Cross-correlation Method for Power Arcing Source Monitoring System

Frank Zoko Ble¹, Matti Lehtonen², Ari Sihvola³, Charles Kim³

Abstract – Power arcs do not only cause important economic loss, but also lead to serious deterioration of the entire power system equipment. With the aging of the distribution networks, the development of power arcs detection and location techniques has been paid more attention. Once an arcing fault has been detected at a monitoring station its location is obtained from the electromagnetic radiation signals. The most common location techniques are based on Time Difference of Arrival (TDOA), Directional Finding (FD) and propagation Attenuation (PA). In this paper, cross-correlation method in connection with TDOA is used to locate power arcing faults. In the experiment, strategically placed antennas and the arrival time’s delay of dominant component of the wide-band electromagnetic signals radiated from the sources are used. The power electric arc was produced by a tree leaning on a current conducting cable. The experiment proves that cross-correlation method combined with TDOA can be used to locate power arcs accurately.

Keywords: cross-correlation, power arc, electromagnetic radiation, antenna, arc source location, signal processing, time delay estimation.

I. Introduction

This paper investigates fault location based on radio signals produced by electric power arcing faults. In power systems networks, arcing faults are frequent and represent at about 80 % of the reported faults in the entire power system network [1]. Arcing faults represent a very complex situation since they are uncontrollable events that occur in the unpredictable environments of the atmosphere. They usually occur when trees are coming into contact with power lines, by dirty insulators, as well as various other types of insulation failures such as insulation electrical strength deterioration. In a certain case the arcs occur due to the over-voltage on the system caused by lightning strikes or switching operations. The human error by technician working on the network due to the failure to remove equipment on the line and incorrect operational procedure may also result in the power arc ignition.

Being so recurrent and due to the devastating effect on the power system equipment the power arcs detection and location techniques have been attracting the attention of the researchers. The difficulty in dealing with such power arcs that they often induce low currents that are undetectable by existing conventional method such as relays, thus placing the entire system at risk [2]. The power arc detection that already exists in power system fault detection method involves the fault impedance and power arc detection that already exists in power system undetectable by existing conventional method such as relays. In the experiment, strategically placed antennas and the arrival time’s delay of dominant component of the wide-band electromagnetic signals radiated from the sources are used. The experiment proves that cross-correlation method combined with TDOA can be used to locate power arcs accurately.

This paper discusses in section II the existing power arc detection and location methods. Next follows the cross correlation method in section III, in which the statistical interpretation of the results is also described. In section IV the arc location experiment description and measurement data are discussed. Subsequently, source location using measured data via radio wave arrival time is described. Finally, in section V, the conclusions are presented with suggested improvements.

II. Power Arc Detection and Location Methods

There are several approaches of RF-signal based electromagnetic radiation source location. Directional Finding (FD) method relies on multiple directional antennas placed around a possible source to decide the location [4, 5, 6]. The basic aspect of the Time Difference of Arrival (TDOA) method is that it determines an RF signal source using the moments (or time differences) of the RF-signals arrival at different antennas. Under this concept, with an antenna i located at \((x_i, y_i, z_i)\), for example, the source location coordinate \((x_s, y_s, z_s)\) can be expressed, using the speed of the RF signal \(c\), which is the same as the light speed, as follows:

\[
\sqrt{(x_s - x_i)^2 + (y_s - y_i)^2 + (z_s - z_i)^2} = D_i \tag{1}
\]
where \( D_{ij} = c \times (t_j - t_{ij}) \) with \( c \) is the speed of light, \( t_{ij} \) signal arrival time difference (namely, \( t_i - t_j \)) between pair of antennas, \( i = 1, 2, 3, 4 \), and the reference antenna \( j \) that is closest to the arc source point. Without using time, \( D_{ij} \) can be expressed as \( D_{ij} = d_j + d_{ij} \), with \( i = 1, 2, 3, 4 \), and \( d_{ij} \) is the 3-dimensional distance difference of arrival (DDOA) between the reference antenna \( j \) and the \( i \)th antenna (namely, \( d_i - d_j \)), where \((x_i, y_i, z_i)\) and \((x_j, y_j, z_j)\) are 3-dimensional coordinates of the antennas \( i \) and \( j \), respectively.

Using (1) an accurate estimate of the arcing source point is determined by measuring the arrival times difference \( t_{ij} \) between pair of antennas and their coordinates \((x_i, y_i, z_i)\) and \((x_j, y_j, z_j)\). The common problem of the above mentioned arrival time based placement is that, due to the noisy RF signals measured at antenna, the exact arrival time point is not always straightforward. To solve the problem, we propose a new method of arrival time approach for arc source location by using the cross-correlation function. An experimental method of obtaining the needed \( t_{ij} \) and the feasibility study of the method proposed using the acquired \( t_{ij} \) are the main subject of the next section.

### III. Cross Correlation Analysis for Power Arc Source Location

In order to estimate the time delay, two antennas are needed to capture the transmitted signal \( s(t) \). Assuming that the signal \( y(t) \) received at antenna \( \text{ant}_2 \) is the replica of \( x(t) \) captured by antenna \( \text{ant}_1 \) but being delayed by time \( t_{12} \). The signals \( x(t) \) and \( y(t) \) received by a pair of antennas separated by distance \((d_{12} = c \times t_{12})\), are expressed as [7]-[11]:

\[
\begin{align*}
x(t) &= s(t) + n_x(t) \\
y(t) &= \alpha s(t + t_{12}) + n_y(t)
\end{align*}
\]

where \( \alpha \) is the signal amplitude attenuation factor, \( n_x(t) \) and \( n_y(t) \) are the wide-sense Gaussian noise processes which are uncorrelated with the signal of interest \( s(t) \).

