CHAPTER 2: Electric Power

AC circuit response: magnitude & phase angle

Complex number Z = x + j y = |Z| < a where a = atan(x/y)

Euler' formula: $e^{jx} = cosx + j sinx$

Euler's identity: $e^{j\pi} = cos\pi + j sin\pi = -1 + j0 = -1$ or $e^{j\pi} + 1 = 0$



cos(x + y) = cos x * cos y - sin x * sin ysin(x + y) = sin x * cos y + cos x * sin y

Derivation for Euler's formula

$$e^{j(x+y)} = \cos(x+y) + j\sin(x+y)$$

$$e^{j(x+y)} = e^{jx}e^{jy}$$

$$= (\cos + j\sin x)(\cos + j\sin y)$$

$$= \cos x * \cos y - \sin x * \sin y + j(\cos x * \sin y + \sin x * \cos y)$$

Phasor

$$v(t) = V * \cos (wt + \theta) = Re\{V * e^{j(wt+\theta)}\} = Re\{V * e^{j\theta} * e^{jwt}\}$$

Let $\overline{V} = V * e^{j\theta}$, \leftarrow Phasor representation of v(t)

Then $v(t) = Re\{\overline{V} * e^{jwt}\}$

If we suspend the "Re" thing, then we can simply state: $v(t) = \overline{V} * e^{jwt}$ and similarly $i(t) = \overline{I} * e^{jwt}$

Phasor for voltage and current: $\overline{V} = V * e^{j\theta_v}$ and $\overline{I} = I * e^{j\theta_i}$

Phasor representation of elements (R, L, and C)

Time-domain	Phasor domain
R	R $R = \frac{\overline{V}}{\overline{L}}$
L	jwL From $v(t) = L * \frac{di}{dt} \to \overline{V}e^{jwt} = L * (jw) * \overline{I} * e^{jwt} \to$
	$\bar{V} = jwL * \bar{I}$
С	1/(jwC) From i(t) = $C * \frac{dv}{dt} \rightarrow \overline{I}e^{jwt} = C * (jw) * \overline{V} *$
	$e^{jwt} \rightarrow \bar{I} = jwC * \bar{V} \rightarrow \bar{V} = \frac{1}{jwC} * \bar{I}$

Effective value ("Root-Mean-Square")

$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}$$

RMS Calculation of a sinusoidal signal



A sinusoidal voltage is given by $v(t) = 300 \cos(120\pi t + 30^\circ)$

- (a) What is the period of the voltage?
- (b) What is frequency of the voltage?
- (c) What is the magnitude of the voltage at t = 2.778 ms?
- (d) What is the rms value of the voltage?

EXAMPLE

Find the <u>Phasor</u> of the following sinusoidal signals (a) $v(t) = 170 \cos (377t - 40^{\circ})$ (b) $v(t) = 10 \sin (1000t + 20^{\circ})$ (c) $v(t) = 5 \cos(wt + 36.87^{\circ}) + 10 \cos (wt - 53.13^{\circ})$

EXAMPLE Find the time-domain signal equation of the following phasors (a) $\overline{V} = 18.6 \angle -54^{\circ}$ (b) $\overline{I} = (20 + j80 - 30 \angle 15^{\circ})$

Impedance

- $R-L \rightarrow Z = R + jwL$
- $R C \rightarrow Z = R 1/(jwC)$

R-L-C → Z = R+ j(wL - 1/(wC))

For the circuit below, find (a) Phasor-domain equivalent circuit and (b) i(t) by Phasor analysis.



