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Least-Squares Estimation of Circuit Element Values from Measured Voltage and Current Waveforms

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***Abstract*— This article discusses an estimation approach for the values of passive elements such as resistance and inductance for a series inductive circuit using measured voltage and current waveforms at the source side. The proposed method finds the unknown two values using the single equation of the circuit by the least-squares approach with the pre-processed data. The pre-processing of the voltage and current waveforms includes concatenation of the discrete samples of voltage and current waveforms with one sampling period shifted. The approach works for steady-state sinusoidal waveforms as well as transients. The proposed approach can find the resistance and the inductance to the transient source in as short as 1/2 cycle length of discrete sample data. The proposed method is tested with waveforms generated from LTspice for a circuit of nominal 12 kV distribution circuit and with real transient waveform from a power distribution circuit. The evaluation result in both cases is good. Its good performance with transient waveforms is particularly significant in that, in such soft fault situations in utility distribution lines as self-clearing faults, the transient lasts just about 1 or 2 cycles before the system returns to the normal state as if nothing happens.**

***Index Terms*— Least-squares estimation, fault-loop, power distribution circuit, fault location, self-clearing fault.**

I. Introduction

MOMEMTARY sag faults in distribution feeders show their distinctive signature behaviors with a few cycles of transient, but often, less than one cycle, before the system returns to normal behavior. Thus they have other common names such as sub-cycle faults, incipient faults, transient faults, and self-clearing faults. A self-clearing transient fault in underground cables has its root cause in a water tree development inside the cable or moisture accumulation in a cable splice, which leads to a momentary insulation breakdown followed by arc, which in turn causes rapid moisture evaporation and temporary insulation recovery [1].

The correct location of self-clearing transitory fault is crucially important in prevention of permanent faults and unscheduled outages. The conventional methods of fault location which rely on steady-state fault waveforms are not effective in such short-lived self-clearing faults which manifests just 1 or 2 cycles of transients before returning to the normal state [2 - 5]. However, the manifested transients represent a distribution circuit; therefore, the location to the transient source can be calculated by the resistance and inductance values of the fault-loop of the circuit.

Previously, an approach was developed which was centered on solving for the unknown location variables (line resistance and line reactance to the transient source) of a fault loop using a discrete inverse time-domain differential equation [6, 7]. Unlike the conventional phasor (or frequency) domain fault location methods developed for permanent faults, the new time-domain inverse approach worked for transient and steady-state waveforms. Despite the above advantage, there was a weakness: the resistance to the fault was ignored due mainly to the fact that there were two unknown variables as stated above in the single fault loop differential equation. Exclusion of the resistance to the fault in certain situations and conductor types would cause unacceptable error in estimating the inductance to the location of transient source.

We propose a new approach which is based on discrete parameter estimation with least squares. The discrete least-squares approach is applied using measurement data of voltage and current signals at the source side of the circuit. The proposed method finds the unknown two values, resistance and inductance, using the single equation of the fault-loop of the circuit by using the concatenated discrete samples of voltage and current as the main components for the least-squares estimation.

The paper is organized as follows. In the next chapter, we briefly discuss the least-squares estimation. Then in Chapter III, we model a distribution circuit and formulate the sample data matrices from the simulated model circuit for least-squares estimation of the circuit element values. Chapter IV discusses the evaluation result of the least-squares method compared with the true values. Chapter V concludes the paper.

II. Least-Squares Estimation

The Least-squares estimation (LSE) was invented by Karl Gauss when he considered inferring the values of the motion parameters of planets and comets from measured data [8]. Consider *M* scalar measurements of a signal y(t) are made at times, t_{k-M}, t_{k-M+1}, ..., t_{k-1}, t_k. Let the measurements of y(t) are assumed to be a linear combination of 2 parameters, β_0 and β_1 with certain signals $x_0(t)$ and $x_1(t)$:

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$$y(t) = x_0(t)\beta_0(t) + x_1(t)\beta_1(t) + e(t),$$

where e(t) is measurement errors with zero mean.

