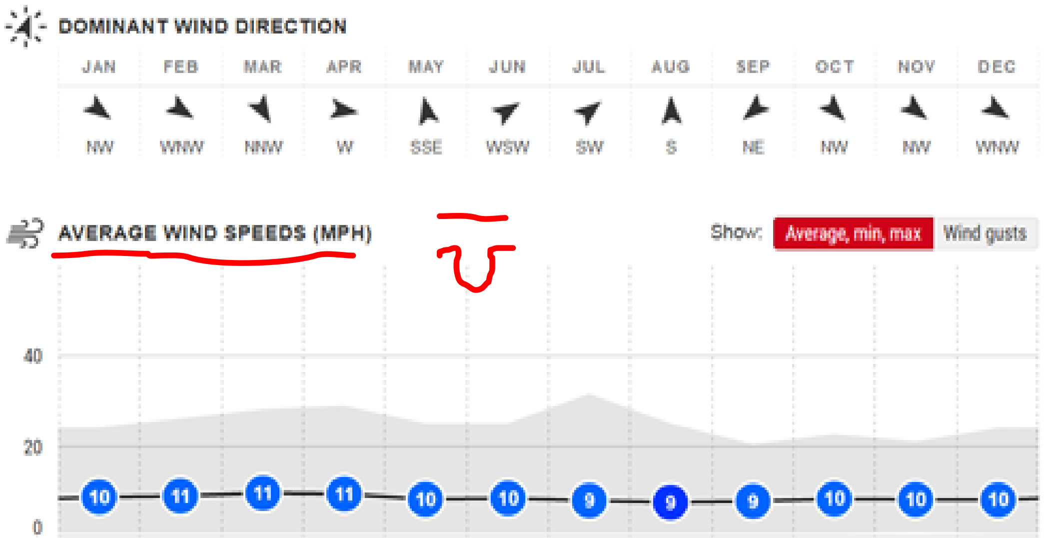
Average Power in the Wind

$$P = \frac{1}{2}\rho A v^3$$

Wind speed averages and directions for R. Reagan Airport/Washington

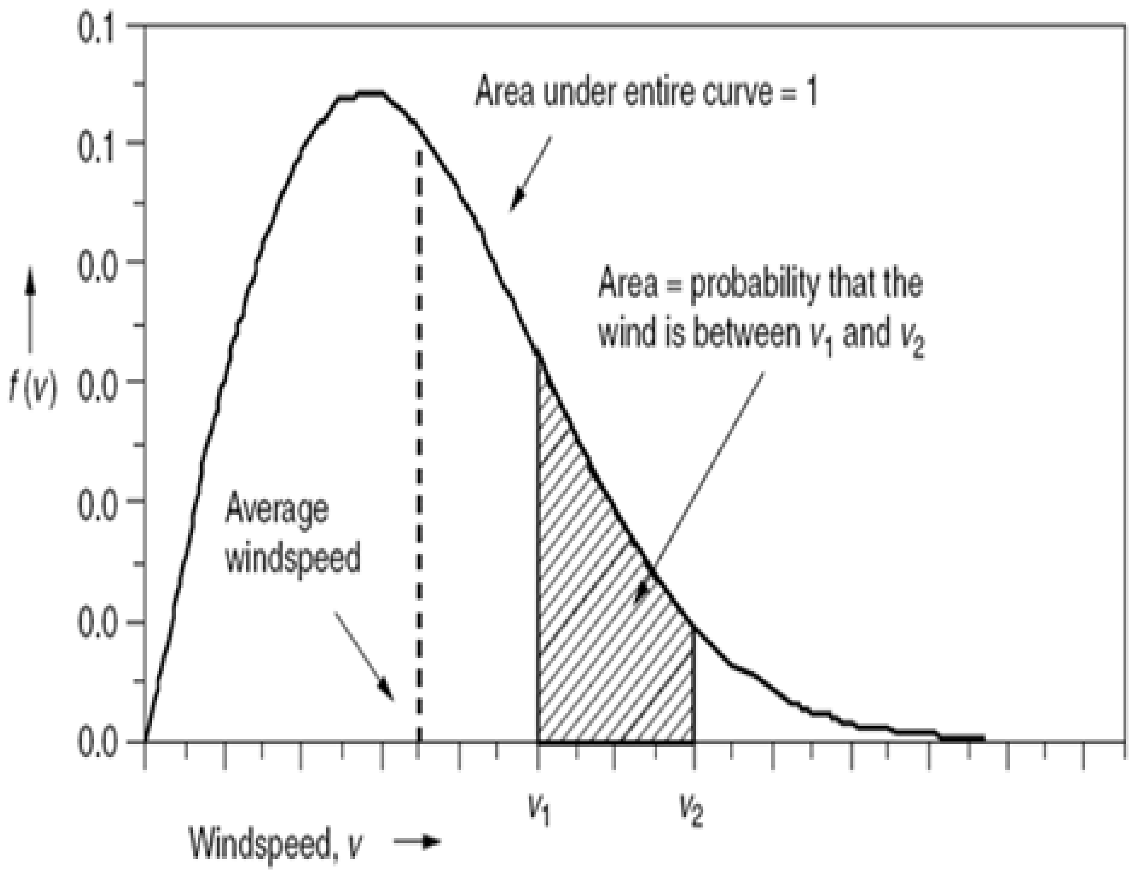
$$\bar{P} \propto \overline{v^3}$$



$\bar{v}$

Average Wind Speed from pdf

Wind Speed Distribution –Rayleigh distribution



The Right Honourable
The Lord Rayleigh
OM PC PRS

John William Strutt, 3rd Baron Rayleigh, OM, PC, PRS (/ˈreɪli/; 12 November 1842 – 30 June 1919), was a British scientist who made extensive contributions to both theoretical and experimental physics.

Rayleigh pdf - Average Speed $\bar{v}$ derivation

$$c \propto \bar{v}$$

$$c = \frac{2}{\sqrt{\pi}} \cdot \bar{v}$$

Rayleigh p.d.f.

$$f(v) = \frac{2v}{c^2} \exp\left[-\left(\frac{v}{c}\right)^2\right]$$

