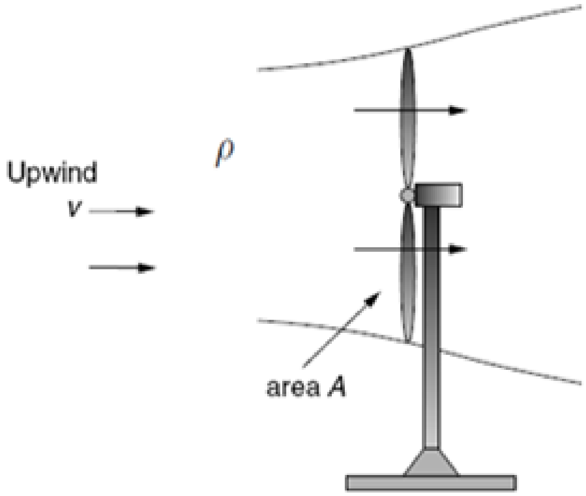
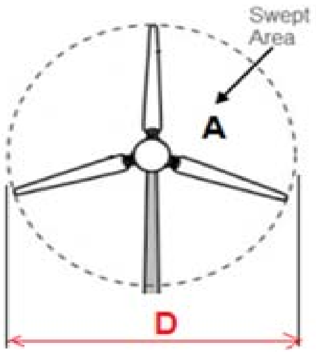
Power in the Wind

v windspeed (m/s)

ρ air density (kg/m^3)

A cross-sectional area (m^2)





Air Density Correction - Temperature

- Air Density (ρ): ρ = 1.225 kg/m^3
  - At 15C (59F) Air Temperature and at 1 atmosphere air pressure

$$\rho = \frac{P_a \times M.W. \times 10^{-3}}{RT}$$

- P_a: Absolute Pressure (atm)
- M.W.: Molecular weight of the gas (g/mol)
- R: Ideal gas constant = 8.2056×10^{-5}
- T: absolute temperature (K): K = C + 273.15

- Molecular Weight of Air (= 28.97 g/mol)
  - 78% of Nitrogen (N^2): 0.7808 * 28.02
  - 20.95% of Oxygen (O_2): 0.2095 * 32
  - 0.93% of Argon (Ar): 0.0093 * 39.95
  - 0.035 % of Carbon Dioxide (CO_2): 0.00035 * 44.01
  - 0.0018% of Neon (Ne): 0.000018 * 20.18

Density of Dry Air at a Pressure of 1 Atmosphere[a]

Temperature (°C)	Temperature (°F)	Density (kg/m^3)	Density Ratio (K_T)
−15	5.0	1.368	1.12
−10	14.0	1.342	1.10
−5	23.0	1.317	1.07
0	32.0	1.293	1.05
5	41.0	1.269	1.04
10	50.0	1.247	1.02
15	**59.0**	**1.225**	1.00
20	68.0	1.204	0.98
25	77.0	1.184	0.97
30	86.0	1.165	0.95
35	95.0	1.146	0.94
40	104.0	1.127	0.92

density of air at 1 atm and 30°C (86°F)

Air Density Correction – Altitude/Pressure

Air Density (ρ): $\rho = 1.225\ kg/m^3$

- At 15C (59F) Air Temperature and at 1 atmosphere air pressure

$$P_a = P_{a0} \cdot e^{-1.185\times10^{-4}H} = 1(\text{atm}) \cdot e^{-1.185\times10^{-4}H}$$

- P_{a0}: Reference Pressure of 1 atm
- H: Height in Meters

$$\rho = \frac{P_a \times \text{M.W.} \times 10^{-3}}{RT}$$

air density at 5°C at 2000 m.

