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Vulnerability Assessment of Power Grid Using Graph Topological Indices

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Abstract

This paper presents an assessment of the vulnerability of the power grid to blackout using graph topological indexes. Based on a FERC 715 report and the outage reports, the cascading faults of summer WSCC 1996 are reconstructed, and the graphical property of the grid is compared between two cases: when the blackout triggering lines are removed simulating the actual sequence of cascading outages and when the same number of randomly selected lines are removed. The investigation finds that the critical path lengths of the triggering events of the July and August outages of 1996 WSCC blackout are higher than those of no-outage and arbitrary events. In addition, the small world-ness index for each of the outage triggering events is much smaller than that of normal or any no-outage scenario, indicating that events of shifting a network from small world to a random network would be more likely cascaded to wide area outage.

KEYWORDS: power grid, graph theory, topological indices, vulnerability, blackout

1. Introduction

Power grid, by the heavily meshed nature, is vulnerable to wide area outages. The source of vulnerability includes natural disasters, equipment failures, human errors, or deliberate sabotage and attacks. Increased electricity demand and transmission bottlenecks make the complex power system even more vulnerable and, with triggering disruption at opportune time, topple it over to blackouts.

The 1996 blackout in the U.S. Western Systems Coordinating Council (WSCC) power grid was caused by a few localized line faults, and spread to neighboring areas and then cascaded to the most western states. There were two outages in the 1996 blackout, apart by about a month. On July 2, 1996 a short circuit on a 345-kV line in Wyoming started a chain of events leading to a break-up of the western North America power grid which resulted in five islands with a blackout in southern Idaho with loss of 11,750MW load. The second one, the cascading failure of August 10, 1996, disintegrated the WSCC grid into four islands resulting in the loss of 30,390 MW load and 7.49 million affected customers in the region [1]. The mechanism of failure for the cascading outages was a transient oscillation on the long distance transmission lines operated near critical loading point. The investigative report on the incident recommended that the grid deploy a better detection and recognition tool for instability signatures in system dynamic activity as an important ingredient in preventing the malaise of blackout [2].

Seven years after the historic blackouts and the operational overhaul mandated by North American Electric Reliability Council and Federal Energy Regulatory Commission, the disaster came again in bigger size. The Northeast Blackout on August 14, 2003, the largest blackout ever with 50 million people affected and nearly 62,000 MW load lost, blackened most of the north-eastern states and the province of Ontario, Canada. The 2003 blackout exposed the still existing problem of the current dynamic analysis and monitoring system in preventing outages. The culprit was, again, the inadequacy of monitoring and situation awareness.

The problem of the conventional planning and operational practices of not being able to rid the power grid of blackouts stimulated researchers to seek solutions from alternative means. One of the approaches, topological analysis, came from the recent research findings in the examination of complex systems. Network topology, which has long been used in the studies of complex systems and non-linear behaviors, received further attention when an article in *Nature* showed that certain networks arising in nature and technology exhibited “small world" phenomenon [3]. The seminal paper by Watts and Strogatz demonstrated, using computer experiments, that introducing a relatively small number of random connections could change the character of a graph dramatically. The new graph, while retaining its properties of being highly clustered, had much shorter average path lengths. This new graph was called a small world network.

In graph theory, the concept of small world describes a semi-random graph which produces a sharp reduction of the average distance between arbitrary nodes while the system is still relatively localized. In essence, small world graph has a high clustering coefficient like complete graphs and a short characteristic path length like random graphs. The wide appeal of this concept is that the small world property seems to be a quantifiable characteristic of many real-world structures and networks. Some of the real-world networks reported as having small world phenomenon include: C Elegans worm neural networks, E. coli graph, food webs, World Wide Web (WWW) sites, Internet domains, and professional article co-authorship [3]. It is also suggested that power grid is an example of a small world.

