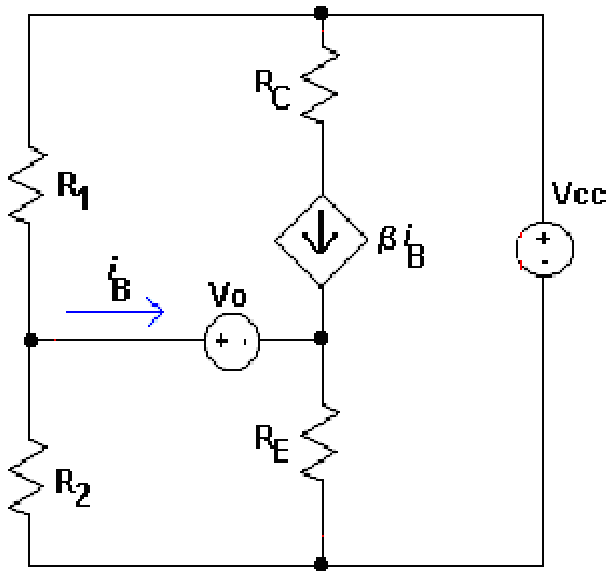
Dependent Source Problem

EECE499-Spring 2022   HW1

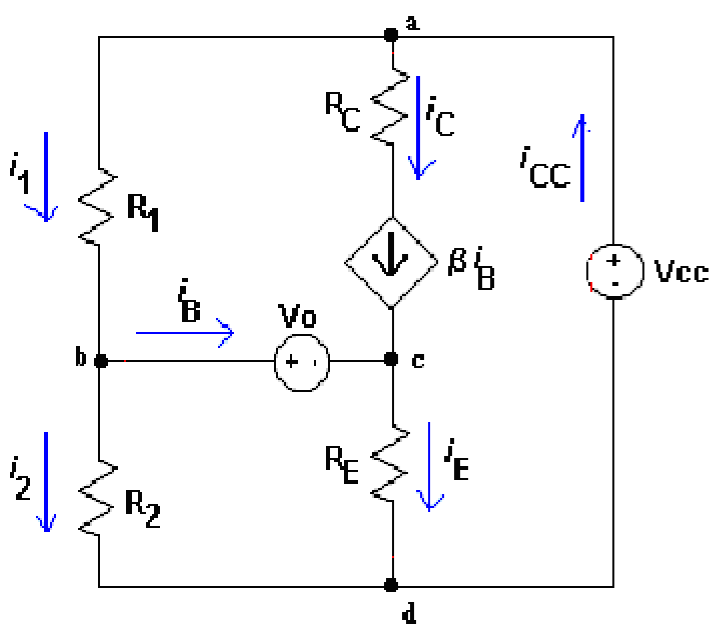
Solution

Problem 2:

In a given circuit below, all the resistances and sources are known.
In other words, the known are: V_{cc}, V_o, R_1, R_2, R_E, and R_C.
And the question is: **Find equation for i_B in terms of all the known values.**



Solution:
1. The fist thing we do is to mark nodes (a, b, c, and d) and flow indication of the currents (i_1, i_2, i_B, i_E, i_C, and i_{CC})



2. Since the current i_B is our target, it must be centered in any equation derivation.

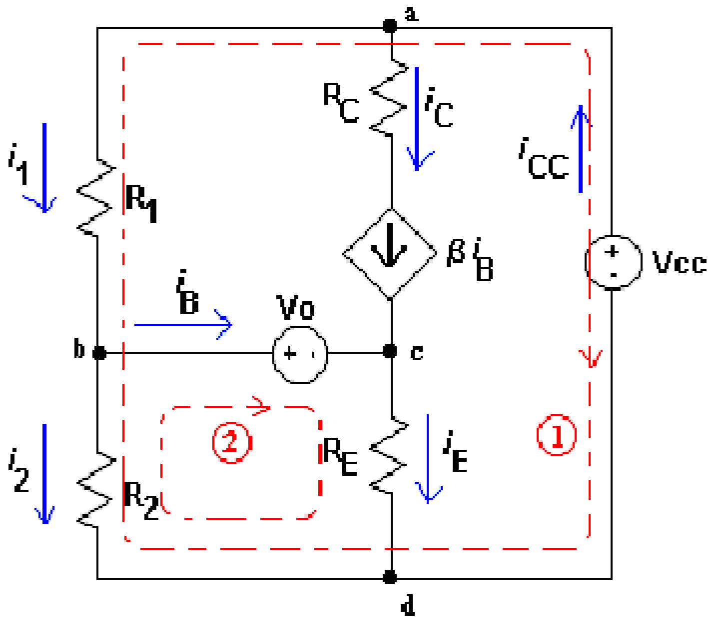
3. At node c, we have a relationship for the current:

