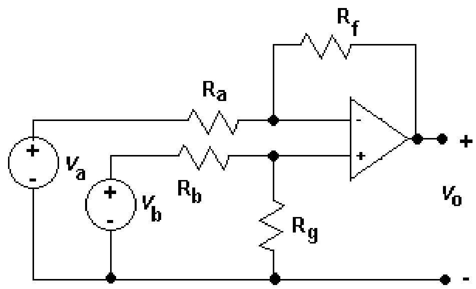


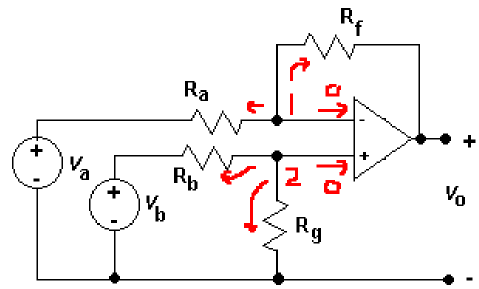


4. Input-Output relationship (see the figure above right for a 3-input summer)
   (a) By ideal op amp model: $V_1=0$ (Why? $V_p=0=V_n=V_1$)
   (b) @ node 1: $\frac{0-V_a}{R_a}+\frac{0-V_b}{R_b}+\frac{0-V_c}{R_c}+\frac{0-V_o}{R_f}=0$ ----> $V_o=-\left(\frac{R_f}{R_a}V_a+\frac{R_f}{R_b}V_b+\frac{R_f}{R_c}V_c\right)$

D. Difference Amplifier

1. A difference amplifier amplifies the difference between two inputs but rejects any signals common to the two inputs.
2. A difference amplifier is also called a differential amplifier.
3. A difference amplifier is also known as the subtractor.





4. Input-Output relationship (see the figure above right for a 3-input summer)

(a) By ideal op amp model: $V_1=V_2$ (Why? $V_p=V_n$)

(b) @ node 1: $\frac{V_1-V_a}{R_a}+\frac{V_1-V_0}{R_f}=0$ ----> $V_o=\left(\frac{R_f}{R_a}+1\right)V_1-\frac{R_f}{R_a}V_a$ -----(1)

(c) @ node 2: $\frac{V_1-V_b}{R_b}+\frac{V_1}{R_g}=0$ -----> $V_1=\frac{R_gV_b}{R_b+R_g}$ ----(2)

(d) Substituting (2) into (1) yields:

$$V_o=\left(\frac{R_f}{R_a}+1\right)\cdot\frac{R_g}{R_b+R_g}V_b-\frac{R_f}{R_a}V_a=\frac{R_f}{R_a}\left(\frac{1+R_a/R_f}{1+R_b/R_g}\cdot V_b-V_a\right) \text{---------(3)}$$

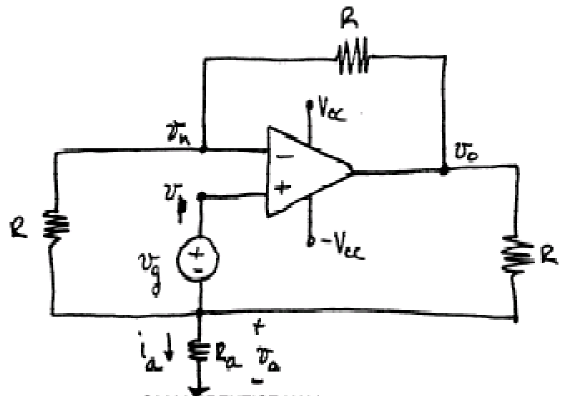
(e) Since a difference amplifier must reject a signal common to the two inputs, it's output must be zero, i.e., $V_o=0$, when $V_a=V_b$. Applying this condition to (3) yields: $\frac{R_a}{R_f}=\frac{R_b}{R_g}$.

(f) Therefore, when $\frac{R_a}{R_f}=\frac{R_b}{R_g}$, the output is: $V_o=\frac{R_f}{R_a}(V_b-V_a)$

(g) When $R_a=R_f$ and $R_b=R_g$, the difference amplifier becomes a subtractor with the output $V_o=V_b-V_a$