For the circuit below, find (a) Phasor-domain equivalent circuit and (b) i2(t) by Phasor analysis.



time $\frac{p}{p_{r-2}}$ phase Instantaneous Power $p(t)$ $\begin{cases} \theta_{T-2} \\ \theta_{T-2} $
$R \rightarrow R$ $p(t) = \eta r(t) \cdot i(t)$ $(V t) = V_{t} \cdot \cos(\omega t + o)$
$L \rightarrow j\omega L$ (i)=7. (as (wt-0)
$C \rightarrow -i \frac{1}{10}$
$= V \cos \omega t \cdot \overline{t} \cosh(\omega t - \theta)$
$U(t) \rightarrow V = \frac{VM}{\sqrt{2}} \frac{U}{\sqrt{2}} \frac{U}{2$
$T = \frac{I_{m}}{P} \left(\frac{\partial u}{\partial u} \right)^{-1} \sqrt{\frac{\partial u}{\partial u}} \sqrt{\frac{\partial u}{\partial u}}$
$U(k) = 1 - \frac{1}{\sqrt{2}} \sum_{n=1}^{\infty} Cos(N-cosp=\frac{1}{2}cos(N-p)+\frac{1}{2}cos(N+p)$
$P(t) \longrightarrow ? [I] = V_m I_m S_{C} a_{S} A + c_{S} (2mI - \theta) \}$
$= \eta f(t) \dot{a}(t)$
$= \cos \frac{\beta}{2} + \sin \frac{\beta}{2}$
I= LANSA LAL - VM IM S COLOR T POR A Signal
Arene $p(t)$ $V = V_{VMS} = \sqrt{2}$ $\frac{1}{2} \left[\cos \theta + \cos \theta + \cos \theta + \sin \theta + \sin \theta \right]$
$(\mathbf{p}(t)) = \frac{V_{m} \mathbf{I}_{m}}{\cos \theta} \cdot \cos \theta \cdot \cos 2wt$
$P(x) = P = \frac{1}{2} \cos \theta$
$= \frac{V_m}{2} \cdot \frac{T_m}{2} \cos \theta = V I \cos \theta [P] + \frac{1}{2} \cdot \frac{1}{2} \sin \theta \cdot \sin \theta = \frac{1}{2} \cdot $
$P = V I \cos \theta$
$P_{a} \downarrow P_{a} uv p r$ $\longrightarrow P_{a}$ R_{a} R_{a}
$\frac{1}{\left(\frac{1}{2}\right)^{1/2}}$ $\left(\frac{1}{\left(\frac{1}{2}\right)^{1/2}}\right)$ $\frac{1}{\left(\frac{1}{2}\right)^{1/2}}$ $\frac{1}{\left(\frac{1}{2}\right)^{1/2}}$
$(VISin\theta = Q)$ $(DIA = VICOS + VICOS $
Readive Power +VISIND. Sin2wt
TV=VL0
$p_{i} = p + p \cdot c_{i} = 1/0^{\circ}$
$\int \Theta = \Theta_{V} $
$\theta = 0$ (150=1) r from γ $U(A)$ 12042
$\overline{Z} = R$ since ∂
$P(t) = VI + VI \cos 2\omega t = VI (1+\cos 2\omega t)$
$(\widehat{\partial} \Theta = 90^{\circ} \qquad \Theta = \Theta_{i} - \Theta_{i} = 90^{\circ} \qquad \Theta_{i} = -90^{\circ}$
$Z = j \omega L cost = 0 \qquad \overline{L} = I \ \underline{/} - 90^{\circ}$

$$P(h) = 0 + 0 \cdot \cos 2\pi i + \sqrt{1 \cdot \sin 2\pi i} \sqrt{1 \cdot \frac{1}{1 \cdot \sin 2\pi i}} \sqrt{1 \cdot \frac{1}{1 \cdot \frac{1}$$

1. Definitions

Instantaneous Power: $p(t) = v(t) \cdot i(t) = P \cos \theta + P \cos 2wt - Q \sin 2wt$ Real Power: $P = \frac{V_m I_m}{2} \cos \theta = V_{rms} I_{rms} \cos \theta$ Reactive Power: $Q = \frac{V_m I_m}{2} \sin \theta = V_{rms} I_{rms} \sin \theta$ Complex Power: $\overline{S} = P + jQ$ (*note: see #3 for deeper discussion) Apparent Power: $S = \sqrt{P^2 + Q^2}$ Power Factor Angle: $\theta = \theta_v - \theta_i$ (phase angle difference between voltage and current)

2. Load Dependence of the Power

Pure R case: $\theta = 0$, $P = \frac{V_m I_m}{2}$, Q = 0 therefore, "R <u>consumes</u> P only" Pure L case: $\theta = 90^\circ$, P = 0, $Q = \frac{V_m I_m}{2}$, therefore, "L <u>consumes</u> Q only" Pure C case: $\theta = -90^\circ$, P = 0, $Q = -\frac{V_m I_m}{2}$, therefore, "C <u>delivers</u> Q" toward the source!!!!!