Then the M measurements of y(t) can be expressed in a matrix equation, with time t_k replaced by just k for notation simplification:

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{E},$$

where

$$Y = \begin{vmatrix} y(k) \\ y(k-1) \\ y(k-2) \\ \vdots \\ y(k-M) \end{vmatrix},$$

$$X = \begin{vmatrix} x_0(k) & x_1(k) \\ x_0(k-1) & x_1(k-1) \\ \vdots & \vdots \\ x_0(k-M) & x_1(k-M) \end{vmatrix},$$

$$\beta = \begin{vmatrix} \beta_0 \\ \beta_1 \end{vmatrix}, \text{ and}$$

$$E = \begin{vmatrix} e(k) \\ e(k-1) \\ e(k-2\_ \\ \vdots \\ e(k-M) \end{vmatrix}.$$

If there are no measurement errors involved, then a deterministic equation is resulted as $\boldsymbol{Y} = \boldsymbol{X\beta}$, and parameter β can be solved as $\boldsymbol{\beta} = \boldsymbol{X}^{-1}\boldsymbol{Y}$. However, if the system is over-determined as in our case with just 2 variables and many more measurements, then Y is approximated as $\hat{Y} = X\hat{\beta}$, where $\hat{\beta}$ is obtained by solving the equation for the value which minimizes the weighted sum of the squares of the equation error, $\tilde{Y} = Y - \hat{Y}$[9]. Then least-squares estimate of β is given by

$$\hat{\beta} = (X^T X)^{-1} X^T Y.$$

III. Least-Squares Estimation of Circuit Element Values

We utilize the LSE in estimating the values of passive element in a circuit during the short-lived transient period. The main focus of this approach is then forming the two matrices **Y** and **X** for the LSE model $\boldsymbol{Y} = \boldsymbol{X\beta}$ from the discrete voltage and current waveforms obtained from the circuit and deriving the element values from $\hat{\beta} = (X^T X)^{-1} X^T Y$. These derived circuit element values can be interpreted as to indicate the resistance and inductance to the self-clearing fault location in a power distribution circuit.

Consider a distribution circuit served by a 12 kV line-to-line voltage substation, one phase of which is as depicted in Fig.1 using LTspice [10] simulation space. The source/internal impedance of the substation transformer is given by R6 and L4. The substation has a capacitor bank C1 of 120 micro-farad. The entire load of the circuit is placed at the circuit end. And the circuit's entire line resistance is given with the sum of R3 and R1, and the line inductance the sum of L3 and L1.

The division of the line by R3 and L3 front and R1 and L1 back is an illustration of a transient source location at which a cycle-long transient of self-clearing event may occur. The locations of the self-clearing event may be adjusted by proportionally allocating the line resistance R3 and R1 and the line inductance L3 and L1, accordingly, assuming that the entire line is of the same conductor type.

R4 and R5 with time-controlled resistance values determine the duration of the self-clearing transient. The path is open with 1 giga-ohm resistance before 150 ms time point from the start of simulation and is shorted with just 1 micro-ohm from 150 ms point to 166 ms. After this 16 ms short, the path is remained open. The voltage and current signals are measured at the point marked "A."

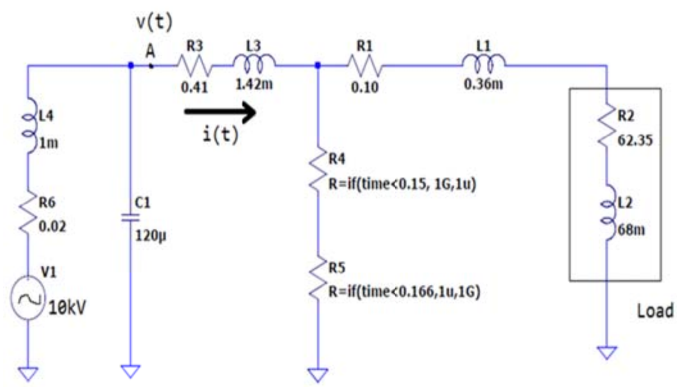

Fig.1. LTspice modeling of a power distribution circuit with a momentary transient path.

Whether the circuit is in a normal state or transient state, the same equation determines the relationship between the voltage and the current:

$$v(t) = R \cdot i(t) + L \cdot \frac{d}{dt}(i(t)).$$

Since the signals are of discrete measurement, the circuit equation is now expressed as a difference equation:

$$v(t) = R \cdot i(t) + L \cdot \frac{i(t+\Delta t) - i(t)}{\Delta t},$$

where Δt is the time between two consecutive signal samples, namely sampling period. We start our matrix formation of LSE using the difference equation. Rearrangement of the difference equation leads to,

$$L \cdot i(t+\Delta t) + i(t)[R \cdot \Delta t - L] = v(t) \cdot \Delta t,$$

and eventually to a matrix equation:

$$[i(t+\Delta t) \quad i(t)] \cdot \begin{bmatrix} L \\ R \cdot \Delta t - L \end{bmatrix} = [v(t)] \cdot \Delta t.$$