general Gaussian Integral

$$\int_0^\infty x^{2n} \cdot e^{-ax^2} dx = \frac{1 \cdot 3 \cdot \ldots (2n-1)}{a^n \cdot 2^{n+1}} \cdot \sqrt{\frac{\pi}{a}}$$

Rayleigh pdf – Derivation for $\overline{v^3}$

$$C = \frac{2}{\sqrt{\pi}} \cdot \bar{v}$$

Rayleigh p.d.f.

$$f(v) = \frac{2v}{c^2} \exp\left[-\left(\frac{v}{c}\right)^2\right]$$

general Gaussian Integral

$$\int_0^\infty x^{2n} \cdot e^{-ax^2} dx = \frac{1 \cdot 3 \cdot \ldots (2n-1)}{a^n \cdot 2^{n+1}} \cdot \sqrt{\frac{\pi}{a}}$$

Average Wind Power Density - Calculation Example

Average Power in the Wind. Estimate the average power in the wind at a height of 50 m when the windspeed at 10 m averages 6 m/s. Assume Rayleigh statistics,

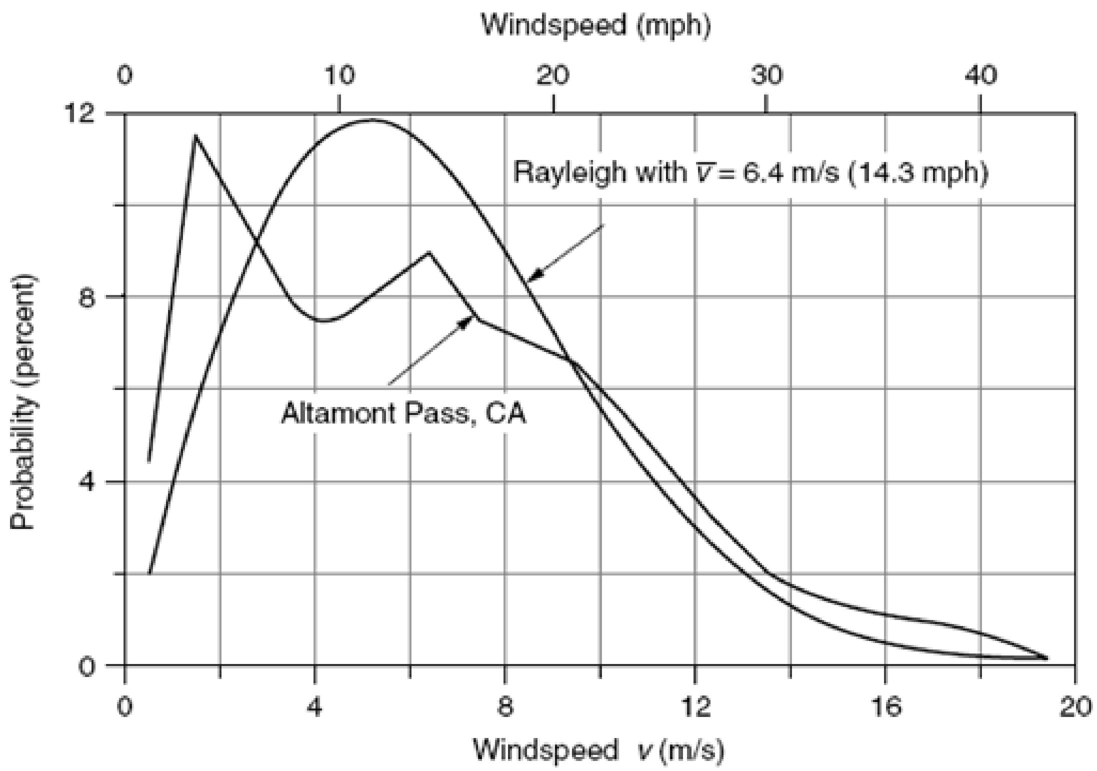
a standard friction coefficient $\alpha = 1/7$,