Examination of power grid in the small world perspective soon followed as an alternative way to assess the vulnerability of the grid. A few published works attempted to explain power grid outages and vulnerability in terms of grid topology. Kim investigated power system stability in small world perspective and concluded that topological estimator could be used as a factor of determining the priority of transmission lines in a power grid [4]. Lu et. al. analyzed the mechanism of cascading failure in bulk power system using the properties of small world network model. They applied a Monte-Carlo simulation and investigated if the topological characteristics and failure dynamic properties of a network strongly correlated with the efficiency and robustness of power grid. Results obtained from the simulation showed that the critical nodes and lines under failure altered the characteristic path length of the network and triggered cascading failure [5]. Surdutovic et. al. applied the difference between characteristic path length of one-link faulted network and the initial not-faulted network to characterize the vulnerability of the total network relative to the unexpected breakdown. Their simulation results showed a strong dependence of the sensitivity of the characteristic path length on the position of a link in the network structure [6]. Motter investigated possible correlation between the local vulnerability of a network and a change in the global characteristics path length in an event of the loss of a network link [7].

The topological analyses of correlating grid topology and vulnerability have been greeted with skepticism especially by those in traditional dynamic analysis. The topological analysis of network is innately static and link focused, ignoring the dynamic natures of the connected elements, therefore, it has not been seriously considered as a metric for changing behavior of power network as it fails. It should be noted, however, that, as structure always affects function, the qualitative nature of a system's topology should not be excluded in determining its dynamic properties. In fact, power grid vulnerability analysis from topology perspective holds a bright prospect of providing system operators fast with easy to interpret information on critical operating point in a disturbed network.

This paper reports one such prospect brought by the topological analysis of the 1996 blackout. By examining the cascading events of the 1996 WSCC blackout, the paper explores the clues to the inherent relationship between the observed topological property of the power grid and the cascading outages experienced in the grid. Nonetheless, the paper's intention is not in the finding of the causes or consequences of the blackout, but only in the explanation of the difference in network topology before the blackout and after. Therefore, the objective of the paper is, using the actual blackout case, to find a dominant topological index that differentiates line faults that led to cascading outages from those did not. In addition, this paper studies several power networks to see if they exhibit small world characteristics.

This paper is organized as follows. In section 2, a review of graph theory and its application in describing the properties of small world network are presented. Section 3 describes the topological analysis of selected power networks and detailed investigation of the WSCC network and its 1996 outages. It discusses about the differentiating topological index for separating cascading line faults from randomly chosen line faults. Section 4 concludes the paper with summary and future works.

2. Graph Theory and Small World Networks

We first review the basics of graph theory only to the extent enough in describing the properties of small world network and, second, we apply the graph theory on IEEE 14-bus system for illustration. This graph theory brief is based on the descriptive introduction in [8]. A discussion follows on the impact of network topology in the power grid analysis.

2.1 Basic Graph Theory

A graph G is defined as comprising of non-empty set of elements, called vertices (or nodes), and a list of unordered pairs of these elements, called edges (or lines). In power system, the vertices and edges of a graph represent such network elements as generators/substations and transmission lines, respectively.

The set of vertices of the graph G is called the vertex set of G denoted by $V(G)$, and the list of edges is called the edge list of G, denoted by $E(G)$. If v and w are vertices of G, then an edge of the form vw is said to "link" v and w. The number of vertices in $V(G)$ is termed the order of the graph, n, and the number of edges in $E(G)$, its size, M. And the number of edges incident with a given vertex v, namely the number of v's adjacent neighboring vertices, is called the degree of

v, denoted by k_v. One characteristic measure frequently used in network classification is the average degree, k, which quantifies the relationship between n and M:

$$k = \frac{2M}{n}. \tag{1}$$

One of the two most important characteristics of a graph is the characteristic path length, $L(G)$, which measures the typical separation between two generic vertices of a graph G. $L(G)$ is defined as follows:

$$L(G) = \frac{1}{n(n-1)} \sum_{i \neq j \in G} d(i,j) \tag{2}$$

where the "distance," $d(i, j)$, here refers not to any separately defined metric space but to the minimum number of edges that must be traversed in order to reach vertex j from another vertex i. In other words, L is the shortest path length between i and j. For a random graph, a reasonable asymptotic approximation of the characteristic path length is:

