499 HW5 SP22 Define Euler identify

Euler(θ) = exp(jθ) = cosθ+jsinθ = 1<θ

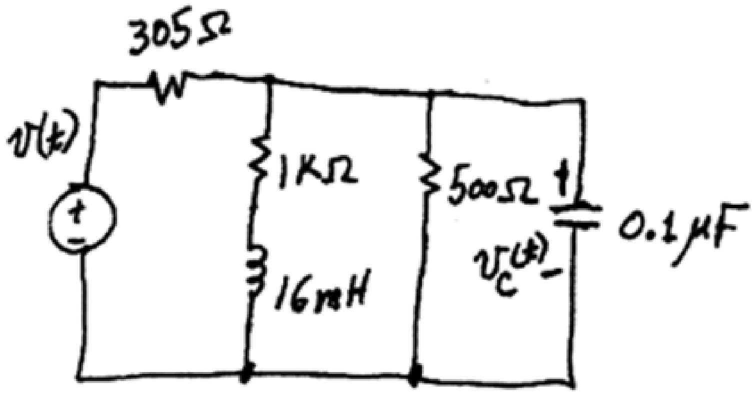
$$\text{Euler}(\text{deg}) := e^{\text{deg}\cdot\frac{\pi}{180}\cdot i}$$

Test

$\text{Euler}(0) = 1$ $\text{Euler}(90) = -3\cdot 10^{-15} + i$ $\text{Euler}(180) = -1 - 6\cdot 10^{-15}\cdot i$

1

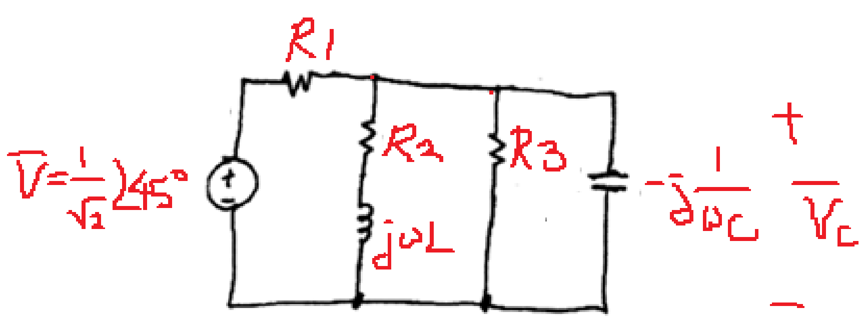
1. Find the voltage $v_c(t)$ by steady-state analysis, where $v(t) = \cos(2\pi f t + 45°)\ [V]$ with $f = 10^4$ [Hz].



$f := 10^4$ $\omega := 2\cdot\pi\cdot f = 62831.8530718$ $L := 16\cdot 10^{-3}$ $C := 0.1\cdot 10^{-6}$

$R1 := 305$ $R2 := 1000$ $R3 := 500$

Phasor Circuit



$$V := \frac{1}{\sqrt{2}}\cdot\text{Euler}(45) = 0.5 + 0.5\cdot i$$

$$|V| = 0.7071068$$

$$\arg(V)\cdot\frac{180}{\pi} = 45$$

Equivalent impedance of the 3 parallel branches: Zp

$$\frac{1}{\omega\cdot C} = 159.154943$$

$$Zp := \frac{1}{\frac{1}{R2 + \omega\cdot L\cdot i} + \frac{1}{R3} + \frac{1}{-\frac{1}{\omega\cdot C}\cdot i}} = 62.9340027 - 145.7381291\cdot i$$

$$\omega\cdot L = 1005.309649$$

Vc can be found by voltage division

$$Vc := V\cdot\frac{Zp}{Zp + R1} = 0.2836423 - 0.0001755\cdot i$$

$$|Vc| = 0.2836423 \qquad \arg(Vc)\cdot\frac{180}{\pi} = -0.0354483$$

$$|Vc|\cdot\sqrt{2} = 0.4011308$$

vc(t) =0.4*cos(ωt-0.035)

2. In a computer center, there are three single-phase computer devices (description listed below) installed in parallel. The magnitude of the voltage of each device is 208 [V].

Disk: 6.157 kVA at pf = 0.79 lag
Drum: 16.93 kW at pf = 0.96 lag
CPU: 22.694 kW while the current through the CPU = 127 [A]

Find the power factor of the combined computer device (i.e., *pf* of the computer center).