3. Complex Power in Phasor

From $\overline{S} = P + jQ$, $\overline{S} = V_{rms}I_{rms}\cos\theta + jV_{rms}I_{rms}\sin\theta = V_{rms}I_{rms}(\cos\theta + j\sin\theta) = V_{rms}I_{rms}e^{j\theta}$ Since $\theta = \theta_v - \theta_i$

 $\overline{S} = V_{rms} I_{rms} e^{j\theta} = V_{rms} I_{rms} \angle \theta = V_{rms} I_{rms} \angle (\theta_v - \theta_i) = V_{rms} I_{rms} \angle \theta_v \cdot \angle - \theta_i = V_{rms} \angle \theta_v \cdot I_{rms} \angle - \theta_i$ Therefore, $\overline{S} = \overline{V_{rms}} \cdot \overline{I}^*_{rms}$

$$(Ex) \quad v(t) = 100\sqrt{2} \cos \omega t$$

$$i'(t) = 10\sqrt{2} \cos \omega t$$

$$0 \sqrt{2} \cos \omega t - 30^{2}$$

$$0 \sqrt{2} \sqrt{2} \cos \omega t - 30^{2}$$

$$0 \sqrt{2} \sqrt{2} \sqrt{2} \cos \omega t - 30^{2}$$

Let's expand the **complex Power S**. See an example circuit below for the discussion.



(1) From
$$\overline{V}_{rms} = Z \cdot \overline{I}_{rms}$$
,
 $\overline{S} = \overline{V}_{rms} \cdot \overline{I}^*_{rms} = \overline{Z}\overline{I}_{rms}\overline{I}^*_{rms} = \overline{Z}I_{rms}^2 = I_{rms}^2(R+jX) = R \cdot I_{rms}^2 + jX \cdot I_{rms}^2$
In this case: $P = R \cdot I_{rms}^2$ and $Q = X \cdot I_{rms}^2$
In other words, *P* relates only to *R*, and *Q*, only to *X*.

(2) Alternatively, from
$$\overline{I}_{rms} = \frac{\overline{V}_{rms}}{\overline{Z}}$$
, $\overline{S} = \overline{V}_{rms} \cdot [\frac{\overline{V}_{rms}}{\overline{Z}}]^* = \frac{V_{rms}^2}{\overline{Z}^*} = \frac{V_{rms}^2}{R - jX} = \frac{V_{rms}^2(R + jX)}{R^2 + X^2}$
By expanding further, $\overline{S} = \frac{V_{rms}^2(R + jX)}{R^2 + X^2} = \frac{V_{rms}^2(R + jX)}{Z^2} = \frac{V_{rms}^2R}{Z^2} + j\frac{V_{rms}^2X}{Z^2}$
In this case: $P = \frac{V_{rms}^2R}{Z^2}$ and $Q = \frac{V_{rms}^2X}{Z^2}$
In other words, again, *P* relates only to *R*, and *Q*, only to *X*.

POWER FACTOR (pf)

1. Definition

Power factor (pf) is defined by "cosine of the angle made by voltage and current." The angle, θ , is defined by $\theta_v - \theta_i$, where θ_v and θ_i are phase angles of the voltage and the current, respectively. This definition indirectly says that the reference in phase domain is the voltage. (See the diagrams below)



"Leading pf" means that the current leads the voltage and therefore the pf angle θ is negative, as shown in (b); "Lagging pf" means that the current lags the voltage and the pf angle θ is positive as in (a).