Now if we include M number of measurements, the matrix expands to,

$$\begin{bmatrix} i(t+M\Delta t) & \cdots & i(t+(M-1)\Delta t) \\ \vdots & \ddots & \vdots \\ i(t+\Delta t) & \cdots & i(t) \end{bmatrix} \cdot \begin{bmatrix} L \\ R\cdot\Delta t - L \end{bmatrix} = \begin{bmatrix} v(t+(M-1)\Delta t) \\ \vdots \\ v(t) \end{bmatrix} \cdot \Delta t$$

If we label the first matrix above as X, and the second β, and the third Y, the above equation is written as:

$$X \cdot \beta = Y \cdot \Delta t,$$

which is the same equation as discussed in the theory of least-squares estimation. Therefrom,

$$\beta = \begin{bmatrix} L \\ R\cdot\Delta t - L \end{bmatrix}$$

is obtained by

$$\beta = (X^T \cdot X)^{-1} \cdot X^T \cdot Y \cdot \Delta t.$$

Finally, the two parameters to the location of interest, L and R, are obtained from the row components of β:

$$L = \beta_0 \text{ , and}$$

$$R = \frac{\beta_0 + \beta_1}{\Delta t}$$

As for the number of measurements (M) in practical application, we use the number of samples for ½ cycle of the signals. Therefore, if the signals are sampled at the rate of 7680 samples per second, 64 measurements are included in the matrix formation for X and Y starting from the onset of transient. At this particular sampling rate, Δt is 0.1302 ms. It is interesting to note that the row elements in the first and second columns of X are identical except each element is just one sampling period shifted, with those in the first column 1 period ahead of those in the second column.

IV. Evaluation of the Least-Squares Approach

To test the least-squares method, we simulate the circuit of Fig.1 in LTspice with the element values and sources as indicated in the model space. The generated voltage and current signals are as depicted in Fig. 2. The voltage is measured at the node "A" and the current measured is that through the line inductor L3. The waveform's saving time is 100ms; therefore, the transient start time appears at 50 ms instead of 150 ms.

The voltage and currents signals are sampled at 128 samples per cycle. From a certain sample point, let say 0, which is usually that of the voltage peak point, the 1/2 cycle length of voltage samples, namely 64, are collected in the order: v[0], v[1], …, v[63]. This 64 samples form the single-column Y matrix. From the same point, the 64 samples of the current signal form the second column of the X matrix: i[0], i[2], …,i[63]. The first column of the X matrix is obtained from the same number of samples but with sampling points shifted by 1 sample point as explained before: i[1], i[2],…,i[64].

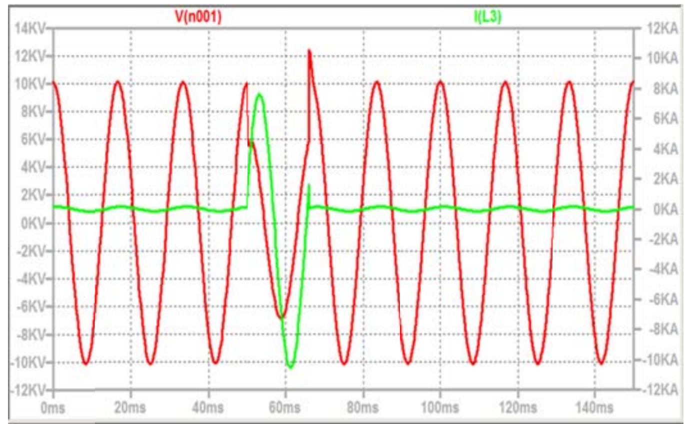

Fig.2. Measured voltage (red trace) and current (green trace) waveforms of the circuit as modelled in Fig. 1.

A. Test with Steady-State Signals

First, we test the least-squares method for the normal steady-state state signal portion to calculate the normal state circuit element values. The normal state waveforms from 0 to 40 ms are depicted in Fig. 3.

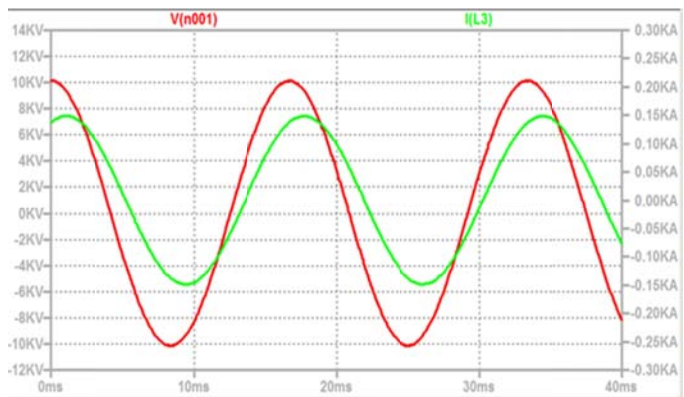

Fig.3. Measured voltage (red trace) and current (green trace) waveforms of the circuit as modelled in Fig. 1 before the transient event.