A common method of estimating the time delay \( t_{ij} \) is to use a cross-correlation function of the received signals. In fact the cross-correlation measures the similarity of two functions \( x(t) \) and \( y(t) \) as the latter is displaced by the time \( t_{ij} \).

![Cross-correlation estimation](image1)

![Auto-correlation estimation](image2)
delay that occurs at the point where $R_{xy}(t_j)$ is maximum, or simply the peak of the function occurs at $t = t_j$.

Having obtained the values of the auto-correlation $R_{xx}(t_j)$ and the cross-correlation $R_{xy}(t_j)$, the attenuation factor is given by [12]:

$$\alpha = \frac{R_{xx}(t_j)}{R_{xy}(t_j)}$$  \hspace{1cm} (4)

In fact, in this study 4 antennas are used to capture the RF signals emitted by the power arc source. Using the cross-correlation function of (3), the time delays between the reference antenna $j$ and the antenna $i$ are computed as $t_{ij}$, (if $j = 1$ then $i = 2, 3, 4$). Then the measured time differences $t_{ij}$ are substituted in (1) to compute the arc source location coordinates $(x_j, y_j, z_j)$. For TDOA estimation it is not necessary to know the absolute time for the signal between the radiation source and the antenna (receiver). From the four antennas we can get three time difference of arrival (TDOA), each of which can be used to solve the (1). In order to find the TDOA we first obtain the cross-correlation of pairs of signals, and then we obtain the auto-correlation of one of them. Having obtained both cross-correlation and auto-correlation, the values of TDOA are finally derived. Figures 1 and 2 illustrate respectively the estimation of cross-correlation and auto-correlation.

The results shown in Table I are directly obtained from the distance calculation using the antennas and actual source 3D Cartesian coordinates. While the values listed in Table II are calculated using the measured signal data when the arc source coordinates have been derived from the solution of (1). The values of the signal distance of arrival (DOA) listed in Table III and IV are directly proportional to the values of respectively illustrated in Tables I and II by a constant $c$ (which is assumed to be the speed of light) and they are defined as signal time of arrival (TOA). From the values of Table III the actual time difference of arrival (TDOA) between the antennas are calculated and listed in Table XI (see appendix). The actual distances between pair of antennas are calculated using their corresponding Cartesian coordinates and the results are illustrated in Table VII (see appendix). From Table VII are derived the actual time between antennas pair as shown in Table XII (see appendix). The measured signal time difference of arrival obtained from the cross-correlation function discussed above are presented in Table X. From these measured TDOA values we obtained the corresponding measured distance difference of arrival (DDOA) as shown in Tables IX (see appendix). Finally the measured distances and times are compared with actual values and the outcome error results observed are shown in the appendix in Tables XIII, XIV and XV. These errors show clearly that the actual and measured values are quite close as it can be observed in Figures 2 and 3. In order to distinguish the actual time between the antennas from the TDOA it can be seen that if two antennas are placed at the same distance from the radiation source, they will have a TDOA equals to zero. This means that two antennas placed at the same distance from the source have no time delay in the traveling wave they receive from the radiation source.

<table>
<thead>
<tr>
<th>Placement</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12.214</td>
<td>10.423</td>
<td>9.748</td>
<td>8.128</td>
</tr>
<tr>
<td>3</td>
<td>12.213</td>
<td>8.700</td>
<td>9.748</td>
<td>6.902</td>
</tr>
<tr>
<td>4</td>
<td>11.211</td>
<td>8.735</td>
<td>10.09</td>
<td>9.748</td>
</tr>
<tr>
<td>5</td>
<td>5.110</td>
<td>6.988</td>
<td>8.193</td>
<td>9.748</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Placement</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12.209</td>
<td>10.420</td>
<td>9.748</td>
<td>8.130</td>
</tr>
<tr>
<td>3</td>
<td>12.209</td>
<td>8.697</td>
<td>9.748</td>
<td>6.904</td>
</tr>
<tr>
<td>4</td>
<td>11.208</td>
<td>8.732</td>
<td>10.089</td>
<td>9.748</td>
</tr>
<tr>
<td>5</td>
<td>5.110</td>
<td>6.988</td>
<td>8.193</td>
<td>9.748</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Placement</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
<th>$t_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40.716</td>
<td>34.745</td>
<td>32.495</td>
<td>33.165</td>
</tr>
<tr>
<td>2</td>
<td>40.712</td>
<td>34.742</td>
<td>32.495</td>
<td>27.095</td>
</tr>
<tr>
<td>3</td>
<td>40.710</td>
<td>29.002</td>
<td>32.495</td>
<td>23.007</td>
</tr>
<tr>
<td>4</td>
<td>37.371</td>
<td>29.116</td>
<td>33.637</td>
<td>32.495</td>
</tr>
<tr>
<td>5</td>
<td>17.033</td>
<td>23.293</td>
<td>27.309</td>
<td>32.495</td>
</tr>
</tbody>
</table>

One should note that TDOA is different from the actual time between the antennas as the first term is based on the concept of signal traveling time while the latter is the ratio of the actual distance between pair of antennas and the speed of light.
III.1. Formulation of a Non-Linear Equation

