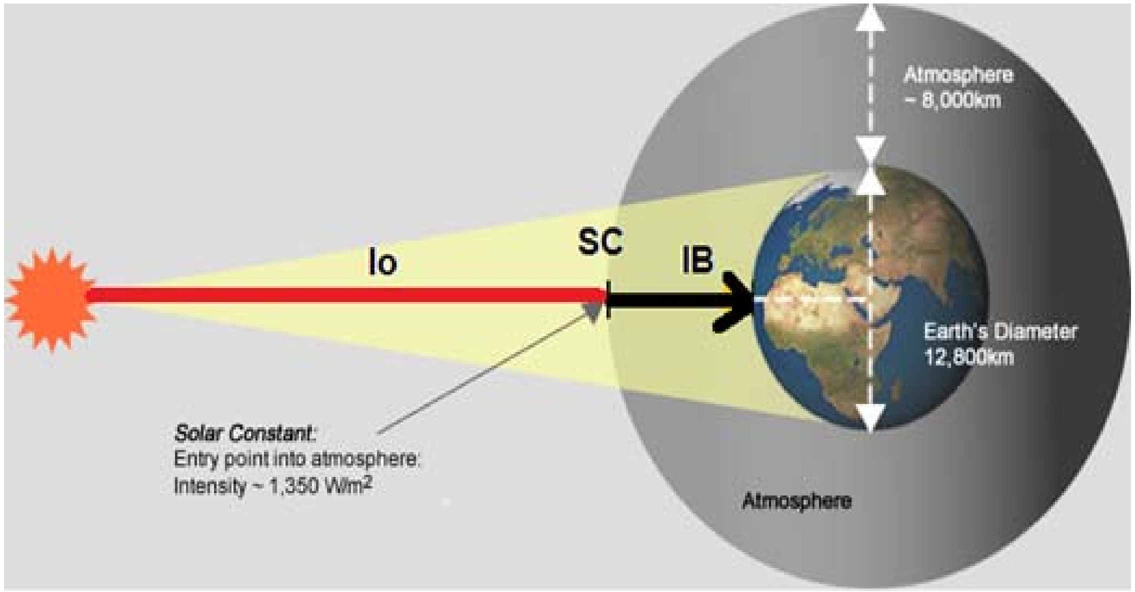
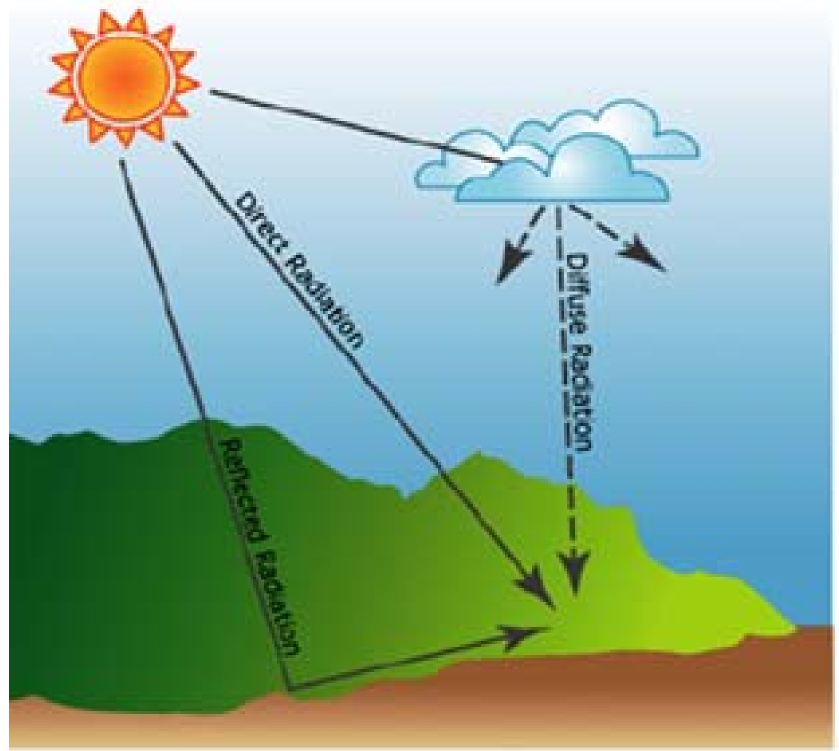
Solar Radiation





