CHAPTER 5

SYNCHRONOUS GENERATORS

Synchronous generators or *alternators* are synchronous machines used to convert mechanical power to ac electric power. This chapter explores the operation of synchronous generators, both when operating alone and when operating together with other generators.

5.1 SYNCHRONOUS GENERATOR CONSTRUCTION

In a synchronous generator, a dc current is applied to the rotor winding, which produces a rotor magnetic field. The rotor of the generator is then turned by a prime mover, producing a rotating magnetic field within the machine. This rotating magnetic field induces a three-phase set of voltages within the stator windings of the generator.

Two terms commonly used to describe the windings on a machine are *field windings* and *armature windings*. In general, the term "field windings" applies to the windings that produce the main magnetic field in a machine, and the term "armature windings" applies to the windings where the main voltage is induced. For synchronous machines, the field windings are on the rotor, so the terms "rotor windings" and "field windings" are used interchangeably. Similarly, the terms "stator windings" and "armature windings" are used interchangeably.

The rotor of a synchronous generator is essentially a large electromagnet. The magnetic poles on the rotor can be of either salient or nonsalient construction. The term *salient* means "protruding" or "sticking out," and a *salient pole* is a magnetic pole that sticks out from the surface of the rotor. On the other hand, a

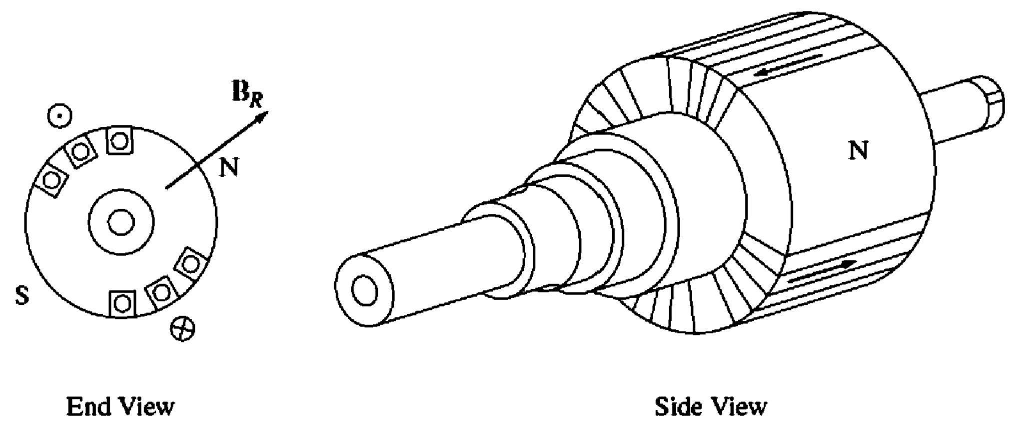


FIGURE 5–1
A nonsalient two-pole rotor for a synchronous machine.

nonsalient pole is a magnetic pole constructed flush with the surface of the rotor. A nonsalient-pole rotor is shown in Figure 5–1, while a salient-pole rotor is shown in Figure 5–2. Nonsalient-pole rotors are normally used for two- and four-pole rotors, while salient-pole rotors are normally used for rotors with four or more poles. Because the rotor is subjected to changing magnetic fields, it is constructed of thin laminations to reduce eddy current losses.

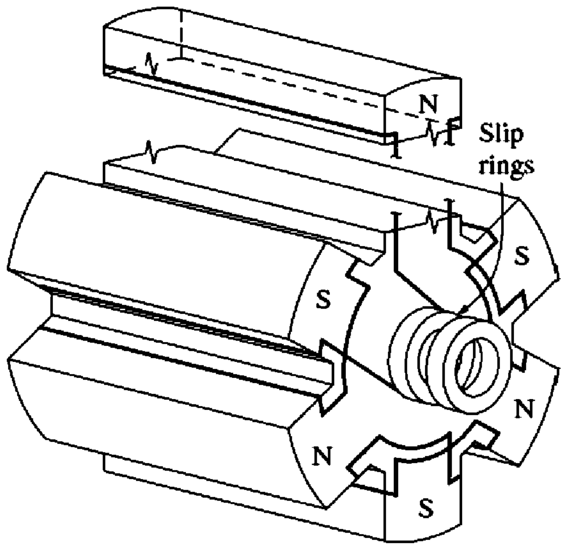
A dc current must be supplied to the field circuit on the rotor. Since the rotor is rotating, a special arrangement is required to get the dc power to its field windings. There are two common approaches to supplying this dc power:

1. Supply the dc power from an external dc source to the rotor by means of *slip rings* and *brushes*.
2. Supply the dc power from a special dc power source mounted directly on the shaft of the synchronous generator.