$$L(G)_{random} \approx \frac{\ln(n)}{\ln(k)}. \tag{3}$$

The other important characteristic of a graph is the clustering coefficient which measures the average connectedness of a vertex. For a vertex v, let's define the sub-graph of neighbors of v, $\Gamma(v)$, as consisting of the vertices adjacent to v (with the exclusion of v itself). Then, the clustering coefficient C_v, is defined by the ratio of the number of edges in the neighborhood of v and the total number of possible edges in that neighborhood. The quantity C_v is written as:

$$C_v = \frac{\eta_v}{\{k_v(k_v - 1)/2\}} \tag{4}$$

where, η_v is number of existing connections in $\Gamma(v)$ and the denominator $k_v(k_v - 1)/2$ is the maximum possible number of edge connections in $\Gamma(v)$.

A measure of clustering over the entire graph, denoted as γ, is defined as the average of C_v over all $v \in V(G)$ and written as follows:

$$\gamma = \frac{1}{n} \sum_{v \in G} C_v \tag{5}$$

Therefore, $\gamma = 1$ would imply a complete graph, and $\gamma = 0$ would imply that no neighbor of any vertex is adjacent with any other neighborhood. In power grid, γ

implies the degree to which substations (or generators or load centers) are connected via transmission lines to others. For a random graph, a reasonable approximation for the clustering coefficient is:

$$\gamma_{random} \approx \frac{k}{n}. \qquad (6)$$

2.2 Illustration of Topological Indices with 14-Bus Network

An example using a simple power network would be appropriate to illustrate the graphical characteristics and their calculation. Shown in Figure 1 is the IEEE 14-Bus network [9] where an example topology calculation is performed.

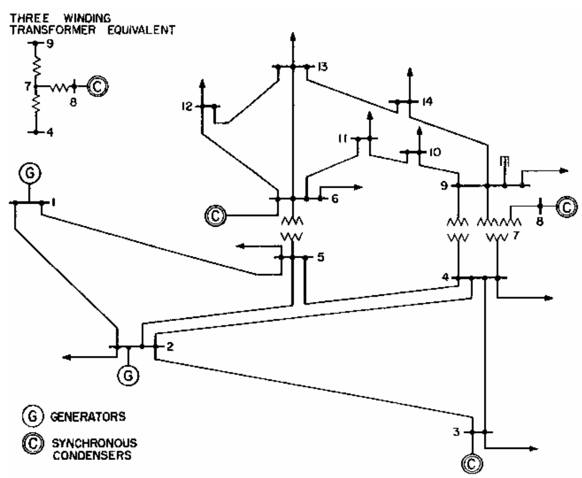

Figure 1. IEEE 14-Bus Network

Consideration of the connections only, regardless of the components whether they are generators, condensers, or transformer, coverts the 14-bus network to a graph of 14 nodes and 20 edges (see Figure 2).

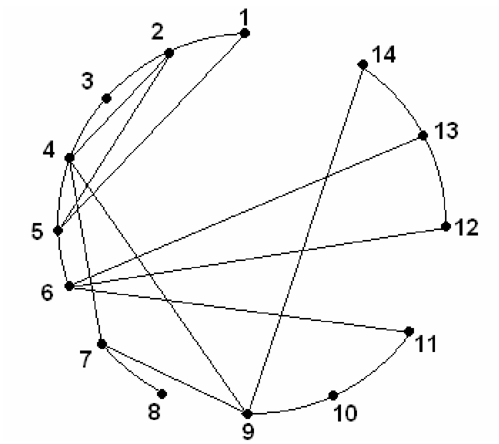

Figure 2. Topology of the 14-Bus Network

The degree (or number of neighbor nodes), the critical path length, and the clustering coefficient of each node are calculated and tabulated in the Table I.