The solution of (1) involves some mathematical transformations such as Taylor series and Newton estimation approach as shown respectively in (6), (7) and (8) [13]-[16]. In real life the power arcs can occur at any unknown time \( t \), and travel to the 4 antennas at different times. However the arc source point \( (x_s, y_s, z_s) \) can always be solved by using a hyperboloid equation expressed in (5) as long as the time differences between antennas pair are measured by finding the roots of nonlinear vector function (6) [17]-[21].

\[
F(X) = 0
\]  

(6)

where \( F(X) \) is a non-linear vector function and \( X = (x, y, z, t) \) is a vector variable. Function \( F(X) \) is expanded using Taylor’s series in vicinity of the root iteration \( X^0 = (x^0, y^0, z^0, t^0) \) expressed as:

\[
F(X^1) = F(X^0) + \sum_{i=1}^{4} \frac{\partial}{\partial X_i} [F(X^0)(X_i^1 - X_i^0)]
\]  

(7)

The iteration operations made from the Least Square method (LSM) is obtained based on two assumptions [22]:

1. \( \sum_{i} F_i(X)^2 \leq \beta \)
2. \( |X^i - X^0| \leq \varepsilon \)

where \( \beta \) is the error limit based on the Least Square method and \( \varepsilon \) is the module vector error making the iteration computation. The solution of (8) is listed in Table IV. As seen in Table IV the measured source per placement is displaced from the actual radiation source point and the corresponding errors are listed Table V. The explanation for these errors in location is found in the statistical analysis in the next paragraph.
TABLE IV
ARC SOURCE POSITION

<table>
<thead>
<tr>
<th>Placement</th>
<th>Actual source x</th>
<th>y</th>
<th>z</th>
<th>Measured source x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.073</td>
<td>8.89</td>
<td>5.1</td>
<td>0.01025</td>
<td>8.88998</td>
<td>5.09999</td>
</tr>
<tr>
<td>2</td>
<td>0.073</td>
<td>8.89</td>
<td>5.1</td>
<td>0.00794</td>
<td>8.88999</td>
<td>5.09999</td>
</tr>
<tr>
<td>3</td>
<td>0.073</td>
<td>8.89</td>
<td>5.1</td>
<td>0.00709</td>
<td>8.88999</td>
<td>5.1</td>
</tr>
<tr>
<td>4</td>
<td>0.073</td>
<td>8.89</td>
<td>5.1</td>
<td>0.00709</td>
<td>8.88999</td>
<td>5.1</td>
</tr>
<tr>
<td>5</td>
<td>0.073</td>
<td>8.89</td>
<td>5.1</td>
<td>0.0177</td>
<td>8.88994</td>
<td>5.09997</td>
</tr>
</tbody>
</table>

TABLE V
ERROR IN ASCC

<table>
<thead>
<tr>
<th>Placement</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.06275</td>
<td>0.00098</td>
<td>0.00099</td>
</tr>
<tr>
<td>2</td>
<td>0.06506</td>
<td>0.00099</td>
<td>0.00099</td>
</tr>
<tr>
<td>3</td>
<td>0.06591</td>
<td>0.00099</td>
<td>0.001</td>
</tr>
<tr>
<td>4</td>
<td>0.06591</td>
<td>0.00099</td>
<td>0.001</td>
</tr>
<tr>
<td>5</td>
<td>0.05530</td>
<td>0.00094</td>
<td>0.00097</td>
</tr>
</tbody>
</table>

III.2. Statistical Analysis for the Impact of Antenna Placement

III.2.1. Multiple Linear Regression

The solution of (6) is the arc source 3D Cartesian coordinates (ASCC). From the Table IV and V it is observed that the calculated arc source point is slightly displaced from the actual source. A linear regression in conjunction with the analysis of variance (ANOVA) is used to analyze the correlation between the error in the actual and calculated ASCC and the placement of the antennas during the experiment. As seen in Table IV the rows show the different antennas placements and the columns are the source coordinates. The correlation between the errors listed in Table V and the antennas Cartesian coordinates per placement is expressed as:

\[ Y_{ij} = \beta_0 + \beta_1 x_{ij} + \beta_2 y_{ij} + \beta_3 z_{ij} \quad (9) \]

where \( Y_{ij} \) denotes the response meaning the errors displayed between the actual and calculated ASCC. The predictor variables are \( X_1, X_2 \) and \( X_3 \) assuming respectively the values of \( x_i \), \( y_i \) and \( z_i \) values of antennas coordinates per placement. The intercept of the model in (9) is \( \beta_0 \). The coefficients \( \beta_1, \beta_2 \) and \( \beta_3 \) are real numbers, the target to be estimated. The variables presented in (9) are defined as:

\[ \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} 1 & X_{11} & X_{21} & X_{31} \\ 1 & X_{12} & X_{22} & X_{32} \\ 1 & X_{13} & X_{23} & X_{33} \\ 1 & X_{14} & X_{24} & X_{34} \\ 1 & X_{15} & X_{25} & X_{35} \end{bmatrix} \]

with \( X \) is a 5x4 matrix, the first member of each row of this matrix \( X \) is 1. The remaining elements of the \( i \)th row for each \( i \) consists of the values assumed by the 3 predictor variables that give rise to the response \( Y_{ij} \) and \( i = 1, 2, 3, 4 \) and \( 5 \). The linear regression results are listed in Tables VI. As seen in Table VI for the linear regression results show R square equals to 0.99988, that is 99.88% of the antennas coordinates are accounted for by the variation observed in the arc source as illustrated in Table IV.