E. Op Amp Circuit Analysis `Problems

1. Show that $i_a = \dfrac{3v_g}{R}$.



$$\frac{v_n - v_a}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_a = v_o$$

$$\frac{v_a}{R_a} + \frac{v_a - v_n}{R} + \frac{v_a - v_o}{R} = 0$$

$$v_a\left[\frac{1}{R_a} + \frac{2}{R}\right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_a\left(2 + \frac{R}{R_a}\right) - v_n = v_o$$

$$v_n = v_p = v_a + v_g$$

$$\therefore \quad 2v_n - v_a = 2v_a + 2v_g - v_a = v_a + 2v_g$$

$$\therefore \quad v_a - v_o = -2v_g \qquad (1)$$

$$2v_a + v_a\left(\frac{R}{R_a}\right) - v_a - v_g = v_o$$

$$\therefore \quad v_a\left(1 + \frac{R}{R_a}\right) - v_o = v_g \qquad (2)$$

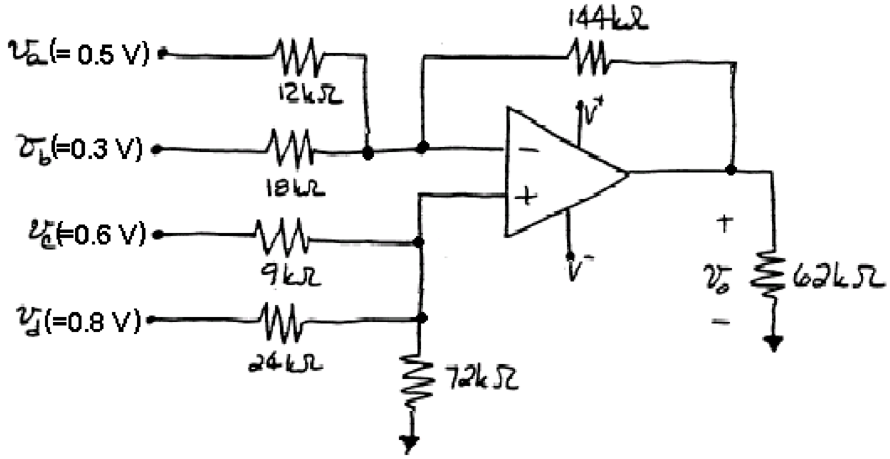
Now combining equations (1) and (2) yields

$$-v_a\frac{R}{R_a} = -3v_g$$

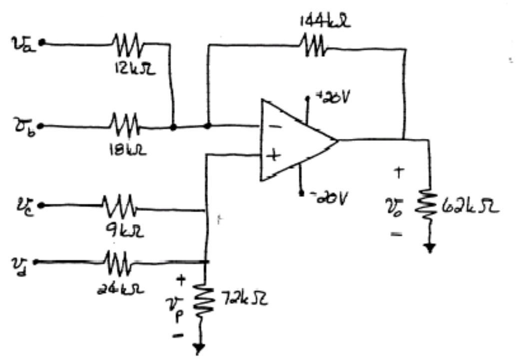
$$\text{or} \quad v_a = 3v_g\frac{R_a}{R}$$

$$\text{Hence} \quad i_a = \frac{v_a}{R_a} = \frac{3v_g}{R} \qquad \text{Q.E.D.}$$

2. Find v_o.



P 5.27 [a]



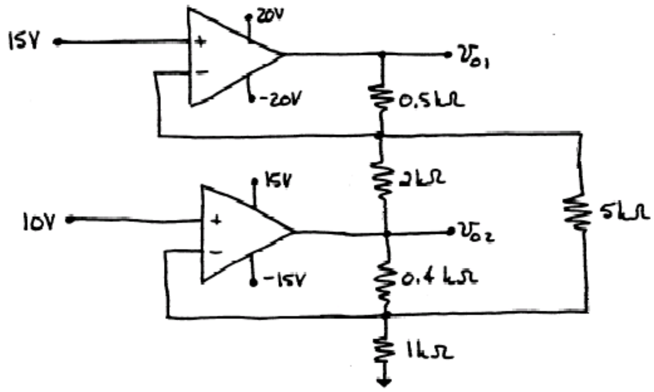
$$\frac{v_p}{72} + \frac{v_p - v_c}{9} + \frac{v_p - v_d}{24} = 0$$

$$\therefore \quad v_p = (2/3)v_c + 0.25v_d = v_n$$

$$\frac{v_n - v_a}{12} + \frac{v_n - v_b}{18} + \frac{v_n - v_o}{144} = 0$$

$$\begin{aligned} \therefore \quad v_o &= 21v_n - 12v_a - 8v_b \\ &= 21[(2/3)v_c + 0.25v_d] - 12v_a - 8v_b \\ &= 21(0.4 + 0.2) - 12(0.5) - 8(0.3) = 4.2 \text{ V} \end{aligned}$$

