Remember: Surge Impedance

\[ Z_c = \sqrt{\frac{L}{C}} \]  

Lossless Line is assumed

If a load is purely resistive equal to the surge impedance

The Load Current is given by (where VL is line-to-line voltage)

\[ Z_{\text{load}} = \sqrt{\frac{L}{C}} \]

\[ |I_L| = \frac{|VL|}{\sqrt{3} \cdot \sqrt{\frac{L}{C}}} \]
Surge Impedance and Loading Power

- If a load is purely resistive equal to the surge impedance,
  \[ Z_{\text{load}} = \sqrt{\frac{L}{C}} \]

- The Power (3-phase) delivered to the load:
  \[
  |I_L| = \frac{|V_L|}{\sqrt{3} \cdot \sqrt{\frac{L}{C}}}
  \]

\[
\begin{align*}
\text{Power} & = \sqrt{3} \cdot \frac{|V_L|}{|I_L|} \\
& = \sqrt{3} \cdot \frac{|V_L|}{\sqrt{3} \cdot \sqrt{\frac{L}{C}}} \\
& = \frac{|V_L|^2}{\sqrt{\frac{L}{C}}} \\
& = \text{SIL (Surge Impedance Loading)} \quad \text{[W]}
\end{align*}
\]

- }
Surge Impedance Loading

\[ V(x) = \frac{V_R + jIRzC}{2} e^{j\alpha} e^{j\beta x} + \frac{V_R - jIRzC}{2} e^{j\alpha} e^{-j\beta x} \]

No reflected voltage!!

\[ V_R = IRzC \]

\[ \frac{V_R + jIRzC}{2} = V_R \Rightarrow V(x) = V_R \]

Voltage at any location \( x \) is the same as the receiving end voltage

Flat Voltage Profile

\[ V = \alpha + j\beta \]

\[ V = \sqrt{z}y \]

\[ j\omega L - j\omega C \]

\[ = j\omega \sqrt{LC} \]

\[ = \alpha + j\beta \]

No attenuation $\alpha = 0$
Role of Transmission Line

- Acts as Shunt Capacitor - Supplying Var to the System
- Acts as Shunt Reactor - Absorbing Var from the System

Practical use of SIL (Surge Impedance Loading)
- A permissible loading of a transmission may be expressed by a fraction of its SIL
- SIL may provide a comparison of load carrying capability of lines
Surge Impedance Loading

Role of Transmission Line

\[ MVar_{\text{Used}} = I^2 X_L \]

\[ MVar_{\text{Produced}} = \frac{V^2}{X_C} \]

Plot Typical for 100 Mile Long 345 kV Line

Thermal Limit

SIL = 450 MW

Figure 1
Surge Impedance Loading of a Transmission Line
Hyperbolic Form or the equations

\[ V(x) = \frac{V_R + I_R \cdot Z_C}{2} e^{\sqrt{X}} + \frac{V_R - I_R \cdot Z_C}{2} e^{-\sqrt{X}} \]

\[ I(x) = \frac{V_R/Z_C + I_R}{2} e^{\sqrt{X}} - \frac{V_R/Z_C - I_R}{2} e^{-\sqrt{X}} \]

more convenient form by hyperbolic functions:

\[ \sinh \theta = \frac{e^{\theta} - e^{-\theta}}{2} \]

\[ \cosh \theta = \frac{e^{\theta} + e^{-\theta}}{2} \]

\[ e^{\theta} = \sinh \theta + \cosh \theta \]

\[ e^{-\theta} = \cosh \theta - \sinh \theta \]
Hyperbolic Form or the equations

\[ V(x) = \frac{V_R + I_R Z_C}{2} e^{\alpha x} + \frac{V_R - I_R Z_C}{2} e^{-\alpha x} \]

\[ = \frac{V_R + I_R Z_C}{2} (\sinh \alpha x + \cosh \alpha x) \]
\[ + \frac{V_R - I_R Z_C}{2} (\cosh \alpha x - \sinh \alpha x) \]

\[ = \frac{V_R + I_R Z_C + V_R - I_R Z_C}{2} \cosh \alpha x \]
\[ + \frac{(V_R + I_R Z_C - V_R + I_R Z_C)}{2} \sinh \alpha x \]

\[ V(x) = V_R \cdot \cosh \alpha x + I_R \cdot Z_C \cdot \sinh \alpha x \]