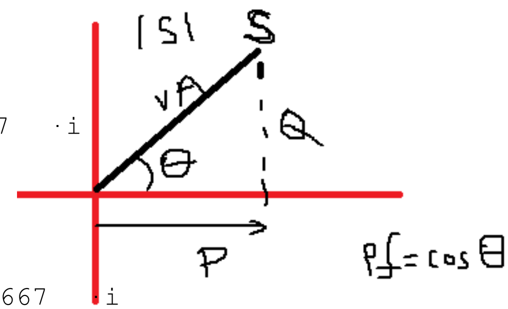
Vmag := 208 Icpumag := 127

$\theta disk := acos(0.79) = 0.6599873$

$Sdisk := 6157 \cdot Euler\left(\theta disk \cdot \frac{180}{\pi}\right) = 4864.03 + 3774.8988277 \cdot i$

$\theta drum := acos(0.96) = 0.2837941$

$Sdrum := 16930 + 16930 \cdot tan(\theta drum) \cdot i = 16930 + 4937.9166667 \cdot i$



$Q = P \cdot tan\,\theta$

$P = VA \cdot cos\,\theta$

$P = |V||I| \cdot cos\,\theta$

$Q = |V||I| \cdot sin\,\theta$

from Pcpu = Vmag* Icpumag*cos(θcpu)

Pcpu := 22694

$\theta cpu := acos\left(\frac{Pcpu}{Vmag \cdot Icpumag}\right) = 0.5372867$

$\theta cpu \cdot \frac{180}{\pi} = 30.7842583$

$Qcpu := Vmag \cdot Icpumag \cdot sin(\theta cpu) = 13519.8897925$

$Scpu := Pcpu + Qcpu \cdot i = 22694 + 13519.8897925 \cdot i$

Combined complex power of the computer devices:

$S := Sdisk + Sdrum + Scpu = 44488.03 + 22232.7052868 \cdot i$

$|S| = 49734.0728038$

$\theta S := arg(S)$

$\theta S \cdot \frac{180}{\pi} = 26.5533974$

$pfS := cos(\theta S) = 0.8945181$	lagging

$acos(0.79) \cdot \frac{180}{\pi} = 37.814489$

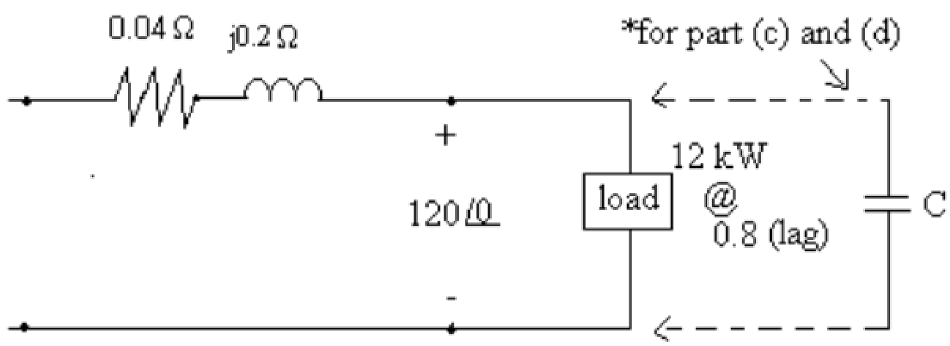
3. A load on a 60-Hz system requires 12kW at 0.8 pf lagging when operated at 120V. The impedance of the feeder supplying the load is 0.04+j0.2 Ω. (See circuit below)

(a) What is the magnitude of the voltage at the source?

(b) What is the power loss in the feeder line?

(c) To improve the pf of the load to 0.96 (lagging), what size capacitor (in microfarads) at the load end is needed?

(d) After the capacitor is installed, what is the magnitude of the voltage at the source, if the load voltage is maintained at 120 V ?



$P1 := 12000$ $pf1 := 0.8$ $V := 120 \cdot Euler(0) = 120$ $f := 60$

$\theta1 := acos(pf1)$ $\omega := 2 \cdot \pi \cdot f = 376.9911184$

$Q1 := P1 \cdot tan(\theta1) = 9000$ $Zt := 0.04 + 0.2 \cdot i = 0.04 + 0.2 \cdot i$

$S1 := P1 + Q1 \cdot i = 12000 + 9000 \cdot i$

From S = V I* I = (S/V)*

$I1 := Re\left(\frac{S1}{V}\right) - Im\left(\frac{S1}{V}\right) \cdot i = 100 - 75 \cdot i$

$|I1| = 125$ $arg(I1) \cdot \frac{180}{\pi} = -36.8698976$

Source voltage = I*Zt + V

$Vs := I1 \cdot Zt + V = 139 + 17 \cdot i$ $|Vs| = 140.0357097$ $arg(Vs) \cdot \frac{180}{\pi} = 6.9727693$

Power loss in the line Pline = |I|^2 * Rline

$Ploss := |I1|^2 \cdot 0.04 = 625$ W

Now load 2 (with Capacitor only) is added to make the total load has pf of 0.96

Since P2=0, therefore the combine complex power S has the same real power (of load 1).