3. Calculate v_{o1} and v_{o2} .



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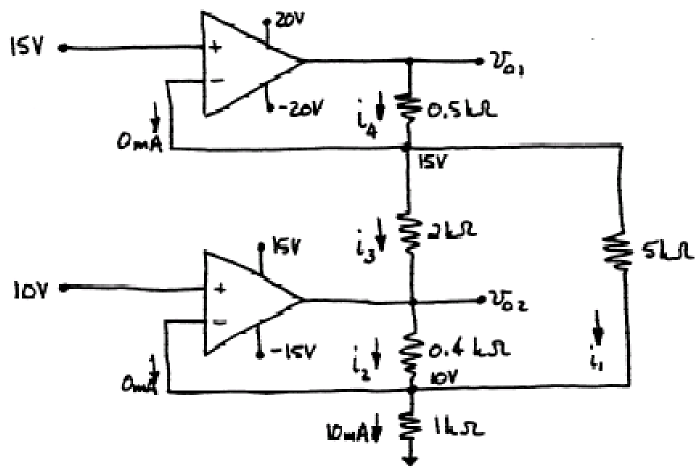


$$i_1 = \frac{15-10}{5} = 1\,\text{mA}$$

$i_2 + i_1 + 0 = 10; \qquad i_2 = 9\,\text{mA}$

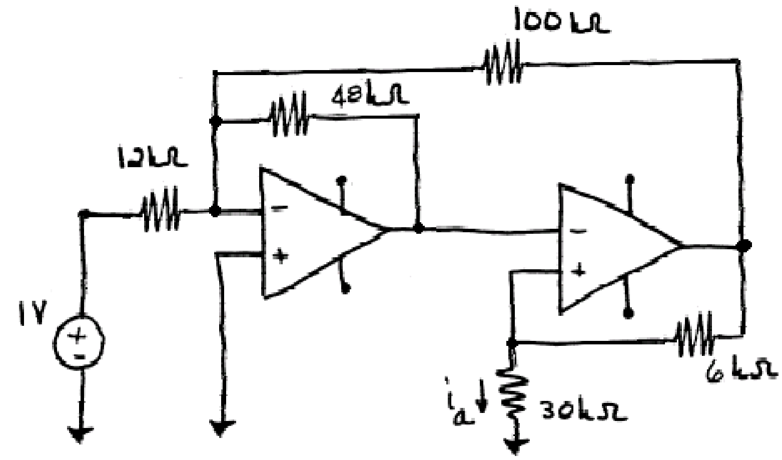
$v_{o2} = 13.6\ \text{V}$

$$i_3 = \frac{15-13.6}{2} = 0.7\,\text{mA}$$

$i_4 = i_3 + i_1 = 1.7\,\text{mA}$

$v_{o1} = 15 + 1.7(0.5) = 15.85\ \text{V}$

4. Find i_a.



P 5.18 Let v_{o1} be the output voltage of the first operational amplifier and v_{o2} the output voltage of the second operational amplifier. Then

$$\frac{0-1}{12} + \frac{0-v_{o1}}{48} + \frac{0-v_{o2}}{100} = 0$$

$$-50 - 12.5v_{o1} - 6v_{o2} = 0$$

$$\frac{v_{o1}}{30} + \frac{v_{o1}-v_{o2}}{6} = 0$$

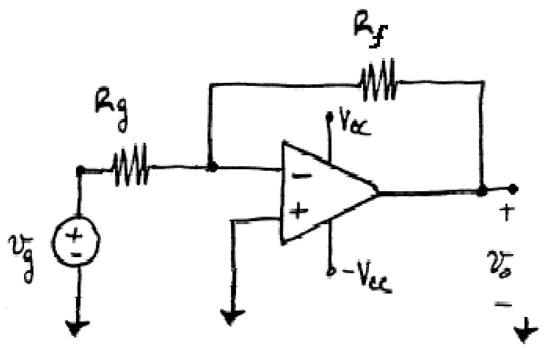
$$\therefore \quad 6v_{o1} = 5v_{o2}$$

$$\therefore \quad -50 - 12.5[(5/6)v_{o2}] - 6v_{o2} = 0 \quad \text{so} \quad v_{o2} = -3.05 \text{ V}$$

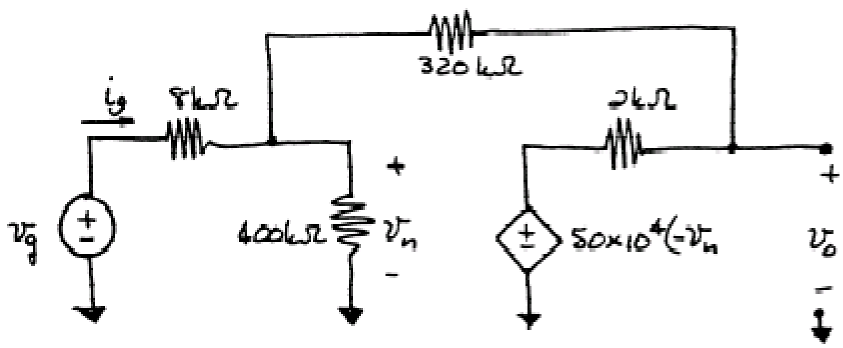
$$i_a = \frac{v_{o2}}{36} = -0.0846 \text{ mA}$$