Similarly,

\[ I(x) = I_R \cdot \cosh \alpha x + \frac{V_R}{Z_C} \cdot \sinh \alpha x \]
Hyperbolic Form or the equations

\[ V_S = V_{(0)} \]
\[ \text{Length} \ 	ext{of line} \ x = 0 \]

\[ V_S = V_R \cdot \cosh \ \nu L + I_R \cdot Z_C \cdot \sinh \ \nu L \]
\[ I_S = \frac{V_R}{Z_C} \cdot \sinh \ \nu L + I_R \cdot \cosh \ \nu L \]

\[ A = \cosh \ \nu L \quad B = Z_C \cdot \sinh \ \nu L \]
\[ C = \frac{\sinh \ \nu L}{Z_C} \quad D = \cosh \ \nu L \]

\[ S = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \]

\[ \begin{aligned}
V_R &= V_S \cdot \cosh \ \nu L - I_S \cdot Z_C \cdot \sinh \ \nu L \\
I_R &= I_S \cdot \cosh \ \nu L - \frac{V_S}{Z_C} \cdot \sinh \ \nu L
\end{aligned} \]

**NOTE:**
\[ V_S, V_R : \text{Line-to-neutral Voltage (V)} \]
\[ I_S, I_R : \text{Line current} \]
What if your calculator does not do Hyperbolic?

\[ V(x) = \frac{V_R + I_R \cdot Z_c}{2} e^{\gamma x} + \frac{V_R - I_R \cdot Z_c}{2} e^{-\gamma x} \]

\[ I(x) = \frac{V_R / Z_c + IR}{2} e^{\gamma x} - \frac{V_R / Z_c - IR}{2} e^{-\gamma x} \]

\[ V_S = \frac{V_R + I_R \cdot Z_c}{2} e^{\gamma S} + \frac{V_R - I_R \cdot Z_c}{2} e^{-\gamma S} \]

\[ I_S = \frac{V_R / Z_c + I_R}{2} e^{\gamma S} - \frac{V_R / Z_c - I_R}{2} e^{-\gamma S} \]
Hyperbolic Form or the equations

More on hyperbolic cos & sin

1. \( \cosh (a + jb) = \cosh a \cos b + j \sinh a \sin b \)
2. \( \sinh (a + jb) = \sinh a \cos b + j \cosh a \sin b \)

Maclaurin’s Series

1. \( \cosh \theta = 1 + \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \frac{\theta^6}{6!} + \ldots \)
2. \( \sinh \theta = \theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \frac{\theta^7}{7!} + \ldots \)

Phase

1. \( \cosh (a + jb) = \frac{e^{a+ib} + e^{-a+ib}}{2} = \frac{1}{2}(e^{a+jb} + e^{-a+jb}) \)
2. \( \sinh (a + jb) = \frac{e^{a+ib} - e^{-a+ib}}{2} = \frac{1}{2}(e^{a+jb} - e^{-a+jb}) \)
Example 5.1

A single-circuit 60-Hz transmission line is 370 km (or 230 mi) long. The conductor’s outside diameter is 0.977 in. And the flat horizontal spacing is 7.25 m (or 23.8 ft) between conductors. The resistance of the conductor is 0.1603 ohm/mile. The load on the line is 125 MW at 215 kV with 1 unity power factor. (a) Find the voltage, current, and power at the sending end. (b) Find the voltage regulation of the line. (c) Determine the wavelength and velocity of propagation of the line.
Example 5.1

* A single-circuit 60-Hz transmission line is 370 km (or 230 mi) long. The conductor’s outside diameter is 0.977 in. And the flat horizontal spacing is 7.25 m (or 23.8 ft) between conductors. The resistance of the conductor is 0.1603 ohm/mile. The load on the line is 125 MW at 215 kV with 1 unity power factor. (a) Find the voltage, current, and power at the sending end. (b) Find the voltage regulation of the line. (c) Determine the wavelength and velocity of propagation of the line.
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\[
\begin{align*}
d &= 0.977 \text{ in} \\
R &= 0.1603 \text{/mile}
\end{align*}
\]
Example 5.1