2. Alternative Definition

Let's expand the original definition of the pf to the complex power S. Complex power is defined:

$$S = \overline{VI}^* = V \angle \theta_v \cdot I \angle -\theta_i = VI \angle (\theta_v - \theta_i) = VI \angle \theta$$

Therefore the angle of the complex power is exactly same as the angle of V and I. In other words, if we know the complex power and present it on the complex plane, we get the power factor angle. Also, since S = P + jQ, power factor angle can also be found once we know P and Q. (See diagrams below) Or, we can draw the general place of a complex power, once we know power factor is leading or lagging. "leading" or "lagging" power factor determines the polarity of reactive power Q.



3. Another Alternative Definition

Let's play a little bit more. This time we will involve load impedance Z. Since load voltage V and load current I determines the load impedance Z, i.e., Z=V/I, we can express the power factor with Z.

$$Z = \frac{\overline{V}}{\overline{I}} = \frac{V \angle \theta_v}{I \angle \theta_i} = \frac{V}{I} \angle (\theta_v - \theta_i) = \frac{V}{I} \angle \theta$$

Therefore, if we locate the load impedance Z, the angle made by the impedance is the power factor angle. Since the impedance is composed of resistance R and reactance X, Z = R + jX, we can relate these elements with power factor as shown below.



4. Final Words

If you compare the phase diagram of S and Z, you may notice that they are places along the same line, either they are lagging or leading. In many problems of power factor calculation, you may want to apply any or all of the definitions presented here. In any power factor related problems, I recommend you draw phase diagram of S, Z, or V & I, before you jump to write your answer. Always careful with the words "leading" and "lagging."

Evaluate the sinusoidal steady-state current for the circuit shown in Fig. 1 by replacing the circuit by its sinusoidal steady state equivalent.



Use phasor techniques to find $v_L(t)$ in the circuit below.



EXAMPLE PROBLEMS

1. As shown below, a voltage source of $250 \angle 0^{\circ}$ (rms) is supplying a load of 39+j26 via a line which has an impedance of 1+j4.

- (a) Find the phasor current and phasor voltage at the load, \bar{I}_L and \bar{V}_L , respectively.
- (b) Calculated power delivered to the load
- (c) Calculate (real) power loss in the line

(d) Calculated power supplied by the source



- 2. Three loads are connected in parallel across a 2400 V (rms) line, as shown below. Load 1 absorbs 18kW and 24 kVar. Load 2 absorbs 60kVA at 0.6 pf (leading). Load 3 absorbs 18kW at unity power factor.
 - (a) Find the impedance that is equivalent to the three parallel loads.
 - (b) Find the power factor of the equivalent load.



3. A factory has an electrical load of 1800kW at a lagging power factor of 0.6. An additional variable power factor load is to be added to the factory. The new load will add 600kW to the real power load of the factory. The power factor of the added load is to be adjusted so that the overall power factor of the factory (with the load and the new load) is 0.96 lagging. Assume that the rms voltage at the input to the factory is 4800V.

(a) What is the rms magnitude of the current into the factory BEFORE the new load is added to the factory?

- (a) Find the reactive power of the added load.
- (b) What is the power factor of the new load?
- (c) What is the rms magnitude of the current into the factory AFTER the new load is added to the factory?

<u>Example –</u>

Example – In-phase condition

The circuit shown below is operating in the sinusoidal steady state. The inductor is adjusted until the current i_g is in-phase with the sinusoidal voltage v_g .

- (a) Specify the inductance (in Henry) if $v_g=100 \cos 500t$ [V].
- (b) Find i_g when L has the value found in part (a).



3-phase system -- SUMMARY

1. Balanced $3-\phi$ system is characterized by:

- 3 voltages are with same magnitude and 120° phase shift
- 3 load impedances are same
- Therefore, the currents are balanced with same magnitude and 120° apart
- 2. There are two types of voltages and two types of currents:
 - Phase Voltage (V_{ϕ}) = "voltage across a phase impedance"
 - Line Voltage (V_l) = "voltage between a (phase) line and another (phase) line"
 - Phase Current (I_{ϕ}) = "current through a phase impedance"
 - Line Current (I_l) = "current through a (phase) line"
- 3. Above definitions have different meaning at different load formation, Y or Δ

<u>*Y-load case:*</u> As shown below, the three phase impedances (Z_A , Z_B , Z_C) form the letter "Y". In the figure, the line connecting 3-phase source to the Y-load is represented by a line impedance, Z_l .