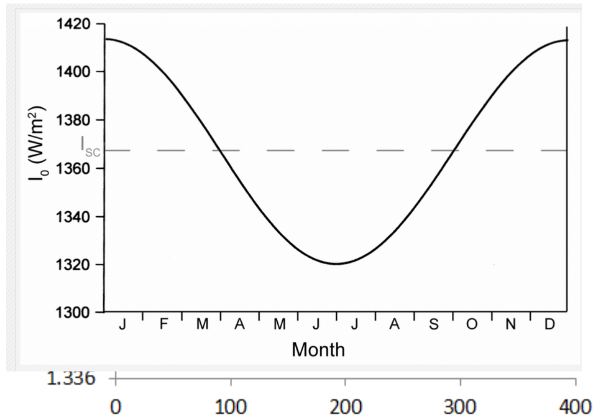
Source: esri.com

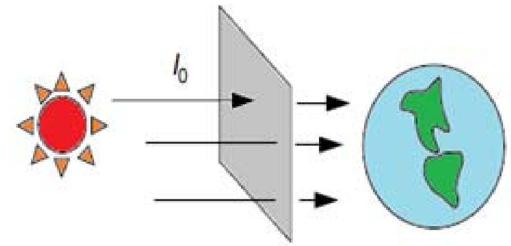
Clear Sky Beam Radiation

- Extraterrestrial Solar Insolation (I_0)

$$I_0 = \mathrm{SC} \cdot \left[1 + 0.034 \cos\left(\frac{360n}{365}\right)\right] \qquad (\mathrm{W/m^2})$$

$$1.377\ \mathrm{kW/m^2}$$





- Portion of the beam reaching the earth horizontal surface (I_B)

$$I_B = Ae^{-km}$$

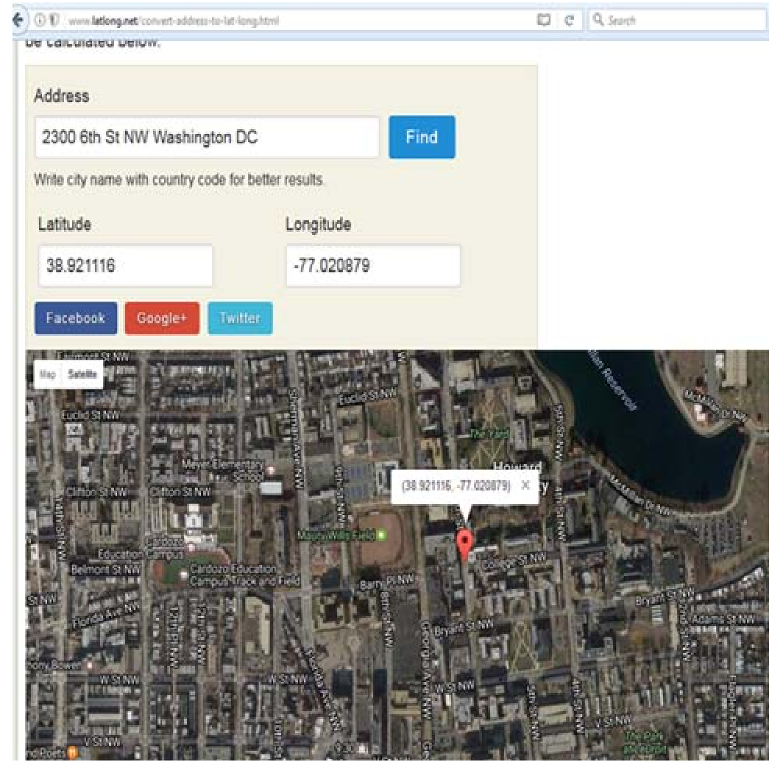
$$A = 1160 + 75 \sin\left[\frac{360}{365}(n - 275)\right] \qquad (\mathrm{W/m^2})$$