A single-circuit 60-Hz transmission line is 370 km (or 230 mi) long. The conductor’s outside diameter is 0.977 in. and the flat horizontal spacing is 7.25 m (or 23.8 ft) between conductors. The resistance of the conductor is 0.1603 ohm/mile. The load on the line is 125 MW at 215 kV with 1 unity power factor. (a) Find the voltage, current, and power at the sending end. (b) Find the voltage regulation of the line. (c) Determine the wavelength and velocity of propagation of the line.

\[ V_s = V_R \cdot \cosh \gamma l + I_R \cdot Z_c \cdot \sinh \gamma l \]

\[ I_s = \frac{V_R}{Z_c} \cdot \sinh \gamma l + I_R \cdot \cosh \gamma l \]

\[ \gamma l = \sqrt{\frac{1}{\sqrt{3}} \cdot \text{len}} = 0.0454922 + 0.4761608 \cdot i \]

\[ Z_c = \sqrt{\frac{Z}{Y}} = 405.2236295 - 38.7148641 \cdot i \]

\[ PR = 125 \cdot 10 \]

\[ VR = \frac{215000}{\sqrt{3}} = 1.2413031 \cdot 10^5 \]

\[ IR = \frac{PR}{3 \cdot VR} = 335.6687612 \]

\[ \cosh(\gamma l) = 0.8896811 + 0.0208595 \cdot i \]

\[ \sinh(\gamma l) = 0.0404456 + 0.4588448 \cdot i \]
Example 5.1

A single-circuit 60-Hz transmission line is 370 km (or 230 mi) long. The conductor's outside diameter is 0.977 in. And the flat horizontal spacing is 7.25 m (or 23.8 ft) between conductors. The resistance of the conductor is 0.1603 ohm/mile. The load on the line is 125 MW at 215 kV with 1 unity power factor. (a) Find the voltage, current, and power at the sending end. (b) Find the voltage regulation of the line. (c) Determine the wavelength and velocity of propagation of the line.

\[ V_S = V_R \cdot \cosh \gamma l + I_R \cdot Z_c \cdot \sinh \gamma l \]
\[ I_S = \frac{V_R}{Z_c} \cdot \sinh \gamma l + I_R \cdot \cosh \gamma l \]
\[ \cosh(\gamma l) = 0.8896811 + 0.0208595i \]
\[ \sinh(\gamma l) = 0.0404456 + 0.4588448i \]

\[ V_S = V_R \cdot \cosh(\gamma l) + I_R \cdot Z_c \cdot \sinh(\gamma l) = 1.2190069 \cdot 10^5 + 64476.1743055i \]
\[ |V_S| = 1.3790198 \cdot 10^5 \quad \text{angle}(V_S) = 27.8754275 \]
\[ I_S = \frac{V_R}{Z_c} \cdot \sinh(\gamma l) + I_R \cdot \cosh(\gamma l) = 297.6084302 + 147.4593272i \]
\[ |I_S| = 332.1371 \quad \text{angle}(I_S) = 26.3575 \]
\[ \text{pf} = \cos(\text{angle}(V_S) - \text{angle}(I_S)) = 0.9996491 \]
\[ P_S = 3 \cdot |V_S| \cdot |I_S| = 1.3740708 \cdot 10^8 \]
Example 5.1

A single-circuit 60-Hz transmission line is 370 km (or 230 mi) long. The conductor’s outside diameter is 0.977 in. And the flat horizontal spacing is 7.25 m (or 23.8 ft) between conductors. The resistance of the conductor is 0.1603 ohm/mile. The load on the line is 125 MW at 215 kV with 1 unity power factor. (a) Find the voltage, current, and power at the sending end. (b) Find the voltage regulation of the line. (c) Determine the wavelength and velocity of propagation of the line.