and standard air density $\rho = 1.225\ \text{kg/m}^3$.

Real vs. Rayleigh Distribution

Altamont Pass, CA

Status	Operational
Commission date	1981
	Wind farm
Type	Onshore
	Power generation
Units operational	4930
Nameplate capacity	576 MW
Annual net output	1.1 TWh



Energy Generation Calculation - Example

A D= 48[m] rotor (Diameter) wind turbine is mounted on a 50 [m] tower in an area with 5 [m/s] average winds at 10 [m] height. Estimate the annual energy delivered (kWh/yr) when it runs 24/7 with an overall efficiency of 30% in converting the power in the wind to electric power. Assume standard air density, Rayleigh statistics, and Class 1 surface roughness (roughness length z = 0.03).

$$\left(\frac{v}{v_0}\right) = \frac{\ln(H/z)}{\ln(H_0/z)}$$

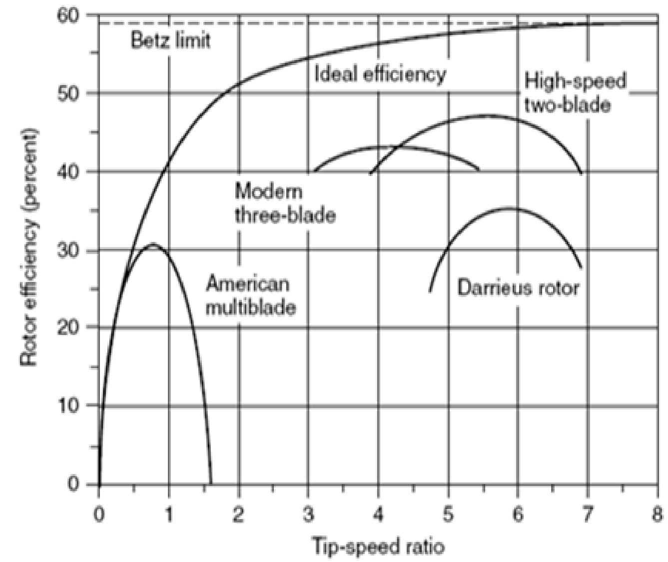
Wind Turbine Efficiency

- Efficiency Determination Factors
  - Machine (rotor, gearbox, generator, tower, control, etc.)
  - Terrain (surface roughness, obstructions, etc.)
  - Wind regime (velocity, timing, and predictability, etc.)
  - Purpose: Individual Wind turbine or Wind Farm

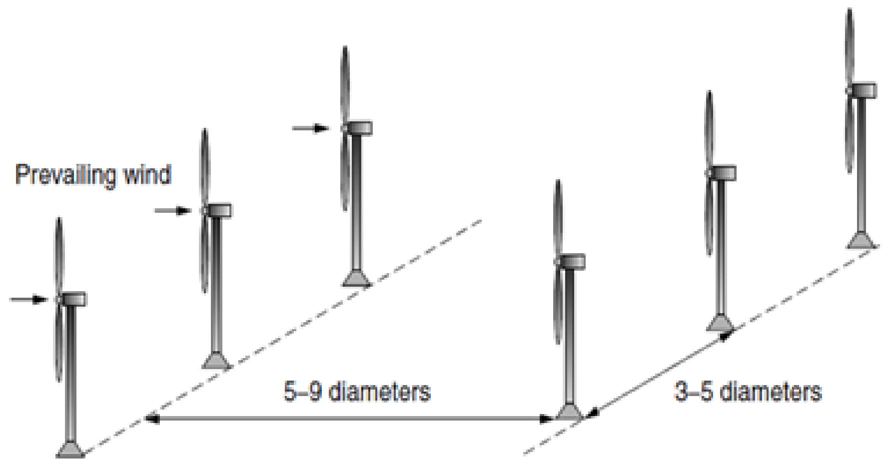
- Wind Turbine Efficiency
  - Max Wind Power Conversion efficiency: 59.3% - Betz Law
  - Max Rotor Efficiency: 75% x Betz = 45%
  - Gearbox and Generator Efficiency: 67%
  - Overall? = (Rotor Efficiency) x (Gear-Generator Efficiency)
    - about 30%