Table I. Graphical Properties of 14-Bus Network

Node Number	Degree, k	Critical Path Length (L)	Clustering Coefficient (γ)
1	2	2.692	1.0
2	4	2.154	0.5
3	2	2.615	1.0
4	5	1.836	0.3
5	4	1.923	0.333
6	4	2.077	0.167
7	3	2.231	0.333
8	1	3.154	0.0
9	4	1.923	0.167
10	2	2.462	0.0
11	2	2.538	0.0
12	2	2.769	1.0
13	3	2.462	0.333
14	2	2.385	0.0
Average	**2.857**	**2.374**	**0.367**

At the bottom of the table, the 14-Bus network reveals its topological properties: average number of neighbors of 2.857, average critical path length (L) of 2.374, and the average clustering coefficient (γ) of 0.367. On the other hand, a random graph with the same number of node, with the same degree of 2.857, would have L_{random}=2.51 and γ_{random}=0.204.

The critical path length of the 14-Bus network is close to that of a random graph, but the clustering coefficient is approximately twice that of the random graph. The 14-Bus network then is not classified as a small world graph. As can be seen in the following section, the number of nodes in the power network seems to determine the characteristics of the network; bigger networks are more of small world networks while smaller ones are usually close to random networks.

2.3 Small World Networks and Impact to Power Grid Analysis

The key topological characteristic of small world phenomenon is the presence of a small fraction of a very long-range, global edges, which contract otherwise distant nodes of the graph, while most edges remain local, thus contributing to the high clustering coefficient. In other words, small world graph is highly clustered yet has path length equivalent to random graphs, and exhibits the following characteristics:

$$L \approx L_{random}(n,k) \tag{7}$$

$$\gamma >> \gamma_{random}(n,k) \tag{8}$$

According to [8], the western states power transmission grid, a large and sparse network with n = 4941 and k = 2.67, is classified as a small world with the values of the critical path length and the clustering coefficient as 18.9 and 0.08, respectively. This clustering coefficient is 160 times larger than the expected value for an equivalent random graph, but the critical path length is only about 1.5 times greater.

The relevant dynamical property of some small world networks, disease spreading in particular, is dominated by the characteristic length of the graph, but that of other small world networks are determined by the clustering coefficient, or both. Of course, there are cases neither one seems to have any relevance with dynamic property [10]. Then, is there any graphical dominant factor in the power network dynamics? And, why does a certain line outage lead to a cascading fault while most become locally contained [11]? How do we quantify the influence of the shape of network to the dynamic stability of the network [12]? Linking the dynamic and the topological property of the network could answer the questions.

In power system, presently, its dynamic properties are mathematically calculated, ignoring the topology of the system. The small world phenomenon now opens a door for topological analysis to look into the vexing problem of cascading outages. But the first question we have to answer is if topology really matters in the network dynamics: are its dynamic properties relevant to its graphical statistic such as the critical path length or the clustering coefficient? Also, we need to draw a conclusion on the power grid's small world-ness. To answer these questions, we need clear understanding of the topological property of a real power network and of the relationship between the topological changes and the failure dynamics. In an effort to provide a better understanding of the role of topology in power network disruption, we trace the topological measures with the sequence of the 1996 cascading events to relate the topology and the inherent instability of power grid. In the analysis presented next, two topological measures, characteristic path length and clustering coefficient, appear as a possible link between the topology and the cascading failures in the power grid.

3. Topological Analysis of Power Grid and 1996 WSCC Outages

We first analyze power grids for small world-ness. Then, we reconstruct the WSCC 1996 outages using the power grid data and the outage report, and compare the graphical property of the grid when the blackout triggering lines are removed simulating the actual sequence of cascading outages with that when the same number of randomly selected lines are removed.