No Load Voltage: \( V_R \) at \( IR = 0 \)

\[
V_{Rnl} = \frac{V_S}{\cosh(\gamma l)} = 1.3863912 \times 10^5 + 69220.5777301 \cdot i
\]

\[
V_{reg} = \frac{|V_{Rnl}| - |V_R|}{|V_R|} \times 100 = 24.8357524 \text{ percent}
\]
Example 5.1

A single-circuit 60-Hz transmission line is 370 km (or 230 mi) long. The conductor’s outside diameter is 0.977 in. And the flat horizontal spacing is 7.25 m (or 23.8 ft) between conductors. The resistance of the conductor is 0.1603 ohm/mile. The load on the line is 125 MW at 215 kV with 1 unity power factor. (a) Find the voltage, current, and power at the sending end. (b) Find the voltage regulation of the line. (c) Determine the wavelength and velocity of propagation of the line.
Class Activity on Transmission Parameters

A 3-phase 60-Hz transmission line is 250 miles long. The voltage at the sending end is 220 kV. The parameters of the line are \( R = 0.2 \, \Omega/\text{mile} \), \( X = 0.8 \, \Omega/\text{mile} \), and \( Y = 5.3 \, \mu\text{S/mile} \). Find the sending-end current when there is no load on the line.

\[
V_S = \frac{V_R + IR \cdot Z_C}{2} e^{\frac{\sqrt{3} \cdot \text{len}}{2}} + \frac{V_R - IR \cdot Z_C}{2} e^{-\frac{\sqrt{3} \cdot \text{len}}{2}}
\]

\[
I_S = \frac{V_R}{2Z_C} e^{\frac{\sqrt{3} \cdot \text{len}}{2}} - \frac{V_R}{2Z_C} e^{-\frac{\sqrt{3} \cdot \text{len}}{2}}
\]

\[\begin{align*}
\text{\( V_L = \sqrt{\frac{y}{z} \cdot \text{len} } \)} \\
\text{\( Z_C = \sqrt{\frac{z}{y} } \)}
\end{align*}\]

For \( IR = 0 \):

\[ VR = \frac{V_S}{\cosh(V_L)} \]

\[ IS = \frac{VR}{Z_C} \cdot \sinh(V_L) \]

\[ \frac{V_S}{\sqrt{3}} \cdot 10^3 \]

\[ \frac{V_S}{\sqrt{3}} \cdot 10^3 \]
What if your calculator does not do Hyperbolic?

\[
V(x) = \frac{V_R + I_R Z_C}{2} e^{\alpha X} + \frac{V_R - I_R Z_C}{2} e^{-\alpha X}
\]

\[
I(x) = \frac{V_R / Z_C + I_R}{2} e^{\alpha X} - \frac{V_R / Z_C - I_R}{2} e^{-\alpha X}
\]

\[N = \alpha + j\beta\]

\[e^{\alpha X} = e^{\alpha X} e^{j\beta X}\]

\[(\cos \beta X + j \sin \beta X)\]

\[\text{[rad]}\]
Can we use Pi-circuit even for a long line?

\[ I = I_R + \frac{Y'}{2} V_R \]
\[ V_S = I Z' + V_R \]
Can we use Pi-circuit even for a long line?

\[ V_s = \left( I_R + \frac{Y'}{2} V_R \right) Z' + V_R \]

\[ I_s = (Z' - Y) V_R + \frac{1}{2} I_R \]

\[ Z' = Z_c \cdot \sinh \alpha l \]

\[ = \frac{\sqrt{3}}{2} \cdot \sinh \alpha l = \frac{\sqrt{3} \cdot \sqrt{3} \cdot l}{\sqrt{3} \cdot \sqrt{3} \cdot l} \cdot \sinh \alpha l \]

\[ Z = Z \frac{\sinh \alpha l}{\sqrt{3} Y l} = Z \frac{\sinh \alpha l}{\sqrt{3} \alpha l} \]

If \( \alpha l \) is small, \( \frac{\sinh \alpha l}{\alpha l} \sim 1 \)

\[ V_c = V_R \cosh \alpha l + \frac{I_R Z_c \sinh \alpha l}{Z_c} \]

\[ I = I_R \cosh \alpha l + \frac{V_c}{Z_c} \sinh \alpha l \]

\[ Z_c = \frac{\sqrt{3}}{2} \]

\[ V = \sqrt{3} V_c \]
Can we use Pi-circuit even for a long line?