$$k = 0.174 + 0.035 \sin\left[\frac{360}{365}(n - 100)\right]$$

$$m = \frac{1}{\sin \beta}$$

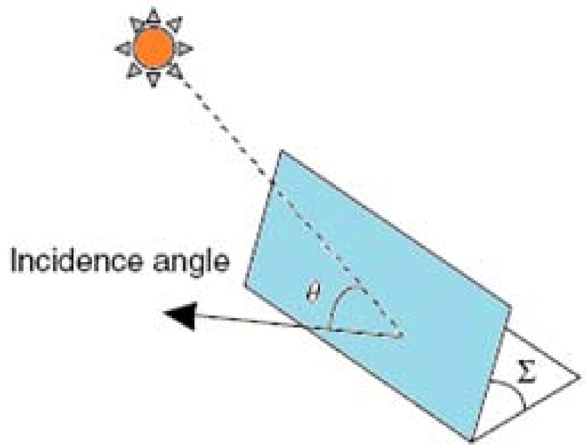
Clear Sky Beam Radiation – Example

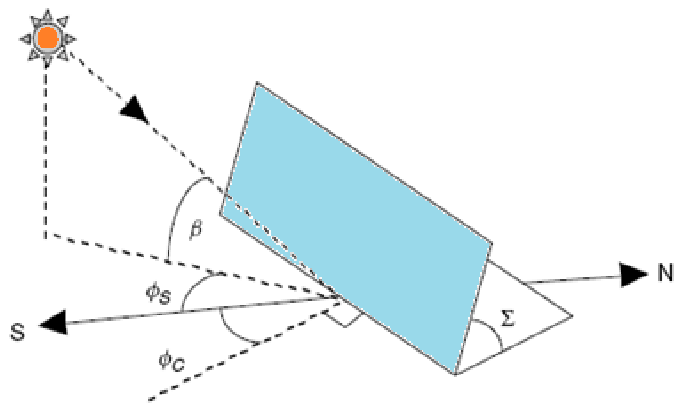
Question: Find the direct beam solar radiation on the horizontal surface at solar noon on a clear day on Howard University campus (latitude ???? degrees) on May 21.



January	$n = 1$	July	$n = 182$
February	$n = 32$	August	$n = 213$
March	$n = 60$	September	$n = 244$
April	$n = 91$	October	$n = 274$
May	$n = 121$	November	$n = 305$
June	$n = 152$	December	$n = 335$

Beam Radiation on **Collector**





Beam Radiation on **Collector - Example**

Question: At solar noon in Atlanta (latitude 33.7) on May 21, the altitude angle of the sun was found to be 76.4 degrees and the clear-sky beam insolation was found to be 902 W/m^2. Find the beam insolation at that time on a collector that faces 20 degrees toward the southeast with tipped angle at 52 degrees.

$$\beta = \sin^{-1}\{\cos L \cos\delta \cos H + \sin L \sin\delta\}$$

$$\phi = \sin^{-1}\left\{\frac{\cos\delta \sin H}{\cos\beta}\right\}$$

January	$n = 1$	July	$n = 182$
February	$n = 32$	August	$n = 213$
March	$n = 60$	September	$n = 244$
April	$n = 91$	October	$n = 274$
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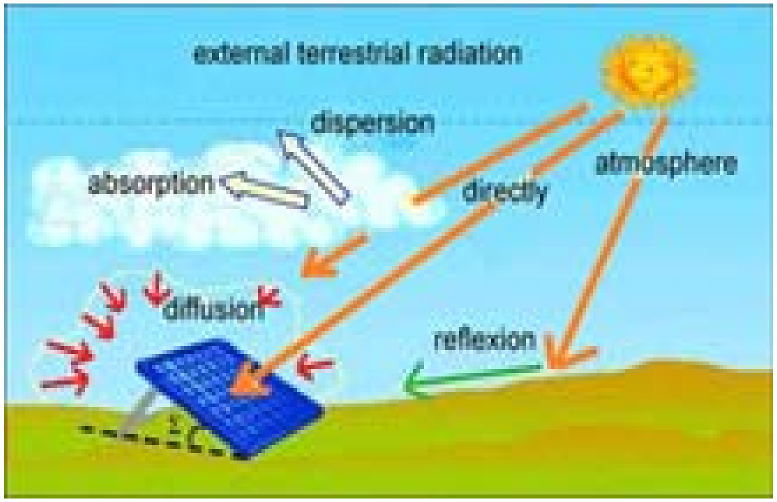
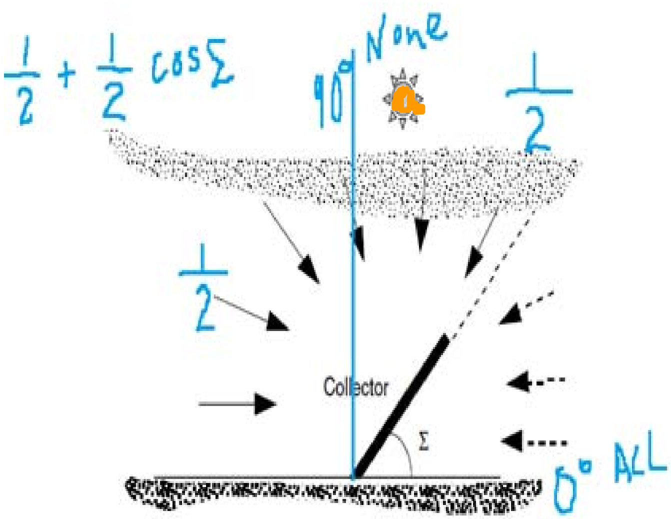
Diffuse Radiation on Collector