KCL @c: $i_B + i_C = i_E$, since $i_C = \beta i_B$ the final equation is: $i_B(1+\beta) = i_E$ (1)

4. Now, we have to find equation of i_E

KCL @ Loop2 or KVL @d-b-c-d: $-i_2 R_2 + V_o + i_E R_E = 0$ (2)

5. Since we have one more variable, i_2, we need another equation which includes i_2



KVL@ Loop1 or KVL @d-b-a-d: $-i_1 R_1 - i_2 R_2 + V_{cc} = 0$ (3)

6. Wait a minute! We now realize that, to solve one equation, we introduce new variables.
So far we have the following 4 unknown variables in 3 equations: i_1, i_2, i_B, and i_E
In other words, we need one more equation. Then we may be in a good shape.

7. At node b, we can have an equation which relates currents i_1 and i_2 in terns of i_B

KCL @b: $i_2 = i_1 - i_B$ (4)

8. Now we have 4 unknowns and 4 equations, so the only thing left is how to manipulate to eliminate some.

Let's have a substitution of (4) to (2):

(4) --> (2): $-(i_1 - i_B)R_2 + V_o + i_E R_E = -i_1 R_2 + i_B R_2 + V_o + i_E R_E = 0$ (2)'

Also, let's have another substitution of (4):

(4) -->(3): $-i_1 R_1 - (i_1 - i_B)R_2 + V_{cc} = -i_1(R_1 + R_2) + i_B R_2 + V_{cc} = 0$ (3)'

9. From (3)', we could get a nice equation of i_1 in terms of i_B and other known values.

(3)'-->: $$i_1 = \frac{V_{cc} + i_B R_2}{R_1 + R_2} \quad (3)''$$

10. Now, if you plug equation (3)" to equation (2)', we have an equation with i_E and i_B only.

(3)"-->(2)': $$-\frac{V_{cc}R_2 + i_B R_2^{\,2}}{R_1 + R_2} + i_B R_2 + V_o + i_E R_E = 0 \quad (2)''$$

11. We now see that with two variables (i_E and i_B), we have two equations (i.e., (1) and (2)"). Let's plug (1) to (2)"

(1) --> (2)": $$-\frac{V_{cc}R_2 + i_B R_2^{\,2}}{R_1 + R_2} + i_B R_2 + V_o + (1+\beta) i_B R_E = 0 \quad (2)'''$$

12. I will leave the final simplification of (2)''' to you.

$$i_B[R_2 + (1+\beta)i_B - \frac{R_2^{\,2}}{R_1 + R_2}] + V_o - \frac{V_{cc}R_2}{R_1 + R_2} = 0$$

$$i_B[(1+\beta)i_B - \frac{R_1R_2 + R_2^{\,2} - R_2^{\,2}}{R_1 + R_2}] + V_o - \frac{V_{cc}R_2}{R_1 + R_2} = 0$$

13. Here is the final solution. Check with yours.

$$i_B = \frac{\frac{V_{cc}R_2}{R_1 + R_2} - V_o}{(1+\beta)R_E + \frac{R_1 R_2}{R_1 + R_2}}$$

Now we use Smath Studio to find the the value:

$Vcc := 15$ $V0 := 200 \cdot 10^{-3}$ $R1 := 20 \cdot 10^{3}$

$RC := 500$ $\beta := 39$ $R2 := 80 \cdot 10^{3}$ $RE := 100$

$$\boxed{iB := \frac{\frac{Vcc \cdot R2}{R1 + R2} - V0}{(1+\beta) \cdot RE + \frac{R1 \cdot R2}{R1 + R2}} = 0.00059} \quad [A]$$

From (3)"

$$i1 := \frac{Vcc + iB \cdot R2}{R1 + R2} = 0.000622$$

Since iC = β* iB

$iC := \beta \cdot iB = 0.02301$

Then at node a by KCL : iCC = iC + i1

$$\boxed{iCC := iC + i1 = 0.023632}$$