\[ V_S = \left( \frac{Z'Y'}{2} + 1 \right) V_R + Z' I_R \]

\[ V_S = V_R \cosh \eta l + I_R Z_C \sinh \eta l \]

\[ \frac{Z'Y'}{2} + 1 = \cosh \eta l \]

From above:

\[ Z' = Z \cdot \frac{\sinh \eta l}{\eta l} \]

\[ Y' \left( Z_C \cdot \sinh \eta l \right) + 1 = \cosh \eta l \]

\[ \frac{Y'}{2} = \frac{\cosh \eta l - 1}{Z_C \cdot \sinh \eta l} = \frac{1}{Z_C} \left( \frac{\cosh \eta l - 1}{\sinh \eta l} \right) \]
Can we use Pi-circuit even for a long line?

\[ \operatorname{tanh}\left(\frac{vl}{2}\right) = \frac{\cosh vl - 1}{\sinh vl} \]

\[ Y' = Y \cdot \frac{1}{Z_c} \cdot \frac{1}{2} \cdot \frac{1}{\cosh \left(\frac{vl}{2}\right)} \]

\[ \frac{1}{Z_c} = \sqrt{\frac{Y}{Y'}} = \sqrt{\frac{Y}{Y'}} = \frac{Y}{\sqrt{Y'}} = \frac{Y}{\sqrt{Y'}} = \frac{Y}{\sqrt{Y'}} \]

Correction factor from nominal PI to long line equivalent PI circuit.
Nominal π to long-line π

- From Nominal Pi-ckt equation (50 < \( l < 150 \))

\[
V_s = \left( \frac{z}{2} \right) V_R + z \cdot I_R
\]

\[
I_s = y \left( \frac{z}{4} + 1 \right) V_R + \left( \frac{z}{2} + 1 \right) I_R
\]

- Long-line equivalent pi-ckt equation (\( l > 150 \) miles)

\[
Z \rightarrow Z \cdot \frac{\sinh \frac{\sqrt{L}}{l}}{\sqrt{L}} \quad (Z')
\]

\[
Y \rightarrow Y \cdot \frac{\tanh \left( \frac{\sqrt{L}}{2} \right)}{\tanh \left( \frac{\sqrt{L}}{2} \right)} \quad (Y')
\]
Example 5.3

* Find the equivalent pi-circuit for the line described in Example 5.1 and compare it with the nominal pi.

\[
\begin{align*}
  z &= R_0 + j \cdot X_L = 0.1603 + 0.8313 \cdot j \ \Omega / \text{mile} & |z| = 0.8465775 \Omega \\
  Z &= z \cdot \text{len} = 36.869 + 191.1903779 \cdot j & \arg(z) \frac{180}{\pi} = 79.0851086 \deg \\
  |z| &= 194.7128238 & \arg(z) \frac{180}{\pi} = 79.0851086 \deg \\
  k &= 8.85 \cdot 10^{-12} \\
  C_n &= \frac{2 \cdot \pi \cdot k}{|\ln(\frac{\text{Dep}}{r})|} = 8.4226 \cdot 10^{-12} \ F / \text{m} \\
  X_C &= \frac{1}{2 \cdot \pi \cdot 60 \cdot C_n \cdot 1.609} = 1.9574 \cdot 10^{-5} \Omega / \text{mile} \\
  y &= \frac{1}{X_C} = 5.1089426 \cdot 10^{-6} \ i \ S / \text{mile} & |y| = 5.1089426 \cdot 10^{-6} \deg \\
  Y &= y \cdot \text{len} = 0.0011751 \cdot j \\
  Y_2 &= \frac{Y}{2} = 0.0005875 \cdot j & |Y_2| = 0.0005875 \deg \\
  Y_1 &= \sqrt{Y \cdot Y} = 0.0454922 + 0.4761608 \cdot j \\
  Z_C &= \sqrt{Z \cdot Y} = 405.2236295 - 38.7148641 \cdot j
\end{align*}
\]
Example 5.3