- Diffuse insolation on a Horizontal surface I_{DH}
- Sky diffuse factor (C)

$$C = 0.095 + 0.04 \sin\left[\frac{360}{365}(n - 100)\right]$$

- Diffuse Radiation on collector I_{DC}



⌘ Question: At solar noon in Atlanta (latitude 33.7) on May 21, the altitude angle of the sun was found to be 76.4 degrees and the clear-sky beam insolation was found to be 902 W/m^2. Find the **diffuse radiation** at that time on a collector that faces 20 degrees toward the southeast with tipped angle at 52 degrees. **Find the Diffuse Radiation** (I_{DC})

$H = 0 \rightarrow \phi_s = 0$ Sun's azimuth

$\phi_c = 20°$ collector's azimuth

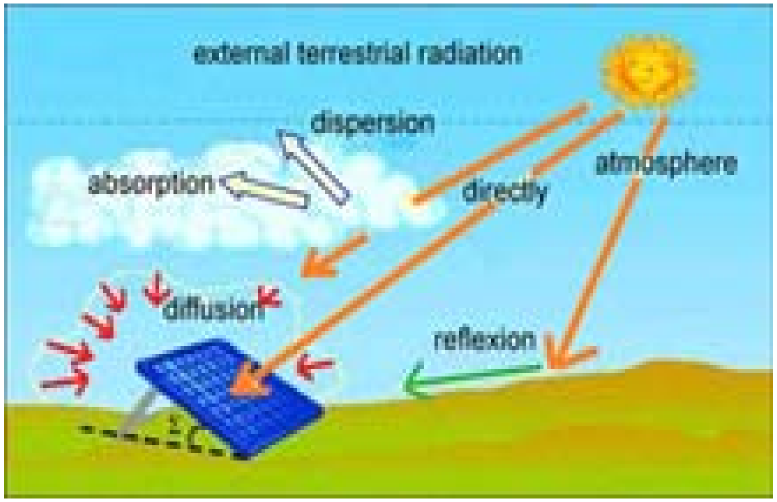
$L = 33.7°$ May 1 $\Rightarrow n = 141$

$\Sigma = 52°$ $\beta_N = 76.4°$

Reflected Radiation on Collector

- Reflection from ground with reflectance (snow, water, etc.)

ground reflectance ρ



$$I_{RC} = \rho I_B (\sin\beta + C)\left(\frac{1-\cos\Sigma}{2}\right)$$

$H = 0 \rightarrow \phi_s = 0$ sun's azimuth

$\phi_c = 20^\circ$ collector's azimuth

$L = 33.7^\circ$ May 1 $\Rightarrow n = 141$

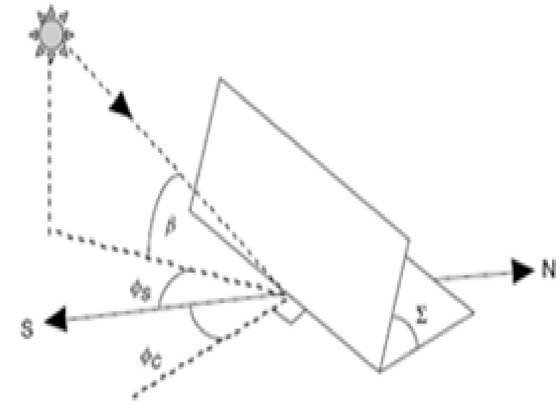
$\Sigma = 52^\circ$ $\beta_N = 76.4^\circ$

Question: At solar noon in Atlanta (latitude 33.7) on May 21, the altitude angle of the sun was found to be 76.4 degrees and the clear-sky beam insolation was found to be 902 W/m^2. Find the **reflected radiation** at that time on a collector that faces 20 degrees toward the southeast with tipped angle at 52 degrees, if the reflectance of the surfaces in front of the panel is 0.2.