TABLE VI
LINEAR REGRESSION

<table>
<thead>
<tr>
<th>Regression Statistics</th>
<th>Coefficients</th>
<th>Standard Error</th>
<th>t Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple R</td>
<td>0.99988</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R Square</td>
<td>0.99975</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R Square</td>
<td>0.49950</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Error</td>
<td>0.00625</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

III.2.2. Analysis of variance (ANOVA)

The analysis of variance (ANOVA) in conjunction to the linear regression model is done to analyze the solution of (6) by comparing the measured arc source point with the actual source in terms of their Cartesian coordinates. To do so we need to compare both arc sources ASCC population means (\( \mu \)) according to differently placed antenna sets by testing:

\[ H_0 : \quad \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 \]

\[ H_1 : \quad \mu_i \neq \mu_j \quad (10) \]

For some \( i \) (which indicates the \( i \)th antenna) and \( j \) (the \( j \)th antenna placement set) based on independent ASCC drawn from the antennas’ placements. Let \( X_{ij} = (x_{ij}, y_{ij}, z_{ij}) \) denotes the calculated ASCC for the
ith antenna in the jth placement set, where \( i = 1, 2, 3, 4 \) and \( j = 1, 2, 3, 4, 5 \). In this comparison procedure the total variation of the location vector \( X_{ij} \) of ASCC per antennas’ placement partitioned into two components which can be attributed to recognizable origins of variation known as coordinate mean square (\( MS_{Co} \)) of the measured arc source and the error mean squared (\( MS_E \)) between the actual and measured arc source points. These two components are useful in testing pertinent hypotheses mentioned (10) since they deal with the practical aspects of the designed experimental studies. The calculated location vector \( X_{ij} \) for the ith ASCC in the jth placement set is expressed as:

\[
X_{ij} = \mu_i + E_{ij}
\]  

(11)

where \( \mu_i \) denotes the mean of the measured ith ASCC per placement and \( E_{ij} \) is the difference between the measured ith ASCC of the jth antenna placement and the corresponding actual arc point. In practical way the theoretical means are replaced by their unbiased estimators \( \bar{X}, \bar{X}_1, \bar{X}_2, \ldots, \bar{X}_k \) respectively, where \( k \) is the number of the calculated arc points per placement. Then the (10) of \( X_{ij} \) is written as:

\[
\bar{X}_{ij} = \bar{X} + (\bar{X}_i - \bar{X}) + (\bar{X}_j - \bar{X}_i)
\]  

(12)

where, \( \bar{X} \) is an estimator for the theoretical mean \( \mu \) of the measured 3D source position, \( \alpha_i = \mu_i - \mu \) is the overall pooled mean effect which is an estimator for \( (\bar{X}_i - \bar{X}) \) which is the effect of the ith ASCC, and \( (\bar{X}_j - \bar{X}_i) \) is an estimator for \( E_{ij} = (X_{ij} - \mu_i) \), the random error or residual. The equation (12) is equivalent to (13):

\[
X_{ij} - \bar{X} = (\bar{X}_i - \bar{X}) + (\bar{X}_j - \bar{X}_i)
\]  

(13)

Taking the square of both sides of (13) gives (14):

\[
\sum_{i=1}^{k} \sum_{j=1}^{n} (X_{ij} - \bar{X})^2 = \sum_{i=1}^{k} \sum_{j=1}^{n} [(\bar{X}_i - \bar{X})]^2 + \sum_{i=1}^{k} \sum_{j=1}^{n} [(\bar{X}_j - \bar{X}_i)]^2
\]  

(14)

with \( n \) is the number of placements and \( k \) is the number of measured arc source points. The equation (14) can be partitioned into the following components:

a) The measure of the total variability in the calculated ASCC which is called total sum of square (\( SS_{Tot} \)):

\[
SS_{Tot} = \sum_{i=1}^{k} \sum_{j=1}^{n} (X_{ij} - \bar{X})^2
\]  

(15)

b) The measure of variability in the calculated ASCC attributed to antenna coordinates which is called coordinate sum of square (\( SS_{Co} \)):

\[
SS_{Co} = \sum_{i=1}^{k} \sum_{j=1}^{n} [(\bar{X}_i - \bar{X}_j)]^2
\]  

(16)

c) The measure of the variability between the actual and the calculated ASCC attributed to the antenna placements which is called residual or sum of square (\( SS_E \)):

\[
SS_E = \sum_{i=1}^{k} \sum_{j=1}^{n} [(\bar{X}_ij - \bar{X}_i)]^2
\]  

(17)

Based on the components defined in (15), (16), and (17), the equation (14) is simply written as:

\[
SS_{Tot} = SS_{Co} + SS_E
\]  

(18)

The analysis of variance procedure uses (18) to test the null hypothesis of the means (\( \mu \)) by comparing the variability in the calculated ASCC attributed to different antenna coordinates (\( SS_{Co} \)) to the variability between the actual and the calculated ASCC attributed to the antenna placements (\( SS_E \)) via a measured F-ratio. To do so, let’s assume that the random errors \( E_{ij} \) are independents and normally distributed random variables with mean (\( \mu \)), and variance (\( \sigma^2 \)), then the theoretical individual measured ASCC vector can also be expressed as:

\[
\bar{X}_i = \sum_{i=1}^{k} (\mu + \alpha_i + E_{ij}) / k = \mu + \alpha_i + E_{ij}
\]  

(19)