Find the equivalent pi-circuit for the line described in Example 5.1 and compare it with the nominal pi.

\[ Z' = Zc \cdot \sinh(y_1) = 34.1536353 + 184.3689102i \]

\[ |Z'| = 187.5056422 \quad \arg(Z') \cdot \frac{180}{\pi} \approx 79.5051445 \]

\[ |Z| = 194.7128238 \]

\[ \frac{|Z'| - |Z|}{|Z|} \cdot 100 = 3.7014417 \quad \text{percent} \]

\[ \frac{\tanh \left( \frac{y_1}{2} \right)}{y_2} = 2.2197466 \times 10^{-6} + 0.0005988i \]

\[ |Y_2'| = 0.0005988 \quad |Y_2| = 0.0005875 \]

\[ \frac{|Y_2'| - |Y_2|}{|Y_2|} \cdot 100 = 1.9142259 \quad \text{percent} \]

\{ \frac{Z'}{Z} : 3.7\% \text{ lower} \} \rightarrow \underline{\text{Ignorable}}

\{ \frac{Y'}{Y} : 2\% \text{ higher} \} \rightarrow \underline{\text{Even Long Lines?!}}
Power Flow

**Power Flow (P & Q)**

- Expressed by $V$, $I$, and $\text{pf}$
- Expressed by ABCD circuit constants
  - Focus: How $V_R$ and $I_R$ affects $V_S$
  - ABCD Constants: All Complex Numbers

\[
\begin{align*}
V_S &= A \cdot V_R + B \cdot I_R \\
I_S &= C \cdot V_R + D \cdot I_R
\end{align*}
\]

Let \[
\begin{align*}
A &= A \angle \alpha \\
B &= B \angle \beta \\
V_R &= V_R \angle \theta \\
V_S &= V_S \angle \delta
\end{align*}
\]
\[ V_s = A \cdot V_R + B \cdot I_R \]

\[ I_R = \frac{V_s - A \cdot V_R}{B} = \frac{V_s}{B} - \frac{A}{B} \cdot V_R \]

Let \( \bar{A} = A/L_x \)
\( B = B/L_y \)
\( V_R = V_R \angle 0^\circ \)
\( V_s = V_s \angle \delta \)

\[ S_R = V_R \cdot I_R = \frac{V_s \cdot V_R}{B} \cdot \frac{1}{B} - \frac{A \cdot V_R^2}{B \cdot 4} \]
\[ S_R = V_R \cdot I_R = \frac{V_s \cdot V_R}{B} \left( \frac{\beta - \delta}{4} - \frac{A \cdot V_R^2}{B} \right) \]
Power Flow: Origin Shift

\[ S = |P + Q| \]

\[ \alpha - \beta \]

\[ \beta - \delta \]

\[ V_A \]

\[ V_S \]

\[ V_B \]

\[ P \]

\[ Q \]

\[ \theta \]

\[ \text{angle} \]

\[ \text{Er} \]

\[ \text{WR} \]
Power Flow: Origin Shift

\[ P_R = \frac{V_s \cdot V_R \cos(\beta - \delta)}{B} - \frac{AV_R^2}{B} \cos(\beta - \delta) \]

\[ \Rightarrow P_R = V_R \cdot I_R \cdot \cos \Theta_R \]

\[ Q_R = \frac{V_s \cdot V_R \sin(\beta - \delta)}{B} - \frac{AV_R^2}{B} \sin(\beta - \delta) \]

\[ \Rightarrow Q_R = V_R \cdot I_R \cdot \sin \Theta_R \]
**Observations**

- Point $n$ is not dependent on $IR$.
- Point $n$ will not change if $VR$ is constant: $(A*VR^2)/B$.
- Distance between $n$ and $k$ is constant for fixed value of $VS$ and $VR$: $(VS*VR)/B$.
- Distance between $o$ and $k$ changes with changing load ($IR$): $\overrightarrow{S} = \overrightarrow{VR} \cdot \overrightarrow{IR}^*$.
Observations

- Because the distance between \( n \) and \( k \) is constant, the distance between \( o \) and \( k \) is constrained to move in a circle whose center is at \( n \).