Total Radiation on Collector

$$I_C = I_{BC} + I_{DC} + I_{RC}$$

$$I_C = Ae^{-km}\left[\cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma + C\left(\frac{1+\cos\Sigma}{2}\right) + \rho(\sin\beta + C)\left(\frac{1-\cos\Sigma}{2}\right)\right]$$



$$I_B = Ae^{-km}$$

$$\delta = 23.45^\circ \sin\left[\frac{360}{365}(n-81)\right]$$

$$\beta = \sin^{-1}\left\{\cos L\cos\delta\cos H + \sin L\sin\delta\right\}$$

$$\phi_S = \sin^{-1}\left\{\frac{\cos\delta\sin H}{\cos\beta}\right\}$$

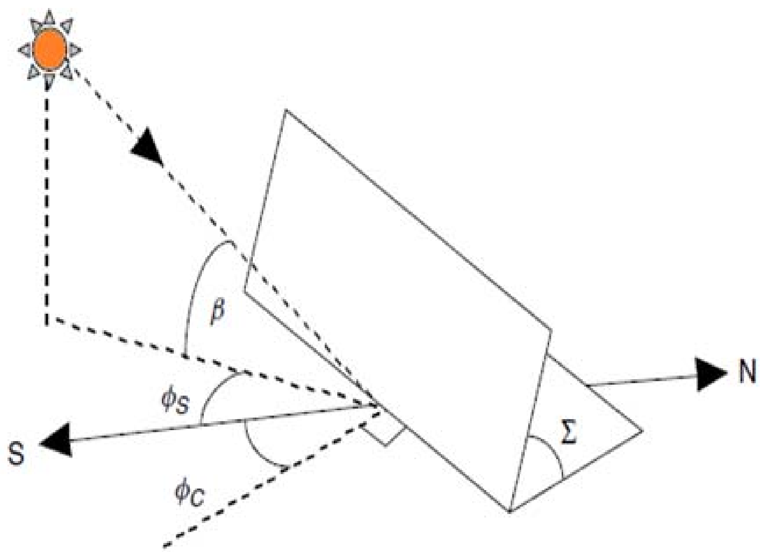
$$A = 1160 + 75\sin\left[\frac{360}{365}(n-275)\right] \qquad (\text{W/m}^2)$$

$$k = 0.174 + 0.035\sin\left[\frac{360}{365}(n-100)\right]$$

$$C = 0.095 + 0.04\sin\left[\frac{360}{365}(n-100)\right]$$

Collector Insolation:

(a) fixed collector (no tracking)
(b) single-axis tracking (follow the sun's azimuth)
(c) two-axis tracking (follow sun''s altitude and azimuth)



$$I_{BC} = I_B \cos\theta$$

$$\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma$$

$$I_{DC} = CI_B\left(\frac{1+\cos\Sigma}{2}\right)$$

$$I_{BH} = I_B\cos(90^\circ - \beta) = I_B\sin\beta$$

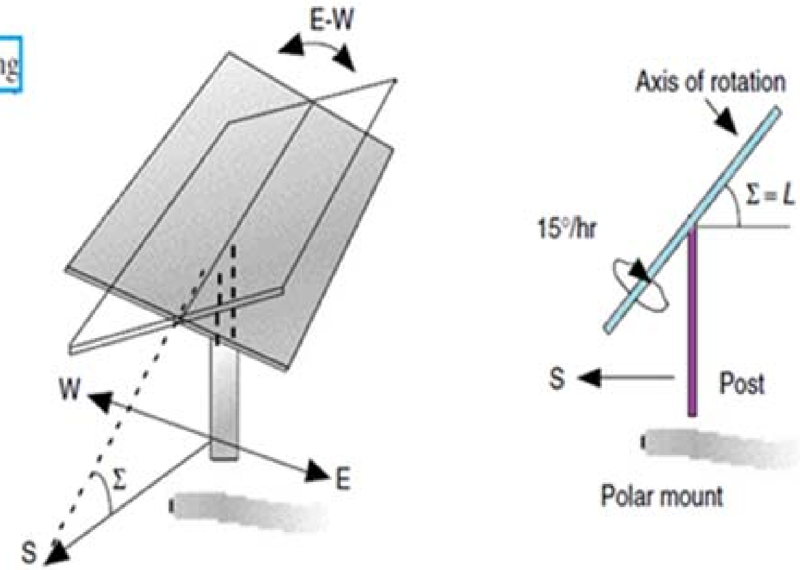
$$I_{RC} = \rho(I_{BH} + I_{DH})\left(\frac{1-\cos\Sigma}{2}\right)$$