3.1 Topological Analysis of Power Networks

Several power networks of various sizes were analyzed to inspect if power network in general is a small world network. The networks analyzed are:

- IEEE standard 118-bus network which has 118 nodes and 179 edges [9];
- MAPP (Mid-Continental Area Power Pool) network of 230kV and above only, which has 575 nodes and 754 edges;
- Nordel network, the interconnected power systems of Finland, Norway, Sweden, and parts of Denmark, of 100 kV and above only, which has 410 nodes and 564 edges;
- KEPCO (Korea Electric Power Corporation) network of 66kV and above only, which has 553 nodes and 783 edges; and
- ERCOT (Electric Reliability Council of Texas) network of 345kV only, which has 148 nodes and 209 edges.

We apply a cellular automata approach as the search tool for neighbor nodes in the algorithm for the calculation of critical path length (L) and the clustering coefficient (γ). This approach saves code spaces and execution times. Table II summarizes the graphical properties of the networks. One thing common to all the networks, regardless of the number of nodes or edges, is that the average degrees (k) are almost the same, ranging from 2.62 to 3.03. This means that a power node has 3 neighboring nodes in average. The L and γ do not show a consistent result: one network's critical path length is close to that of a random graph while its clustering coefficient is no bigger than that of a random graph, or one with a bigger clustering coefficient has twice bigger critical path length than that of a random graph. It is clear, however, that bigger networks have higher clustering coefficients and, at the same time, slightly longer critical path lengths, and are closer to the category of small world network.

Table II. Graphical Properties of Selected Power Networks

	node, n	edge, M	degree, k	L/L_{random}	γ/γ_{random}
118Bus	118	179	3.03	1.47	6.8
MAPP[1]	575	754	2.62	2.39	18.4
Nordel[2]	410	564	2.75	2.37	21.4
KEPCO[3]	553	783	2.83	1.24	23.5
ERCOT[4]	148	209	2.82	1.47	7.3

[1] 230kV and above only; [2] 100kV and above only; [3] 66kV and above only; [4]345kV only

3.2 Reconstruction of 1996 WSCC Power Outage

According to a FERC 715 report [13], WSCC network of year 1996 consists of over 6500 nodes and 9000 transmission/distribution lines with voltage levels ranging from 6 to 550 kV. The network with 100kV and higher voltage only contains 4610 nodes and 6244 lines. If only voltage of 200kV and higher are considered, it has 1646 nodes and 2348 lines. If we include only the components of 300kV and above, WSCC becomes much smaller network with 352 nodes and 449 lines.

Table III shows the calculated results of WSCC topology for the sub-networks of 100kV and above and of 300kV and above. Our topological analysis of WSCC grid shows that it could be classified as a small world, even though there are no clear cut-off lines in the ratios of L/L_{random} and γ/γ_{random} to become a small world network. The result of the WSCC with 100kV and above is not very different from the work reported in [8] which classifies WSCC as a small world network. However, the reduced network with 300kV and above is much smaller and its graphical properties are of neither small world nor random network. Since the ratio of clustering coefficient is about seven, we take the 300 kV circuit as a quasi-small world network.

Table III. Graphical Properties of the WSCC Network

WSCC	node, n	edge, M	degree, k	L	γ	L/L_{random}	γ/γ_{random}
>100 kV	4610	6244	2.71	25.25	0.046	2.87	78.23
>300 kV	352	449	2.51	17.28	0.048	2.56	7.10

With the notion that WSCC is a small world network, with its sub-network of 300kV as a pseudo-small world, we investigate the WSCC outages. Based on the FERC 715 network of WSCC and the outage reports [1, 12], the cascading faults of summer WSCC 1996 are reconstructed. Only the lines and nodes (substations and generators) of above 300kV are considered in the study. Figure 3 illustrates the nodes and lines of the area where the 1996 failures initiated. The node names are the same as in the FERC 715 report, but the line numbers (numbers next to lines) are generated by the authors for the analysis. The line faults which triggered the July and the August outages are marked by X's (*X*) and check's (√), respectively. The boxed number next to each of the X marked lines indicates the sequential order of the July outage; the circled number next to each of the checked (√) lines is the order of the August outage. The nodes encircled with dotted lines are generating stations.