Wind Turbine Efficiency Calculation - Example

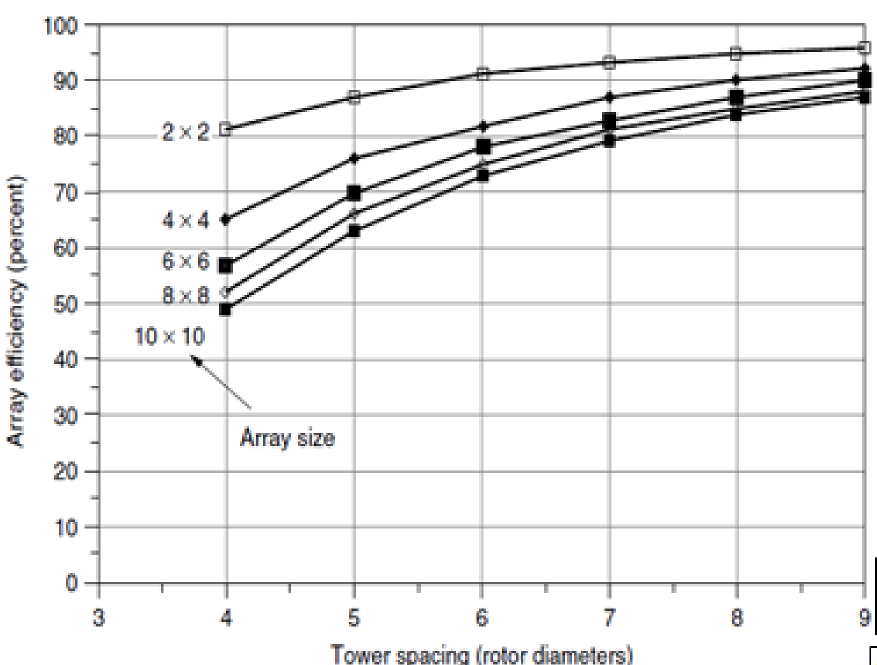
A three-blade wind turbine of 40-m blade diameter produces 1 MW at an average wind speed of 14 m/s. Air density is the standard 1.225 kg/m^3. Assume Rayleigh statistics. (Q1) If the TSR is 4.0, what is the speed of the rotor of the wind turbine? (Q2) If the generator needs to turn at 1800 rpm, what gear ratio is needed to match the rotor speed to the generator speed? (Q3) Referring the Rotor Efficiency-TSR chart below, what is efficiency of the **gear box + generator** under these conditions ?



Wind Turbine Efficiency and Energy delivery – Wind Farm



Optimum spacing of towers is estimated to be 3–5 rotor diameters between wind turbines within a row and 5–9 diameters between rows.



Impact of tower spacing and array size on performance of wind turbines.
Source: Data in Milborrow and Surman (1987), presented in Grubb and Meyer (1993).

- Wind farm or wind park: clustering wind turbines together at a windy site
  - Reduced site development costs
  - Simplified transmission connection
  - Centralized O&M
- Array Efficiency
  - Downwind is slower than upwind
  - **(Array) Efficiency**
  - Some distance between turbines is required
- Wind Farm Spacing - Rule of Thumb
  - 3 – 5 rotor diameters (D) separating towers within a row
  - 5 – 9 diameters (D) between rows

Wind Farm

- Overall Efficiency = (Wind Turbine Efficiency)*(Array Efficiency) = (Electric Power)/(P in the wind)

Wind Farm Efficiency Calculation – Example

Example: Suppose that a wind farm, located in the area of an average wind speed of 10 m/s, has 10 wind turbines running 24/7 without stopping. Each turbine's blade diameter is 10-m. Assume the wind turbine efficiency is 25%. Rayleigh statistics and the standard air density assumed. If the Electric energy generated from the wind farm is 1800 MWh per year,

(Q) Find the array efficiency.

Wind Farm Efficiency Calculation – Example 2

Example: Suppose that a wind farm, located in the area with wind power density of 500 W/m^2, has 4-rotor-diameter tower spacing along its row, with 7-diameter spacing between rows (4D x 7D) as arranged below. Assume that all the turbines run 24/7 without stopping. Also assume that the wind turbine efficiency is 30%. If the electric energy generated per year per unit land area is 23.5 kWh, find

(Q1) the land area per turbine

(Q2) the array efficiency.