$$I_{DH} = CI_B$$

One-Axis Polar Mount Tracking

- Single-axis
  - fixed tilted angle
  - follow the sun's azimuth

A single-axis tracking



$$I_{BC} = I_B \cos\delta$$

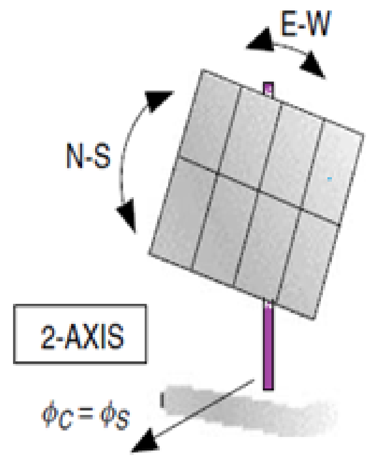
$$I_{DC} = C\, I_B \left[\frac{1 + \cos(90^\circ - \beta + \delta)}{2}\right]$$

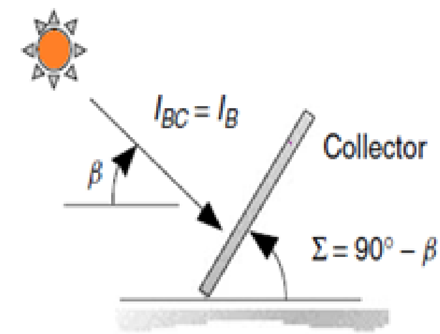
$$I_{RC} = \rho(I_{BH} + I_{DH})\left[\frac{1 - \cos(90^\circ - \beta + \delta)}{2}\right]$$

Two-Axis Tracking

- Two-Axis
  - Follow the sun's altitude
  - Follow the sun's azimuth

Two-Axis Tracking:





$$I_{BC} = I_B$$

$$I_{DC} = C I_B \left[\frac{1 + \cos(90^\circ - \beta)}{2}\right]$$

$$I_{RC} = \rho(I_{BH} + I_{DH}) \left[\frac{1 - \cos(90^\circ - \beta)}{2}\right]$$

Clear Sky Insolation Equations – Handout & Homework

Summary of Clear-Sky Solar Insolation Equations

$$I_0 = 1370\left[1 + 0.034\cos\left(\frac{360n}{365}\right)\right] \text{ (W/m}^2\text{)}$$

$$m = \frac{1}{\sin\beta}$$

$$C = 0.095 + 0.04\sin\left[\frac{360}{365}(n - 100)\right]$$

$$I_B = Ae^{-km}$$

$$I_{DC} = I_{DH}\left(\frac{1+\cos\Sigma}{2}\right) = I_B C\left(\frac{1+\cos\Sigma}{2}\right)$$

$$A = 1160 + 75\sin\left[\frac{360}{365}(n - 275)\right] \text{ (W/m}^2\text{)}$$

Homework

Question 1: Calculate the clear sky insolation on a fixed titled angle of 40° facing south collector located at 40° latitude at (a) solar noon, (b) at 10:00am, and (c) 2:00pm on the summer solstice. Ignore the reflected insolation.

Question 2: Calculate the clear sky insolation on a single-axis polar mount collector located at 40°latitude at (a) solar noon , (b) at 10:00am, and (c) 2:00pm on the summer solstice. Ignore the reflected insolation.

Question 3: Calculate the clear sky insolation on a two-axis tracking collector located at 40°latitude at (a) solar noon, (b) at 10:00am, and (c) 2:00pm on the summer solstice. Ignore the reflected insolation.