The July 2 failure is triggered by an event of flashover to a tree on the Bridger-Goshen line (line #106). Then, the Bridger-Kinport line (#100) is tripped. These two line losses trip the Bridger generating units and, consequently, the line #95 is removed from the service. Later, lines #105 and #219 are also removed from the service [14].

In the August 10 failure, the Allston-Keeler line (line #143) sags to a tree and flashes over and sparks off the event. However, two forced outages of John Day-Marion (line #202) and Big Eddy-Ostrander (line # 151) lines precede the Allston-Keeler line fault. On the failure date, after the loss of line #143, the Keeler-Pearl line (line #191) opens by the Keeler breaker operation. At the same time thirteen McNary units, connected at 230kV, sequentially trip, which causes system power and voltage oscillations [1].

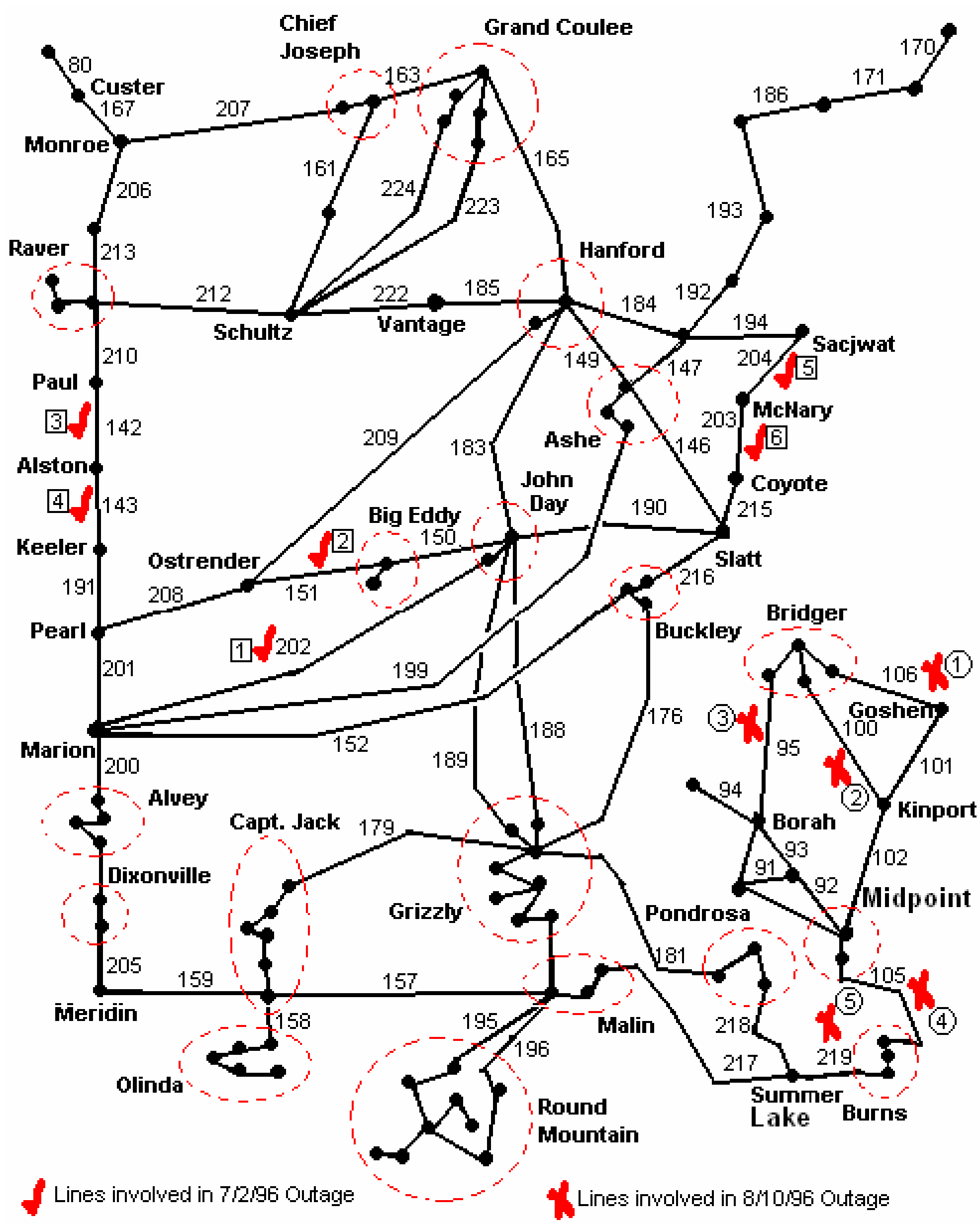

Figure 3. Reconstructed outages of 1996 WSCC blackout.

3.3 Comparison of Topological Indices

Using the reconstructed WSCC 1996 outages, we calculate the graphical properties of the WSCC grid when the blackout triggering lines are removed and when the same number of randomly selected lines are removed. The two indices, the critical path lengths and the clustering coefficients, are calculated for the topological property, for three cases: grid with no line removal, grid with the removal of 4 outage triggering lines, and grid with the removal of arbitrarily selected 4 lines. The three cases are further divided into the following nine scenarios (see Table IV), with one normal gird, four removals of outage triggering lines, and four removals of randomly selected lines.

Table IV. Nine Scenarios for Graphical Property Comparison