- If \( VR \) is held constant, a different circle can be drawn for different values of \( VS \)’s.
Power Flow for Max Power

If VR is held constant, a different circle can be drawn for different values of VS's.

\[ P_{\text{max}} = \frac{V_s \cdot V_R}{B} - \frac{A \cdot V_R^2}{B} \cos(\beta - \alpha) \]
Power Flow for Max Power

\[ (\beta - \delta) \]
When PR has to be maintained, moving from a to b involves:

- Sending end voltage reduces from VS4 to VS3
- (To keep the VR intact), due to the VS decrease, the Var has to be decreased, which means negative reactive power must be supplied by parallel capacitors
Reactive Compensation of Transmission Line

Reactive Compensation by:

- **Series Compensation:** Capacitor placed in each conductor reduces the series impedance

- **Shunt Compensation:** Placement of inductors between conductor and neutral reduces the susceptance (admittance)

**Pi-Circuit Case**

\[ V_s = \left( \frac{Z_Y}{Z} + 1 \right) V_R + \frac{Z}{Z_R} I_R \]

\[ I_s = Y \left( \frac{Z_Y}{Z} + 1 \right) V_R + \left( \frac{Z_Y}{Z} + 1 \right) I_R \]

\[ V_s = V_R \cdot \cosh \frac{qL}{2} + I_R \cdot Z_c \cdot \sinh \frac{qL}{2} \]

\[ I_s = \frac{V_R \cdot \sinh \frac{qL}{2} + I_R \cdot \cosh \frac{qL}{2}}{Z_c} \]
Reactive Compensation of Transmission Line

- **Pi-Circuit Case**

- **Maximum Power Equation**

\[
P_{R_{\text{max}}} = \frac{V_s \cdot V_R}{B} - \frac{A \cdot V_R^2}{B} \cos(\beta - \alpha)
\]

**Dominant Factor**

\[
B = \left\{ \begin{array}{l}
\frac{Z}{Z_c \cdot \sinh \frac{V}{l} = \sqrt{\frac{V}{l}} \cdot \frac{Z_c}{l} \cdot \sinh \frac{V}{l}} \\
= \frac{X_c}{X_L} \sinh \frac{V}{l} = \frac{Z_c}{V}
\end{array} \right.
\]

"Compensation Factor" = \[
\frac{X_c}{X_L}
\]

Total Inductive Reactance of the Line

Total Capacitive Reactance

**Equivalent Circuit**

\[
\begin{align*}
V_s &= \left( \frac{Z_c}{4} + 1 \right) V_R + Z_c I_R \\
I_s &= \left( \frac{Z_c}{4} + 1 \right) V_R + \left( \frac{Z_c}{2} + 1 \right) I_R
\end{align*}
\]
In the southwestern part of the United States series compensation is especially important because large generating plants are located hundreds of miles from load centers and large amounts of power must be transmitted over long distances. The lower voltage drop in the line with series compensation is an additional advantage. Series capacitors are also useful in balancing the voltage drop of two parallel lines.
EXAMPLE 5.4: Reactive Compensation of Transmission Line

In order to show the relative changes in the B constant with respect to the change of the A, C, and D constants of a line when series compensation is applied, find the constants for the line of Example 5.1 uncompensated and for a series compensation factor of 70%
EXAMPLE 5.4: Reactive Compensation of Transmission Line

In order to show the relative changes in the B constant with respect to the change of the A, C, and D constants of a line when series compensation is applied, find the constants for the line of Example 5.1 uncompensated and for a series compensation factor of 70%.

From 5.1

\[ \text{len} = 230 \]
\[ z = 0.1603 + 0.831263 \cdot i \quad y = 5.108943 \cdot 10^{-6} \cdot i \]
\[ Z_c = 405.223629 - 38.714864 \cdot i \quad y_l = 0.045492 + 0.476161 \cdot i \]

For the Uncompesated Line:

\[ A = \cosh(y_l) = 0.889681 + 0.020859 \cdot i \]
\[ D = A \]
\[ B = Z_c \cdot \sinh(y_l) = 34.153635 + 184.36891 \cdot i \]
\[ |B| = 187.505642 \]
\[ C = \frac{\sinh(y_l)}{Z_c} = -8.295422 \cdot 10^{-6} + 0.001132 \cdot i \]
EXAMPLE 5.4: Reactive Compensation of Transmission Line

In order to show the relative changes in the B constant with respect to the change of the A, C, and D constants of a line when series compensation is applied, find the constants for the line of Example 5.1 uncompensated and for a series compensation factor of 70%.