$P := P1 = 12000$ $pf := 0.96$ $P2 := 0$

$\theta := acos(pf) = 0.2837941$ $tan(\theta) = 0.2916667$

$\tan(\theta) = 0.2916667$

Then $Q := P \cdot \tan(\theta) = 3500$

$S := P + Q \cdot i = 12000 + 3500 \cdot i$

Since Q = Q1 + Q2, Q2 = Q - Q1 This is required Q from Load 2

$Q2 := Q - Q1 = -5500$

$S2 := P2 + Q2 \cdot i = -5500 \cdot i$

$\omega = 376.9911184$

Now current through Load 2 is: From S=VI* --> I =(S/V)*

$I2 := \mathrm{Re}\left(\frac{S2}{V}\right) - \mathrm{Im}\left(\frac{S2}{V}\right) \cdot i = 45.8333333 \cdot i$

Then the impedance of load 2

$Z2 := \frac{V}{I2} = -2.6181818 \cdot i$

Since Z2 = - j (1/wC)

$C := \frac{1}{\omega \cdot |Z2|} = 0.0010131$ F

$C \cdot 10^{6} = 1013.1391053$ uF

Now the total current is I1 + I2

$I := I1 + I2 = 100 - 29.1666667 \cdot i$

$Vs := V + I \cdot Zt = 129.8333333 + 18.8333333 \cdot i$

$|Vs| = 131.192183$

$\arg(Vs) \cdot \frac{180}{\pi} = 8.2536288$

$Ploss2 := |I|^{2} \cdot 0.04 = 434.0277778$ W

4. A three-phase line has an impedance of 0.8 + j 2.4 Ω each phase. The line feeds two balanced three-phase loads that are connected in parallel. The first load is absorbing a total of 144 kW and 108kVar. The second load is Δ-connected and has an impedance of 144 – j 42 Ω each phase. The line-to-neutral voltage at the load end of the line is 2400 V. What is the magnitude of the *line voltage* at the source end of the line?

3 phase load 1

$S31 := 144000 + 108000 \cdot i$

Load 2

$Z\Delta := 144 - 42 \cdot i$

Load Line voltage

$VAN := 2400$

Line impedance

$Zline := 0.8 + 2.4 \cdot i$

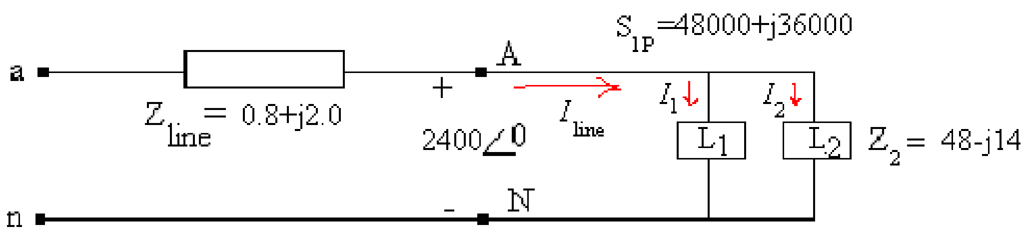
per -phase analysis

single phase power of Load 1

$S1 := \frac{S31}{3} = 48000 + 36000 \cdot i$

Δ to Y conversion

$ZY := \frac{Z\Delta}{3} = 48 - 14 \cdot i$



From S=VI* --> I=(S/V)*

$I1 := Re\left(\frac{S1}{VAN}\right) - Im\left(\frac{S1}{VAN}\right) \cdot i = 20 - 15 \cdot i$

$I2 := \frac{VAN}{ZY} = 46.08 + 13.44 \cdot i$

$Iline := I1 + I2 = 66.08 - 1.56 \cdot i$

$Van := VAN + Iline \cdot Zline = 2456.608 + 157.344 \cdot i$

$|Van| = 2461.6417286$

$arg(Van) \cdot \frac{180}{\pi} = 3.6647482$

Since Line voltage is sqrt(3) bigger and 30 deg angle shift

$Vab := \sqrt{3} \cdot Van \cdot Euler(30) = 3548.6480989 + 2363.5009351 \cdot i$

$\boxed{|Vab| = 4263.688544}$

$arg(Vab) \cdot \frac{180}{\pi} = 33.6647482$