For \( \bar{X} = \mu + \bar{E} \), we can rewrite \( SS_{Co} \) as (20) by substituting (19) into (16):

\[
SS_{Co} = \sum_{i=1}^{k} k\alpha_i^2 + 2\sum_{i=1}^{k} k\alpha_i \bar{E}_i + \sum_{i=1}^{k} k\bar{E}_i^2 - k\bar{E}^2
\]  

(20)
The expectation of $SS_{Co}$ is given as:

$$E[SS_{Co}] = (k-1)\sigma^2 + \sum_{i=1}^{k}k\alpha_i^2$$

(21)

by dividing (16) by $(k - 1)$ the coordinates mean square $MS_{Co}$ is obtained as:

$$MS_{Co} = SS_{Co} / (k - 1)$$

(22)

Similarly to obtain the unbiased estimator for the variance $\sigma^2$ of the calculated ASCC, $SS_{E}$ is divided by $(N - k)$ where $N$ is the total number of observations per placements, resulting in so called error mean squared $MS_{E}$ as:

$$MS_{E} = SS_{E} / (N - k)$$

(23)

And finally the values of $MS_{Co}$ and $MS_{E}$ are used to make a $H_0$

- $H_0$ is true when F-ratio $MS_{Co} / MS_{E} = 1$
- $H_0$ is not true when F-ratio $MS_{Co} / MS_{E} > 1$

The entire results of ANOVA analysis is presented in Table XVI (see appendix).

An experimental set-up for the arc source location and the interpretations of the results of the calculated ASCC statistics analysis are the main subject of the next section.

IV. Arc Location Experiment

In order to evaluate the performance of the proposed algorithm, we performed a set of arc location experiments as shown in Figures 5 and 7. As seen in Figure 7 we present 2 types of topologies in term of antennas placements such as horizontally and vertically placed antennas. These two types of placements will help to choose the suitable antennas arrangement that could be adopted for power arc fault detection in power distribution network.

IV.1. Experiment set-up

The experiment set-up consisted of four antennas placed at known distances from the arc source. The antennas were connected through coaxial cable of 3 m to a multichannel LeCroy digitizer of 2 GHz sampling rate. As for arc staging, a pine tree of total height 9 m was lent on a metallic rod (as a conductor) to make an arcing contact at about 5.1 m above the floor as shown in Figure 5 in which the antennas are labeled as $ant_1$, $ant_2$, $ant_3$, and $ant_4$.

![Fig. 5. The antennae used are Yagi – Uda (Yagi) antennae which cover a frequency range of 47 - 862 GHz.](image)

The antennas used are Yagi – Uda (Yagi) antennas which cover a frequency range of 47 - 862 GHz. As seen in Figure 7 the placements 1 to 4 show that the antennas are horizontally configured, when in placement 5 they are vertically placed. The arc current passing through the rod was also recorded. It was determined that the tree had a resistance of 316 k. The signals were captured at the sample rate of 20000 samples per microsecond. A total of 100 measurements were made with $N = 20$ observations per antenna placement. A high voltage AC source of 20 kV was used to generate the power arcs and the supplied voltage levels used for placement 1, 2, 3, 4 and 5 are respectively 3.925, 3.46, 3.415, 2.84 and 2.705 kV. Figure 6 shows the signals captured by the 4 antennas connected to the digitizer during the experiment. The current signal passing through the tree is shown above in Figure 6 and below it are the arc radiation signals captured by the antennas named $ant_i$ (with $i = 1, 2, 3, 4$). In Figure 7, the antennas the antennas close to the arc source point are considered as reference points such as antennas 3 and 4 respectively for the placement 1 and 2. Placement 3 and 4 present slightly similar configuration but the reference antenna is placed in different location, where the antenna 4 is used as reference point in placement 3 and the antenna 2 is the reference point of placement 4. Finally the antennas configuration is changed to a vertical position in placement 5 with antenna 1 used as reference point. From these 5 different
placements, the signal time and distance difference of arrival are calculated as discussed above. The algorithm

\[ d_i^2 - d_0^2 = |X_i - X_s|^2 - |X_0 - X_s|^2 \]  (24)

The right side of (24) can be expressed as

\[ (x_i - x_s)^2 + (y_i - y_s)^2 + (z_i - z_s)^2 - \left[ (x_0 - x_s)^2 + (y_0 - y_s)^2 + (z_0 - z_s)^2 \right] \]  (25)

The left side of (24) is expressed as

\[ d_i^2 - d_0^2 = (d_{0i} + d_0)^2 - d_0^2 \]

Substituting (25) and (26) into (24) yields to (27)

\[ d_{0i}^2 - 2d_0d_{0i} = x_i^2 - x_0^2 - 2x_i(x_i - x_0) + y_i^2 - y_0^2 - 2y_i(y_i - y_0) + z_i^2 - z_0^2 - 2z_i(z_i - z_0) \]  (27)

Grouping the known terms in (27) together yields to (29), then \( F(X) \) mentioned before in (6) is expressed as:

\[ F(X) = -(d_{0i}^2 - 2d_0d_{0i}) + x_i^2 - x_0^2 - 2x_i(x_i - x_0) + y_i^2 - y_0^2 - 2y_i(y_i - y_0) + z_i^2 - z_0^2 - 2z_i(z_i - z_0) \]  (29)

The solution of (29) is solved as:

\[ AX_s = U \]  (30)

where
According so that the reference antenna is referred are solved simultaneously.