Air Pressure at 15°C as a Function of Altitude

Altitude (meters)	Altitude (feet)	Pressure (atm)	Pressure Ratio (K_A)
0	0	1	1
200	656	0.977	0.977
400	1312	0.954	0.954
600	1968	0.931	0.931
800	2625	0.910	0.910
1000	3281	0.888	0.888
1200	3937	0.868	0.868
1400	4593	0.847	0.847
1600	5249	0.827	0.827
1800	5905	0.808	0.808
2000	6562	0.789	0.789
2200	7218	0.771	0.771

Example – Wind power density (with height and temperature compensation)

$$P\,[\mathrm{W}] = \frac{1}{2}\rho A v^3$$

- Find the power density (W/m^2) in 10 m/s wind at an elevation of 1500 m and a temperature of 32 °F.

$$\rho = \frac{P_a \times \mathrm{M.W.} \times 10^{-3}}{RT}$$

- P_a: Absolute Pressure (atm)
- M.W.: Molecular weight of the gas (g/mol)
- R: Ideal gas constant = 8.2056×10^{-5}
- T: absolute temperature (K): K = C + 273.15

$$P_a = P_{a0} \cdot e^{-1.185\times10^{-4}H} = 1(\mathrm{atm}) \cdot e^{-1.185\times10^{-4}H}$$

- P_{a0}: Reference Pressure of 1 atm
- H: Height in Meters

Molecular Weight of Air (= 28.97 g/mol)

Impact of Tower Height – Friction Coefficient α

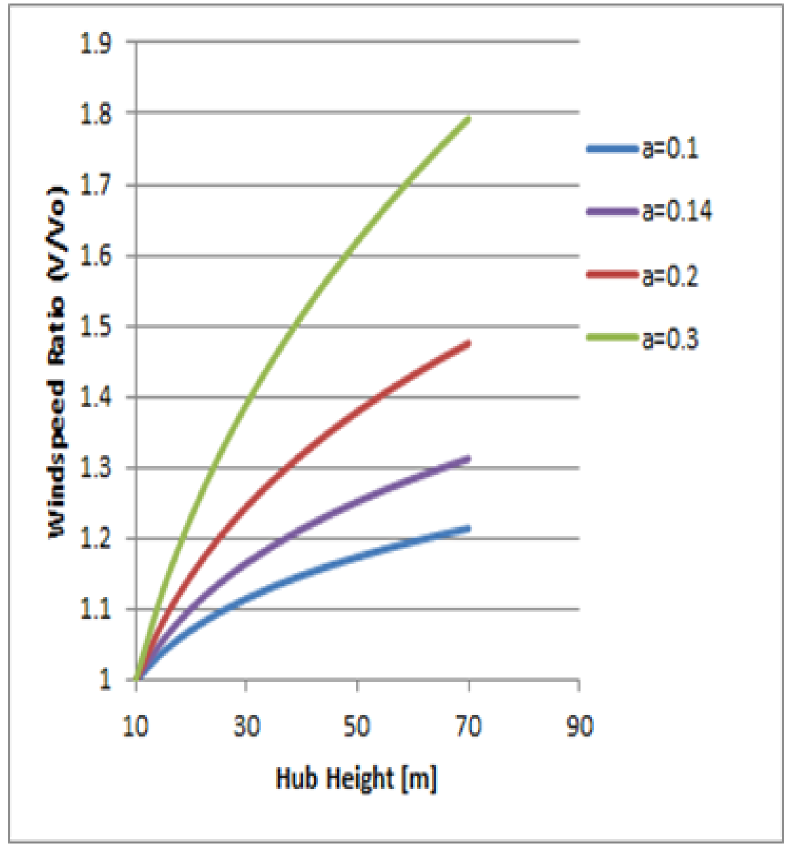
- Impact of the roughness of the earth's surface on wind speed – Friction Coefficient

$$\left(\frac{v}{v_0}\right) = \left(\frac{H}{H_0}\right)^{\alpha}$$

- v: wind speed at height H
- v_0: wind speed at height H_0 (H_0 is usually **10 m** as reference)
- α : friction coefficient