In the normal state the circuit seen by the voltage and current waveforms are the sum of the line resistance and inductance and the load as modeled in Fig. 1, so the correct total resistance and inductance values are R = 62.86 ohm and L = 69.78 mH, respectively.

As an intermediate calculation output, the matrix X^TX is obtained as

$$X^T X = \begin{pmatrix} 7.0825 \times 10^5 & 7.074 \times 10^5 \\ 7.074 \times 10^5 & 7.0826 \times 10^5 \end{pmatrix},$$

and the final output for parameter as

$$\beta = \begin{pmatrix} 0.0698 \\ -0.0615 \end{pmatrix}.$$

Therefore, with Δt = 1.302 x 10^{-4}, the resistance and inductance are calculated as:

$$L_{LSE} = \beta_0 = 0.0698 \text{ and}$$

$$R_{LSE} = \frac{\beta_0+\beta_1}{\Delta t} = 63.5111 \text{ .}$$

These two calculated values are very close to the true values of L = 0.06978 H and R = 62.86 ohm, respectively.

B. *Test with Transient Signals*

Next we test the least-squares method to the transient portion of the waveforms as shown in Fig. 4 in its expanded view.

In the transient period, the resistance and inductance the waveforms see is the line resistance and inductance to the momentary switching path through the time-controlled resistors of R4 and R5 whose values are negligible during the transient period. Therefore, the total resistance and inductance of the circuit is R=0.41 ohm and L = 1.42 mH, respectively.

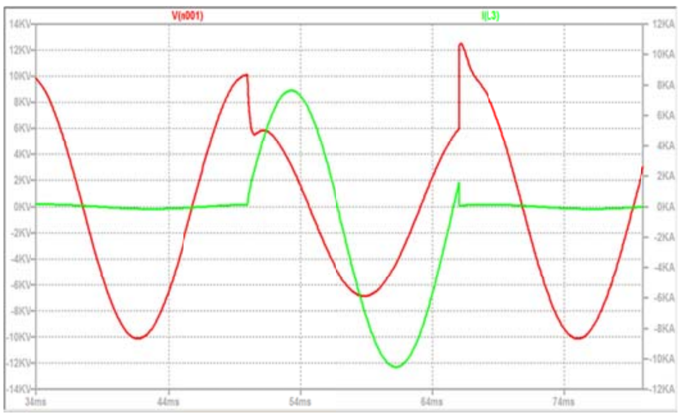

Fig.4. Measured voltage (red trace) and current (green trace) waveforms of the circuit as modelled in Fig. 1 during the transient period of 15 ms.

From the 1/2 cycle length LSE calculation, the matrix X^TX is obtained as

$$X^TX = \begin{pmatrix} 1.7641 \times 10^9 & 1.7435 \times 10^9 \\ 1.7435 \times 10^9 & 1.7304 \times 10^9 \end{pmatrix},$$

and the final output for parameter as

$$\beta = \begin{pmatrix} 0.001421 \\ -0.0013651 \end{pmatrix}.$$

Therefore, with Δt = 1.302 x 10^{-4}, the resistance and inductance are calculated as:

$$L_{LSE} = \beta_0 = 1.421 \text{ mH and}$$

$$R_{LSE} = \frac{\beta_0+\beta_1}{\Delta t} = 0.4291 \text{ ohm.}$$

These two calculated values are very close to the true values of L = 1.42 mH and R = 0.41 ohm, respectively.

The good performance with transient is particularly significant in that, in such soft fault situations in utility distribution lines as self-clearing faults, the transient lasts just about 1 or 2 cycles before the system returns to the normal state as if nothing happens.

Further tests are performed with similar 1/2 cycle transients resulted from different locations of momentary switching. Table I summarizes the true values of R and L to the switching point and the corresponding calculated values by the LSE along with their respective error rates. Across the varying values of R3 and L3 which are the resistance and inductance to the momentary switching point, the error on R is much higher than on L. The error on R is 3 - 5% while that on L is 0 - 1 %. The higher error on R is thought to be from the ignorable amount resistance in the switching path. However, the performance on L is excellent and this performance would not be impacted even by the small amount of resistance in the path. Overall, the performance of the LSE is acceptable.

TABLE I
EVALUATION OF THE LSE-CALCULATED RESISTANCE AND INDUCTANCE TO THE SWITCHING POINTS.