$$A = \begin{bmatrix} x_0 - x_1 & y_0 - y_1 & z_0 - z_1 & d_{10} \\ x_0 - x_2 & y_0 - y_2 & z_0 - z_2 & d_{20} \\ x_0 - x_i & y_0 - y_i & z_0 - z_i & d_{i0} \end{bmatrix}$$

$$X_s^T = \begin{bmatrix} x_s - x_0 & y_s - y_0 & z_s - z_0 & d_0 \end{bmatrix}^T$$

$$U^T = \begin{bmatrix} u_{01} & u_{02} & u_{03} \end{bmatrix}^T$$

Note that $u_{0i} = \frac{1}{2}(d^2_i - d_i^2 + d_{0i}^2)$ and the calculated arc source Cartesian coordinates vector $X_s$ is calculated as:

$$AX_s = U \Leftrightarrow X_s = A^{-1}U$$

(31)

![Figure 8](image)

Fig. 8. The estimated arc source 3D position compared with the actual source

One should note that there are 4 antennas and if the reference antenna is changed to the antenna 1, for example, then the indexes of the antennas should be changed accordingly so that the reference antenna is always indexed as 0. Finally the calculated 3-D power arc source position illustrated in Figure 8 is obtained after 4 iterations. In fact if the matrix $A$ is not singular then the Cartesian coordinates for the point source and the distance from it to the referent are solved simultaneously by solving the linear system (30). One might note that there were some arrangements of the antennas for which the matrix is not singular. The most suitable of these arrangements is the antennas aligned with a uniform spacing.

However for antennas with random spacing the matrix is virtually always non-singular. If the matrix is singular, then the equation (29) will generate a least-squares error results. If $A^{-1}A$ is singular, then a normalized $QR$ decomposition of matrix $A = QR$ could be used, with $Q$ an orthonormal matrix and $R$ is an upper-triangular matrix. The performance of this method is quite promising as its outcomes for the arc location calculation as illustrated in Figure 9 with the arc source placed at the center of a circle of a radius of 1 m and the measured arc source results accordingly placed in this unit circle.

In Figure 9 the read filled red dot represents the actual 2D source point while the calculated arc source points in placement 1, 2, 3, 4 and 5 are respectively marked with filled color star dots except for the source point of placement 3 with unfilled blue circle for better observation. It can also be observed that the arc source in placement 5 is quite close to the actual source followed respectively by placements 1, 2, 3 and 4.

In Table XVI the t-test in conjunction with R square value shows that there is a statistical significance of the designed regression and that can be accepted. From Table XVI, the final linear regression model designed is expressed as:

$$Y|_{X_1, X_2, X_3} = -0.0054X_1 - 0.0089X_2 + 0.6425X_3$$

(32)

From (32) it can be observed that the variation in 1 unit in antenna $x$-value will displace the arc source 0.00053 units from the actual source point. Similarly a variation in 1 unit of $y$- and $z$-value will displace the arc source location respectively in 0.00049 units and 0.20980 units.
from the actual source point. The coefficient 0.20980 of the explanatory variable $X_{3}$ (the antennas z-coordinate) is quite too large compared to the coefficients of $x$ and $y$. This large coefficient observed in z-value is due to the fact that the antennas height of 1.1 m is too close to the floor level causing the signal to be reflected by the floor. The answer is found in the ANOVA statistics results as discussed below.

It can be said from Table XVI that the null hypothesis for the 5 antennas' placements cannot be rejected since their corresponding F-ratio are close to 1. This means that there is strong evidence that the antennas placement is significant in calculation errors in arc source location. But the F-ratios of the antennas Cartesian coordinates per placement is larger than 1. Therefore F-ratio indicates that there is no statistical evidence of the differences observed in the antennas different location. Then the comparison of their corresponding variance ($\sigma^2$) required in order to test the $H_1$ hypothesis. That will tell the exact percentage errors attributed to the antennas coordinates. From the ANOVA results listed in Table XVI, the unbiased estimates for the variance of the antennas coordinates attributed to the antennas coordinate in placement 1 is as follows:

$$\hat{\sigma}^2 = MS_E = 4.265$$

$$\hat{\sigma}_{Co}^2 = (MS_{Co} - MS_E)/n_0$$

= (8.0144 − 4.265)/20

= 0.1875

The estimated total error due to the antennas' coordinates is:

$$\hat{\sigma}_{Tot}^2 = (\hat{\sigma}^2 + \hat{\sigma}_{Co}^2) = 4.4524$$

The proportion of total error in antennas' coordinates due to the placements is:

$$\frac{\hat{\sigma}_{Co}^2}{\hat{\sigma}_{Tot}^2} = 0.441$$

That is 4.21 % of total error observed in antennas' coordinates attributed to the antennas location. Similarly the estimated total errors observed in placement 1 are 0 %. Based on these statistical evidences we conclude that the method of cross-correlation approach for arc source location depends on the antennas' placements. From the linear regression model and the ANOVA analysis, it is observed that the antennas heights are too low and their proximity from the floor level causes the signal reflection from the floor is affecting the location accuracy with an error of 4.21 %. In Figure 10 the measured source is displaced from the actual position with an accuracy of

6.3, 6.5, 6.6, 6.6 and 5.5cm respectively for the placements 1, 2, 3, 4 and 5.

However from the statistical analysis, the 5 placements methodology in this experiment produces quite satisfactory results. It can be said that the placement 5 presents an accurate prediction followed respectively by placements 1, 2, 3 and 4 as shown in Figure 10. The placement 1 which is perfectly horizontally aligned seems to be more accurate than the others. Placements 3 and 4 which are almost similar produced a measured source at the same distance from the actual source. However it can be partially concluded that the perfectly vertical and horizontal alignment topographies are much suitable for this type of power arc fault location.