Now the compensation with COMpensation Factor of 70%: \( \frac{X_c}{X_L} = 0.7 \)

\[
\text{Comp} = 0.7
\]

\[
X_c = \text{Comp}(-i) \cdot \text{XL} \cdot \text{len} = -133.833265i
\]

\[
\text{XL} = 0.831263
\]

Now new B constant:

\[
B_{\text{new}} = B + X_c = 34.153635 + 50.535646i
\]

\[
|B_{\text{new}}| = 60.994445
\]

\[
\left|\frac{B_{\text{new}}}{B}\right| = 0.325294
\]
EXAMPLE 5.4: Reactive Compensation of Transmission Line

**Other Constants**

\[ z = 0.1603 + 0.831263 \cdot i \]
\[ Z = z \cdot \text{len} = 36.869 + 191.190378 \cdot i \]
\[ Z_{\text{new}} = \text{Re}(z \cdot \text{len}) + i \cdot \text{Im}(z \cdot \text{len}) \cdot |1 - \text{Comp}| = 36.869 + 57.357113 \cdot i \]
\[ z_{\text{new}} = \frac{Z_{\text{new}}}{\text{len}} = 0.1603 + 0.249379 \cdot i \]
\[ y_{\text{new}} = \sqrt{z_{\text{new}} \cdot y \cdot \text{len}} = 0.079759 + 0.271587 \cdot i \]
\[ A_{\text{new}} = \cosh(y_{\text{new}}) = 0.966412 + 0.021419 \cdot i \quad |A_{\text{new}}| = 0.96665 \]
\[ |A| = 0.889926 \]
\[ \arg(A_{\text{new}}) \frac{180}{\pi} = 1.269661 \]
\[ Z_{\text{new}} = \sqrt[6]{\frac{z_{\text{new}}}{y}} = 231.126571 - 67.876998 \cdot i \]
\[ C_{\text{new}} = \frac{\sinh(y_{\text{new}})}{Z_{\text{new}}} = -8.427465 \cdot 10^{-6} + 0.001162 \cdot i \quad |C_{\text{new}}| = 0.001162 \]
\[ B_{\text{new}} = Z_{\text{new}} \cdot \sinh(y_{\text{new}}) = 36.044308 + 56.97852 \cdot i \quad |B_{\text{new}}| = 0.001132 \]
EXAMPLE 5.4: Reactive Compensation of Transmission Line

In order to show the relative changes in the B constant with respect to the change of the A, C, and D constants of a line when series compensation is applied, find the constants for the line of Example 5.1 uncompensated and for a series compensation factor of 70%.

\[
\begin{align*}
|B_{\text{new}}| &= 60.994445 \\
|B_{\text{new2}}| &= 67.422132 \\
|B| &= 187.505642
\end{align*}
\]

Consider the maximum power equation:

\[
P_{\text{max}} = \frac{V_s V_r}{B} - \frac{A V_r^2}{B} \cos(\beta - \alpha)
\]
Charging Current Reduction with Shunt Compensation

\[ I_{ch} = j \omega C_v v_n = \frac{v_n}{j \omega} \]

\[ \rightarrow I_{ch} = \frac{v_n}{|X_c|} = B_C \cdot v_n \]

Connect 2 Inductors from Line to Neutral with amount \( \frac{2}{B_L} \) \( \text{inductive susceptance} \)

After Shunt Compensation:

\[ \rightarrow I_{ch} = (B_C - B_L) \cdot v_n \]

\[ = (B_C - B_L) \frac{B_C}{B_C} \cdot v_n = B_C \cdot v_n \left(1 - \frac{B_L}{B_C}\right) \]

"Shunt Compensation Factor"