R3 (True)	L3 (True)	R3(by LSE)	L3 (by LSE)	R3 error (%)	L3 error (%)
0.051	0.178	0.053	0.179	3.922	0.833
0.102	0.355	0.105	0.359	2.941	0.974
0.255	0.889	0.262	0.898	2.745	1.059
0.357	1.244	0.377	1.241	5.602	0.244
0.408	1.422	0.429	1.421	5.147	0.053

C. *Test with Real Power Distribution Signals*

Evaluation with real waveforms, however, is the ultimate test of the proposed method. In efforts to evaluate with real data from power distribution feeders and circuits, we are acquiring self-clearing fault signals from utility companies which were measured either at substation bus or circuit. However, it is quickly found that the real circuit transient signals are very difficult to obtain. Fortunately, here we present one real self-clearing transient which was captured for events which eventually led to an underground cable fault. The outage report on the event listed the location as 1.3923 mH without mentioning the resistance to the fault. However, since, in the report, the inductance to a location included the initial (or source) inductance of 0.9814 mH at the monitoring point, the true inductance to the fault from the monitoring device was 0.4109 mH.

Fig. 5 shows the voltage and current waveforms of the faulted phase sampled at 128 samples per cycle.

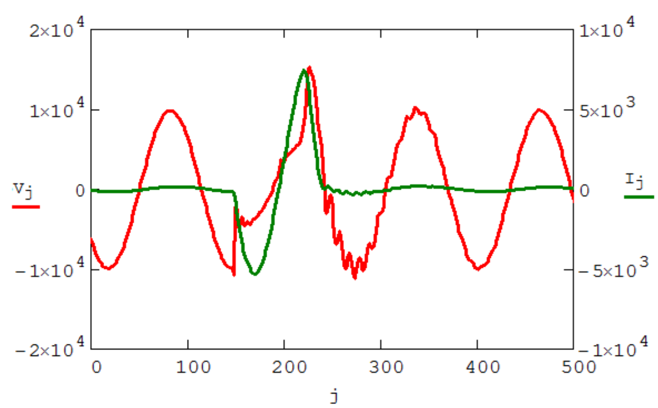

Fig.5. Measured voltage (red trace) and current (green trace) waveforms from a real power circuit of self-clearing fault at the location of L = 0.4109 mH.

The starting point for voltage and current samples for X and Y matrices is at the sample point j = 147 which coincides with the negative peak point of the voltage signal. The same 1/2 cycle length is used for forming the matrices.

The intermediate matrix X^TX is obtained as

$$X^TX = \begin{pmatrix} 2.9916 \times 10^9 & 2.9303 \times 10^9 \\ 2.9303 \times 10^9 & 2.8866 \times 10^9 \end{pmatrix},$$

and the final output for parameter as

$$\beta = \begin{pmatrix} 0.00041083 \\ -0.00036009 \end{pmatrix}.$$

Therefore, with $\Delta t = 1.302 \times 10^{-4}$, the resistance and inductance are calculated as:

$$L_{LSE} = \beta_0 = 0.41083 \text{ mH} \text{ and}$$

$$R_{LSE} = \frac{\beta_0 + \beta_1}{\Delta t} = 0.3897 \text{ ohm}.$$

The calculated inductance is unbelievably close to the true inductance.

We will continue our evaluation work of the concatenated discrete LSE approach with real transient waveforms from utility distribution circuits as they are obtained. As we gather and test more waveforms and evaluation statistics, we plan to publish the result subsequently.

V. Conclusions

We applied the least-squares method to estimate the value of passive elements such as resistance and inductance for a general series inductive circuit using the measured voltage and current waveforms at the source side. The approach was formulated with concatenated measurement matrices of voltage and current in discrete form for the differential equation of the fault loop. The proposed method was tested with waveforms generated from an LTspice model for nominal 12 kV distribution circuits. The approach worked well for steady-state sinusoidal waveforms as well as transients. The proposed approach could find the resistance and the inductance to the transient source with reasonable accuracy using 1/2 cycle length of discrete sample data. Its good performance with 1/2 cycle transient waveform is particularly significant in that, in self-clearing faults, the transient lasts just about 1 or 2 cycles before the system returns to the normal state as if nothing happens. We have tested just one real transient signal of self-clearing fault and the calculation result was very good. We will continue our evaluation work with real self-clearing fault signals from utility companies by acquiring and testing them as they are obtained. When we have reasonable amount of waveform data and evaluation statistics, we may have much better sense of the effectiveness of the proposed concatenated discrete least-squares approach for estimating circuit element values from the short-lived transients. We hope we publish the result soon.

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