Friction Coefficient for Various Terrain Characteristics

Terrain Characteristics	Friction Coefficient α
Smooth hard ground, calm water	0.10
Tall grass on level ground	0.15
High crops, hedges and shrubs	0.20
Wooded countryside, many trees	0.25
Small town with trees and shrubs	0.30
Large city with tall buildings	0.40



Impact of Friction and Tower Height - Example

$$\left(\frac{v}{v_0}\right) = \left(\frac{H}{H_0}\right)^{\alpha}$$

- An anemometer mounted at a height of 10 m above a surface with crops, hedges, and shrubs (i.e., $\alpha = 0.2$) shows a wind speed of 5 m/s. Estimate the wind speed and the specific power (or Power Density) in the wind at a height of 50 m. Assume 15°C and 1 atm of pressure.

Example 2

- Case: A Wind Resource Manual for a wind turbine installation candidate site lists the following data measured at 60°F (with no impact of height on the air pressure - the same 1 atm at any elevation):
  - Wind Speed at 10 m = 6 [m/s]
  - Power Density in the Wind at 50 m = 500
- Q: What is the friction coefficient of the candidate site?

Impact of Tower Height – Roughness Length (z)

- Impact of the roughness of the earth's surface on wind speed –"Roughness Length" – Europe Style

$$\left(\frac{v}{v_0}\right) = \frac{\ln(H/z)}{\ln(H_0/z)}$$

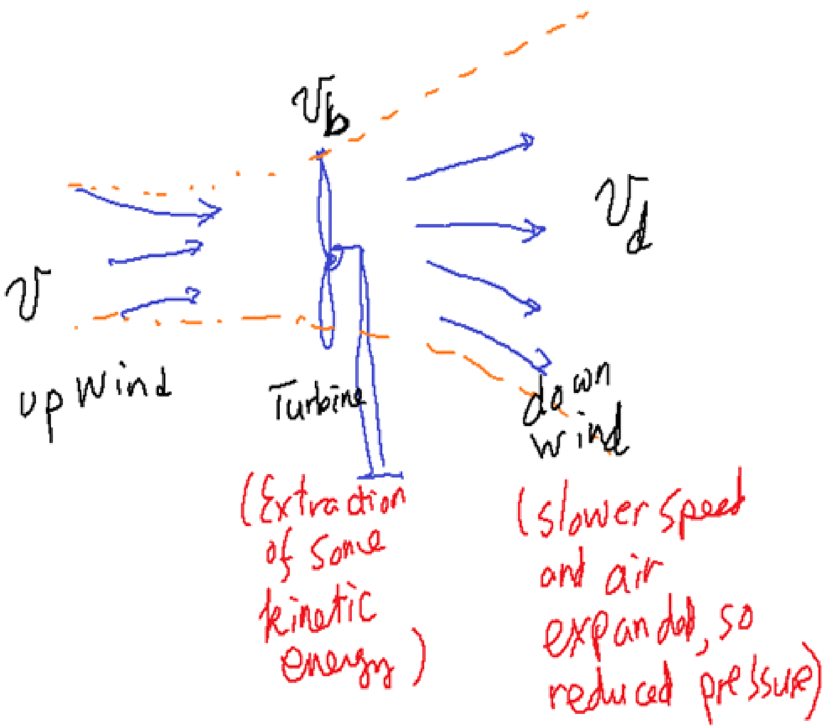
- *v*: wind speed at height H
- v_0: wind speed at height H_0 (H_0 is usually **10 m** as reference)
- z: roughness length

Roughness Class	Description	Roughness Length $z(m)$
0	Water surface	0.0002
1	Open areas with a few windbreaks	0.03
2	Farm land with some windbreaks more than 1 km apart	0.1
3	Urban districts and farm land with many windbreaks	0.4
4	Dense urban or forest	1.6