No	Outage	Line Removal Principle	Lines Removed (#)
1	No outage	N/A	None
2	August 10	Outage Triggering Lines	142, 203, 143, 204
3	August 10	Outage Triggering Lines	151, 202, 142, 203
4	July 2	Outage Triggering Lines	106, 100, 95, 219
5	July 2	Outage Triggering Lines	106, 100, 95, 105
6	No outage	Arbitrarily Selected	100, 200, 300, 400
7	No outage	Arbitrarily Selected	101, 202, 303, 404
8	No outage	Arbitrarily Selected	103, 203, 304, 405
9	No outage	Arbitrarily Selected	10, 20, 30, 40

From the illustrated result in Figure 4, we can see that the critical path lengths for the July outage (scenarios 4 and 5) show much higher than other scenarios of no-outage and arbitrary line removals. We also see a little increase in the critical paths of the two August outage scenarios (2 and 3). It is significant that the critical path lengths of the both July and August scenarios are higher than those of no-outage and arbitrary scenarios. The clustering coefficient, however, does not show any differential behavior between the outage and the no-outage scenarios.

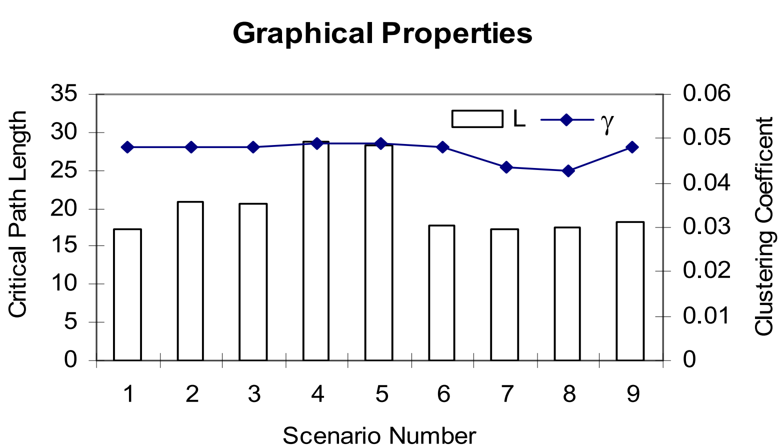

Figure 4. Illustration of the graphical property changes in different scenarios.

Next, we compare the small world-ness of the network of the scenarios. Compared with a random network, a small world network has the feature of the same characteristic path length and of much higher clustering coefficient, therefore, we define the "small world-ness index" as:

$$\frac{\gamma / \gamma_{random}}{L / L_{random}}. \qquad (9)$$

The small world-ness index would increase as a network gets closer to small world. Figure 5 shows the small world-ness index of each of the nine scenarios.