IV.3. Discussion on the Result of Arc Source Location

In this present paper, only single source location is considered. The time difference of arrival (TDOA) is estimated using the cross-correlation method. The classical process of locating a near field radiating source involves an estimation of TDOA between all antennas pairs followed by the localization, requiring the solution of a set of nonlinear equations, which is a challenging task in a noisy environment. The experiment shows that the proposed algorithm works at a reasonable level of accuracy proving that the cross-correlation method has a potential in locating an arc source with multiple antennas placed around it. The accuracy of the solution implies the applicability of this method to the actual power system network.

The errors displayed in the analysis seem to come from the antennas placements since perfect vertical and horizontal topologies produce better source location results compared to the other placements. Next the errors are due to the antennas heights above the floor level as shown by the coefficient of z-coordinate in the (32), inducing some signal reflection that affects the measurement integrity. Also, since the antennas in all the
placement sets, are quite close to each other, it seems to influence the signal integrity and add an additional source of error.

V. CONCLUSION

This paper reported an experimental investigation of power arc source location using radio frequency measurements. A digitizer equipped with the antennas and connected to a PC proved to be a useful device in power arcing fault detection and location. According to the measurements it seems that electromagnetic radiation from an arc source point can be evaluated by measuring the signals in time domain. The difference between signal wave times of arrival can help with power arc source location. This proposed method could become a useful technology of the power arc location as well as distance estimation from the point of view of the cost and accuracy.

The proposed source localization realized based on time delay of arrival (TDOA) estimation using antenna array, proved accurate and efficient. Of course, measurements of power arcs on-site will be disturbed by different noises. Therefore, it might be useful to create a database of different signals, which could serve for pattern recognition purpose, and that will be the main topic of our future research paper. This paper has introduced an improved power arc location determination system where the time difference of arriving signals can be determined using cross-correlation method. When used in conjunction with a suitable location algorithm, the errors associated with the location of an arcing fault source need to be further reduced; therefore our future works will investigate the arc fault location using other types of fault detection algorithms.

Appendix

TABLE IX
MEASURED DDOA BETWEEN THE ANTENNAS [m]

<table>
<thead>
<tr>
<th>Placement</th>
<th>$d_{12}$</th>
<th>$d_{13}$</th>
<th>$d_{14}$</th>
<th>$d_{23}$</th>
<th>$d_{24}$</th>
<th>$d_{34}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.789</td>
<td>2.460</td>
<td>2.257</td>
<td>0.671</td>
<td>0.468</td>
<td>0.203</td>
</tr>
<tr>
<td>2</td>
<td>1.789</td>
<td>2.460</td>
<td>4.078</td>
<td>0.671</td>
<td>2.289</td>
<td>1.618</td>
</tr>
<tr>
<td>3</td>
<td>3.511</td>
<td>2.460</td>
<td>5.305</td>
<td>1.051</td>
<td>1.793</td>
<td>2.844</td>
</tr>
<tr>
<td>4</td>
<td>2.476</td>
<td>1.119</td>
<td>1.459</td>
<td>1.358</td>
<td>1.017</td>
<td>0.341</td>
</tr>
<tr>
<td>5</td>
<td>1.878</td>
<td>3.083</td>
<td>4.638</td>
<td>1.205</td>
<td>2.760</td>
<td>1.556</td>
</tr>
</tbody>
</table>

TABLE X
MEASURED TIME DIFFERENCE OF ARRIVAL BETWEEN THE ANTENNAS (TDOA) [ns]

<table>
<thead>
<tr>
<th>Placement</th>
<th>$t_{12}$</th>
<th>$t_{13}$</th>
<th>$t_{14}$</th>
<th>$t_{23}$</th>
<th>$t_{24}$</th>
<th>$t_{34}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.963</td>
<td>8.201</td>
<td>7.524</td>
<td>2.238</td>
<td>1.561</td>
<td>0.677</td>
</tr>
<tr>
<td>2</td>
<td>5.963</td>
<td>8.201</td>
<td>13.594</td>
<td>2.238</td>
<td>7.632</td>
<td>5.393</td>
</tr>
<tr>
<td>3</td>
<td>11.704</td>
<td>8.201</td>
<td>17.682</td>
<td>3.503</td>
<td>5.978</td>
<td>9.481</td>
</tr>
<tr>
<td>5</td>
<td>6.260</td>
<td>10.276</td>
<td>15.461</td>
<td>4.016</td>
<td>9.201</td>
<td>5.185</td>
</tr>
</tbody>
</table>

TABLE XI
ACTUAL TIME DIFFERENCE OF ARRIVAL BETWEEN THE ANTENNAS (TDOA) [ns]

<table>
<thead>
<tr>
<th>Placement</th>
<th>$t_{12}$</th>
<th>$t_{13}$</th>
<th>$t_{14}$</th>
<th>$t_{23}$</th>
<th>$t_{24}$</th>
<th>$t_{34}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.971</td>
<td>8.222</td>
<td>7.552</td>
<td>2.230</td>
<td>1.580</td>
<td>0.670</td>
</tr>
<tr>
<td>2</td>
<td>5.970</td>
<td>8.217</td>
<td>13.617</td>
<td>2.248</td>
<td>7.647</td>
<td>5.400</td>
</tr>
<tr>
<td>5</td>
<td>6.260</td>
<td>10.276</td>
<td>15.461</td>
<td>4.016</td>
<td>9.201</td>
<td>5.185</td>
</tr>
</tbody>
</table>