$$i_a = -84.6\,\mu\text{A}$$

5. The op amp has an input resistance of 400 kΩ, and output resistance of 2kΩ, and an open-loop gain of 500,000. Calculate the closed-loop voltage gain $A_v = \frac{v_o}{v_g}$ when $R_g = 8\,\text{k}\Omega$ and $R_f = 320\,\text{k}\Omega$.



P 5.40 [a]



$$\frac{v_n}{400} + \frac{v_n - v_g}{8} + \frac{v_n - v_o}{320} = 0$$

$\therefore \quad 41.8v_n - v_o = 40v_g$

$$\frac{v_o - 500{,}000(-v_n)}{2} + \frac{v_o - v_n}{320} = 0$$

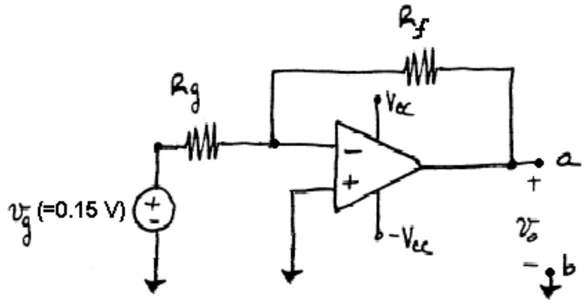
$\therefore \quad 80 \times 10^6 v_n + 161v_o = 0$

$$\Delta = \begin{vmatrix} 41.8 & -1 \\ 80 \times 10^6 & 161 \end{vmatrix} = 80{,}006{,}729.8$$

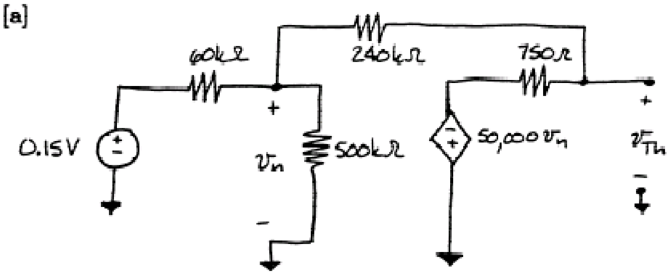
$$N_o = \begin{vmatrix} 41.8 & 40v_g \\ 80 \times 10^6 & 0 \end{vmatrix} = -32 \times 10^8 v_g$$

$$v_o = \frac{N_o}{\Delta} = -39.997v_g; \qquad \text{so } \frac{v_o}{v_g} = -39.997$$

6. The op amp has an input resistance of 500 kΩ, and output resistance of 750 Ω, and an open-loop gain of 50,000. Find the Thevenin equivalent circuit with respect to the output terminals ***a*** and ***b*** when $R_g = 60\,\mathrm{k\Omega}$ and $R_f = 240\,\mathrm{k\Omega}$.



P 5.43 [a]

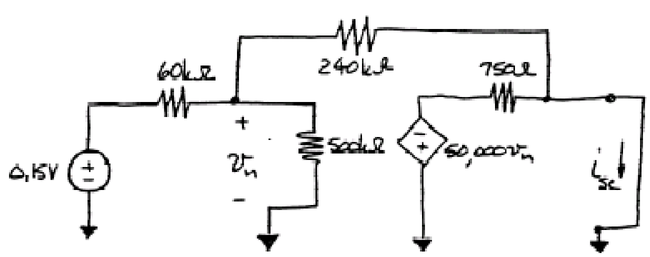


$$\frac{v_n - 0.15}{60} + \frac{v_n}{500} + \frac{v_n - v_{Th}}{240} = 0$$

$$\frac{v_{Th} + 5 \times 10^4 v_n}{0.75} + \frac{v_{Th} - v_n}{240} = 0$$

Solving, $v_{Th} = -0.6$ V

Short-circuit current calculation:



$$\frac{v_n}{500} + \frac{v_n - 0.15}{60} + \frac{v_n - 0}{240} = 0$$

$\therefore\ v_n = 0.1095$ V

$$i_{sc} = \frac{v_n}{240{,}000} - \frac{5 \times 10^4}{750} v_n = -7.3 \text{ A}$$

$$R_{Th} = \frac{v_{Th}}{i_{sc}} = 82.2\,\mathrm{m\Omega}$$