 $V_{\phi}=V_{AN}, V_{BN}, \text{ and } V_{CN}$ (1)

i.e., phase voltage is same as the voltage between a (phase) line and the neutral (marked as "N") (2)
V_l= V_{AB}, V_{BC}, and V_{CA} (3)
I_φ =I_{AN}, I_{NN}, and i_{CN}
I_l = I_{aA}, I_{bB}, and i_{cC} (also, I_{AN}=I_{aA},etc)
Conclusion of Y-load:

(i) I_φ=I_l
(ii) V_φ ≠ V_l (instead, V_l = √3V_φ∠30°)

 Δ Load Case: As shown below, the 3 phase loads (Z_{AB}, Z_{BC}, and Z_{CA}) form a Delta shape. As in Y-load, the line connecting 3-phase source to the Y-load is represented by a line impedance, Z_l. Note that there is no neutral point.



 $V_{\phi}=V_{AB}, V_{BC}, \text{ and } V_{CA}$ (4) $V_{l}=V_{AB}, V_{BC}, \text{ and } V_{CA}$ (5) $I_{\phi}=I_{AB}, I_{BC}, \text{ and } i_{CA}$ $I_{l}=I_{aA}, I_{bB}, \text{ and } i_{cC}$

Conclusion of Δ -load:

(i)
$$I_{\phi} \neq I_{l}$$
 (instead $I_{l} = \sqrt{3}I_{\phi} \angle -30^{\circ}$) (ii) $V_{\phi} = V_{l}$

4. Single-Phase Equivalent Circuit

- In a balanced 3-phase system, voltage and current magnitudes are same.
- In a balanced 3-phase system, voltage and current are 120° apart from each other
- Therefore, once a phase value is known, the other two are also known
- **<u>NOTE</u>**: Single-phase equivalent circuit is formed so that a phase impedance is connected *between a phase (line) and the neutral*.

Y-load case: From the Y-load figure, let's delete two phases (B, and C), then the remaining circuit looks like below:



 Δ -Load Case: There is a slight problem here, since there is no neutral point. So we have to convert the load to Y-load equivalent. By the usual Δ -Y Transformation, we could get the Y impedance, in terms of Delta-load, as $Z_Y = \frac{Z_A}{3}$. Then the single-phase circuit looks like this:



NOTE: The voltage across the impedance in this single-phase circuit is **not** the actual phase voltage across the impedance. V_{AB} is the actual voltage across a impedance. So we have to convert the voltage, after your calculation of V_{AN} , to V_{AB} for a delta-load phase voltage.

$$V_{\phi} = V_{AB} = \sqrt{3}V_{AN} \angle 30^{\circ} \text{ and}$$
$$I_{\phi} = I_{AB} = \frac{I_{aA}}{\sqrt{3}} \angle 30^{\circ}$$

3-phase example problems

EX#1.

A balanced three-phase Y-connected generator has a voltage of 120 V/ ϕ . A balanced 3-phase Δ -load is fed from the source through a distribution line having an impedance of $0.5 + j \ 1.4 \ \Omega/\phi$. The load impedance is 118.5 + j 85.8 Ω/ϕ . Use the a-phase voltage of the generator as the reference.

(a) construct 3-phase circuit

(b) construct a single-phase equivalent circuit

(c) calculate the line currents I_{aA} , I_{bB} , I_{cC} .

(d) calculate the phase voltages at the load.

(e) calculate the total complex power delivered to the load

(f) calculate what percentage of the real power at the sending end of the line is delivered to the load?

EX#2.

A balanced 3-phase Y-load requires 480kW at a lagging power factor of 0.8. The load is fed from a line having an impedance of $0.005+j0.025 \Omega/\phi$. The line voltage at the terminals of the load is 600V.

- (a) construct 3-phase circuit
- (b) construct a single-phase equivalent circuit
- (c) magnitude of the line current
- (d) magnitude of the line voltage at the sending end of the line
- (e) power factor at the sending end of the line