TABLE XII
ACTUAL TIME OF ARRIVAL BETWEEN THE ANTENNAS [ns]

<table>
<thead>
<tr>
<th>Placement</th>
<th>$t_{12}$</th>
<th>$t_{13}$</th>
<th>$t_{14}$</th>
<th>$t_{23}$</th>
<th>$t_{24}$</th>
<th>$t_{34}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.23</td>
<td>24.5</td>
<td>31.167</td>
<td>12.267</td>
<td>18.933</td>
<td>6.667</td>
</tr>
<tr>
<td>3</td>
<td>14.09</td>
<td>24.5</td>
<td>33.433</td>
<td>14.123</td>
<td>19.608</td>
<td>13.82</td>
</tr>
<tr>
<td>5</td>
<td>8.5</td>
<td>13.233</td>
<td>19.033</td>
<td>4.733</td>
<td>10.533</td>
<td>5.8</td>
</tr>
</tbody>
</table>

TABLE XIII
ERROR IN TDOA [ns]

<table>
<thead>
<tr>
<th>Placement</th>
<th>$e_{12}$</th>
<th>$e_{13}$</th>
<th>$e_{14}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00850</td>
<td>0.02057</td>
<td>0.02743</td>
</tr>
<tr>
<td>2</td>
<td>0.00659</td>
<td>0.01595</td>
<td>0.02246</td>
</tr>
<tr>
<td>3</td>
<td>0.00423</td>
<td>0.01424</td>
<td>0.02108</td>
</tr>
<tr>
<td>4</td>
<td>0.00148</td>
<td>0.00557</td>
<td>0.01167</td>
</tr>
<tr>
<td>5</td>
<td>0.00005</td>
<td>0.00006</td>
<td>0.00007</td>
</tr>
</tbody>
</table>

Copyright © 2007 Praise Worthy Price S.r.l. - All rights reserved
International Review of Electrical Engineering, Vol. xx, n. x
The authors gratefully acknowledge the contributions of Tatu Nieminen and Joni Klüss, for their work on building the laboratory experiment.

Acknowledgements

The authors gratefully acknowledge the contributions of Tatu Nieminen and Joni Klüss, for their work on building the laboratory experiment.

References

Authors’ information

Frank Zoko Ble obtained a B.Sc. in Physics in Ivory coast National University, Abidjan in 1997. He received M.Sc. in Electrical Engineering in Helsinki University of Technology (TKK), Espoo, Finland in 2010. Currently he is working toward his PhD degree in Aalto University, School of Electrical Engineering. His research interests are in electric power arcs detection using radio frequency measurements. He is a researcher in the Department of Electrical of Aalto University, School of Electrical Engineering, Finland.

Matti Lehtonen (1959) was with VTT Energy, Espoo, Finland from 1998 to 2003, and since 1999 has been a professor at the Helsinki University of Technology (TKK), where he is now head of Electrical Engineering department. Matti Lehtonen received both his Master’s and Licentiate degrees in Electrical Engineering from Helsinki University of Technology, in 1984 and 1989 respectively, and the Doctor of Technology degree from Tampere University of technology in 1992. The main activities of Professor Lehtonen include power system planning and asset management, power system protection including earth fault problems, harmonic related issues and applications of information technology in distribution systems. He is a Professor in Aalto University, School of Electrical Engineering, Finland.

Charles Kim received a PhD degree in electrical engineering from Texas A&M University (College Station, TX) in 1989. Since 1999, he has been with the Department of Electrical and Computer Engineering at Howard University. Previously, Dr. Kim held teaching and research positions at Texas A&M University and the University of Suwon. Dr. Kim’s research includes failure detection, anticipation, and system safety analysis in safety critical systems in energy, aerospace, and nuclear industries. Several inventions of his in the research area have been patent filed through the university’s intellectual property office. Dr. Kim is a senior member of IEEE and the chair of an IEEE chapter in Washington Baltimore section.

Ari Sihvola was born on October 6th, 1957, in Valkeala, Finland. He received the degrees of Diploma Engineer in 1981, Licentiate of Technology in 1984, and Doctor of Technology in 1987, all in Electrical Engineering, from the Helsinki University of Technology (TKK), Finland. Besides working for TKK and the Academy of Finland, he was visiting engineer in the Research Laboratory of Electronics of the Massachusetts Institute of Technology, Cambridge, in 1985–1986, and in 1990–1991, he worked as a visiting scientist at the Pennsylvania State University, State College. In 1996, he was visiting scientist at the Lund University, Sweden, and for the academic year 2000–01 he was visiting professor at the Electromagnetic and Acoustics Laboratory of the Swiss Federal Institute of Technology, Lausanne. In the summer of 2008, he was visiting professor at the University of Paris XI, France. Ari Sihvola is professor of electromagnetic in Aalto University School of Electrical Engineering (former name before 2010: Helsinki University of Technology) with interest in electromagnetic theory, complex media, materials modeling, remote sensing, and radar applications. He is Chairman of the Finnish National Committee of URSI (International Union of Radio Science) and Fellow of IEEE. He also served as the Secretary of the 22nd European Microwave Conference, held in August 1992, in Espoo, Finland. He was awarded the ve-year Finnish Academy Professor position starting August 2005. He is also director of the Finnish Graduate School of Electronics, Telecommunications, and Automation (GETA).