Maximum Rotor Efficiency Betz' Law

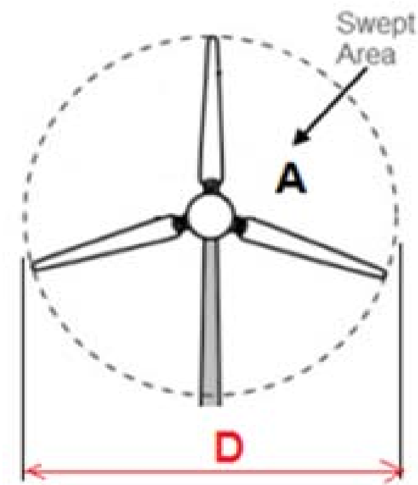
- Fundamental Constraints that restrict the maximum possible conversion
- Maximum power that a turbine can extract from the wind – formulated by ***Albert Betz*** (German Physicist) in 1919, with concept of Stream Tube.

(Power in the wind → Power in the Rotor Blade)

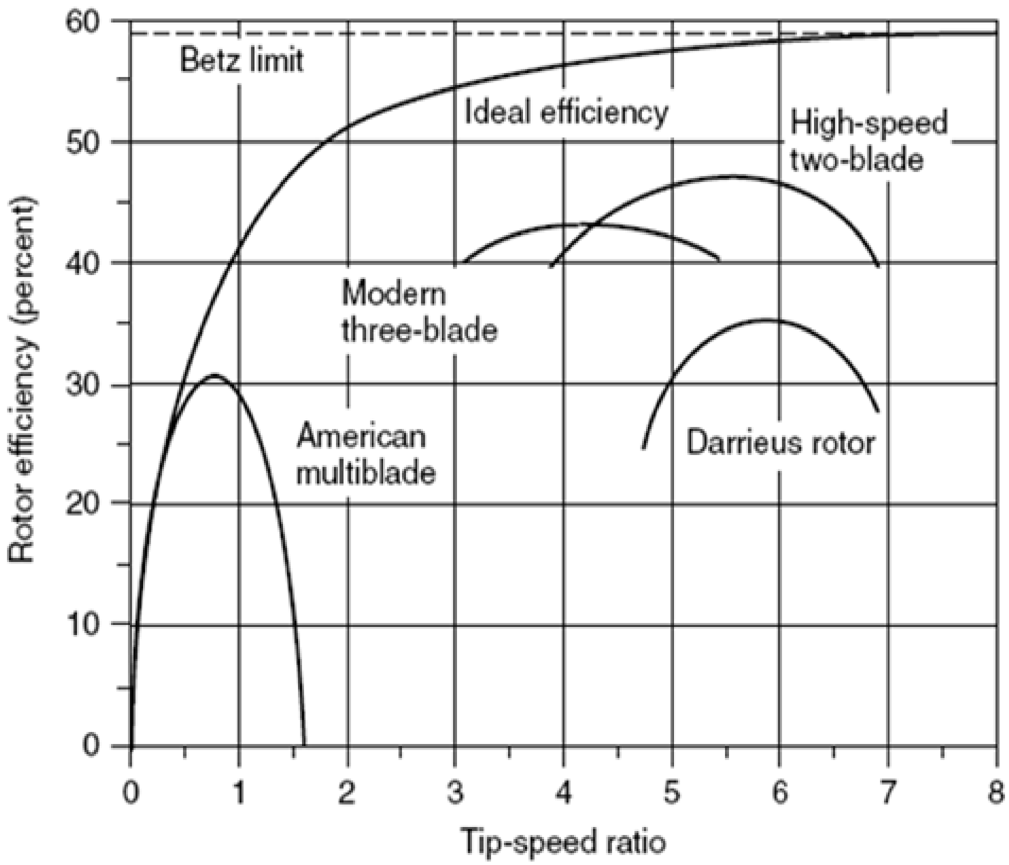


Blade Tip Speed & Tip Speed Ratio (TSR)

- The usual way to illustrate rotor efficiency is to present it as a function of TSR.



Efficiency in terms of TSR



Perfect TSR

Perfect TSR?

Empirical equation — by power output measurement

$$\lambda_{(max\ power)} = \frac{4\pi}{n}$$

← # of blades