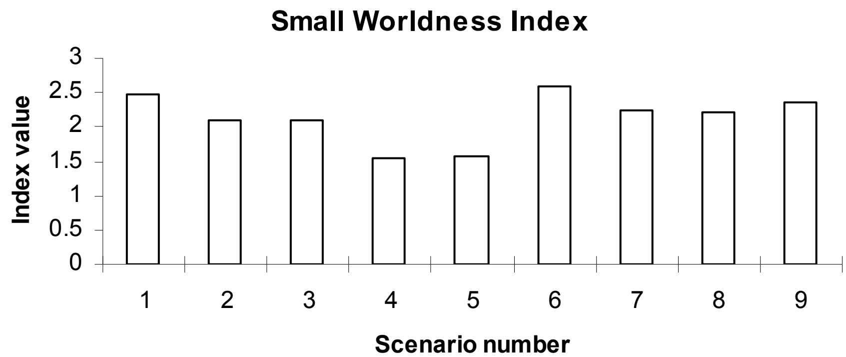

Figure 5. Illustration of Small World-ness Indexes of 9 scenarios

As in the characteristic path length featured in Figure 4, the small world-ness indexes of the actual outages, July ones in particular, are distinctive: scenarios 4 and 5 show much lower values. The two August outages (scenarios 2 and 3) have lower indexes than normal and other arbitrary scenarios. A conclusion we draw, though not without reservation, is that the line faults which collectively shift a network away from the normal small world-ness may be related to cascading outage.

The overall results from the scenarios are quite interesting and we are expanding the research efforts of investigation into other scenarios and other outages. Particularly, with the 2003 Northeast blackout data, we are currently investigating the significance of the sequence of the line removal and its impact to the changes in the critical path length, the clustering coefficient, and the small world-ness index.

We admit that the study presented in the paper does not reflect the actual, overall grid structure of the WSCC; the study is performed on the sub-network of the grid. However, the study results provide new insights into the possible connection of cascading outages with static topological measures. Also, they open the door wider of applying topological index, along with the traditional dynamic indices, in assessing vulnerability of power grid. A suggested use of the finding is monitoring the topological indices and subsequent small world-ness index and, when the critical path length gets higher than usual and the small world-ness index lower than normal, alerting operators for immediate check of the system status for possible cascading effect of a system disruption. The static topological analysis can provide the information much faster than the conventional calculation-heavy dynamic analyses.

4. Conclusions

This paper presented a new and interesting approach for power grid outages and vulnerability assessment by topological estimators, i.e., critical path length and clustering coefficient. The investigation found that the critical path lengths of the triggering events of the July and August outages of 1996 WSCC blackout were higher than those of no-outage and arbitrary events. In addition, the small world-ness index for each of the outage triggering events was much smaller than that of normal or any no-outage scenario, indicating that events of shifting a network from small world to a random network would be more likely cascaded to wide area outage. The finding suggests that utility monitor the topological index real time and alert operators for immediate check of the system dynamics for possibility of cascading from otherwise seemingly routine line outages. Another use of the finding is the prioritization by the index of each transmission line in a grid in terms of triggering probability, if faulted, of a cascading failure and outage. The highly prioritized lines should be first considered in maintenance, repair, and upgrade. Another off-line application area would be in the route and connection planning stage when new transmission lines are to be added or old ones are to be removed. The planning for construction or removal of transmission lines would have better perform a topological study and assess the vulnerability and susceptibility of the network to the expected changes.

The overall result from the investigation is encouraging and we are currently applying the same topological indices to the 2003 Northeast blackout to verify the conclusion we drew from the 1996 WSCC outages. Our long term goal is to do the same analysis on the overall North American power grid to further evaluate the merit of topological approach in the practical vulnerability assessment of the complex power grid.

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