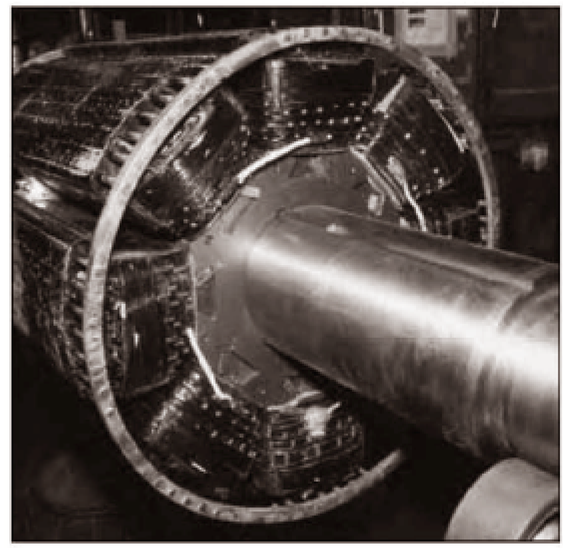
Slip rings are metal rings completely encircling the shaft of a machine but insulated from it. One end of the dc rotor winding is tied to each of the two slip rings on the shaft of the synchronous machine, and a stationary brush rides on each slip ring. A "brush" is a block of graphitelike carbon compound that conducts electricity freely but has very low friction, so that it doesn't wear down the slip ring. If the positive end of a dc voltage source is connected to one brush and the negative end is connected to the other, then the same dc voltage will be applied to the field winding at all times regardless of the angular position or speed of the rotor.

Slip rings and brushes create a few problems when they are used to supply dc power to the field windings of a synchronous machine. They increase the amount of maintenance required on the machine, since the brushes must be checked for wear regularly. In addition, brush voltage drop can be the cause of significant power losses on machines with larger field currents. Despite these problems, slip rings and brushes are used on all smaller synchronous machines, because no other method of supplying the dc field current is cost-effective.

On larger generators and motors, *brushless exciters* are used to supply the dc field current to the machine. A brushless exciter is a small ac generator with its



(a)



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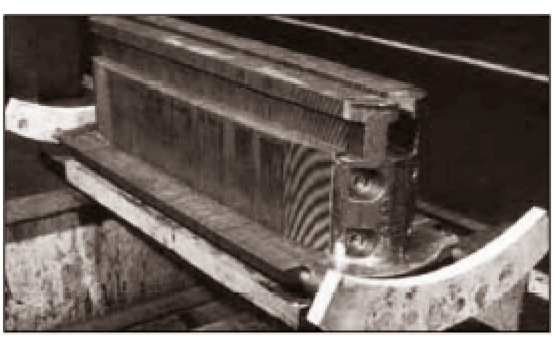

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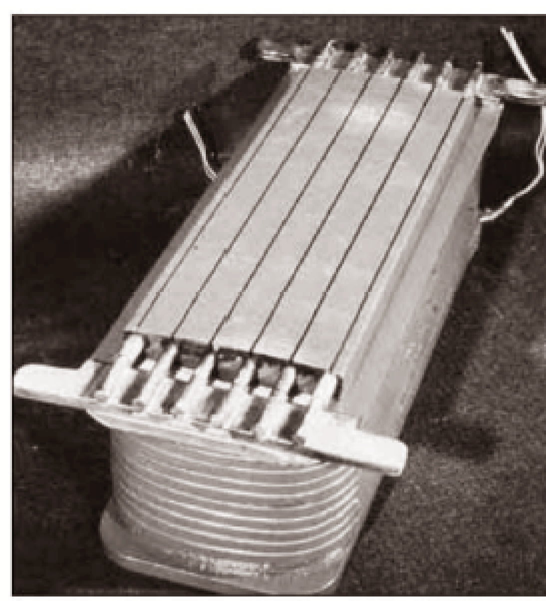


(d)

FIGURE 5–2
(a) A salient six-pole rotor for a synchronous machine. (b) Photograph of a salient eight-pole synchronous machine rotor showing the windings on the individual rotor poles. (*Courtesy of General Electric Company.*) (c) Photograph of a single salient pole from a rotor with the field windings not yet in place. (*Courtesy of General Electric Company.*) (d) A single salient pole shown after the field windings are installed but before it is mounted on the rotor. (*Courtesy of Westinghouse Electric Company.*)

field circuit mounted on the stator and its armature circuit mounted on the rotor shaft. The three-phase output of the exciter generator is rectified to direct current by a three-phase rectifier circuit also mounted on the shaft of the generator, and is then fed into the main dc field circuit. By controlling the small dc field current of the exciter generator (located on the stator), it is possible to adjust the field current on the main machine ***without slip rings and brushes***. This arrangement is shown schematically in Figure 5–3, and a synchronous machine rotor with a brushless exciter mounted on the same shaft is shown in Figure 5–4. Since no mechanical contacts ever occur between the rotor and the stator, a brushless exciter requires much less maintenance than slip rings and brushes.

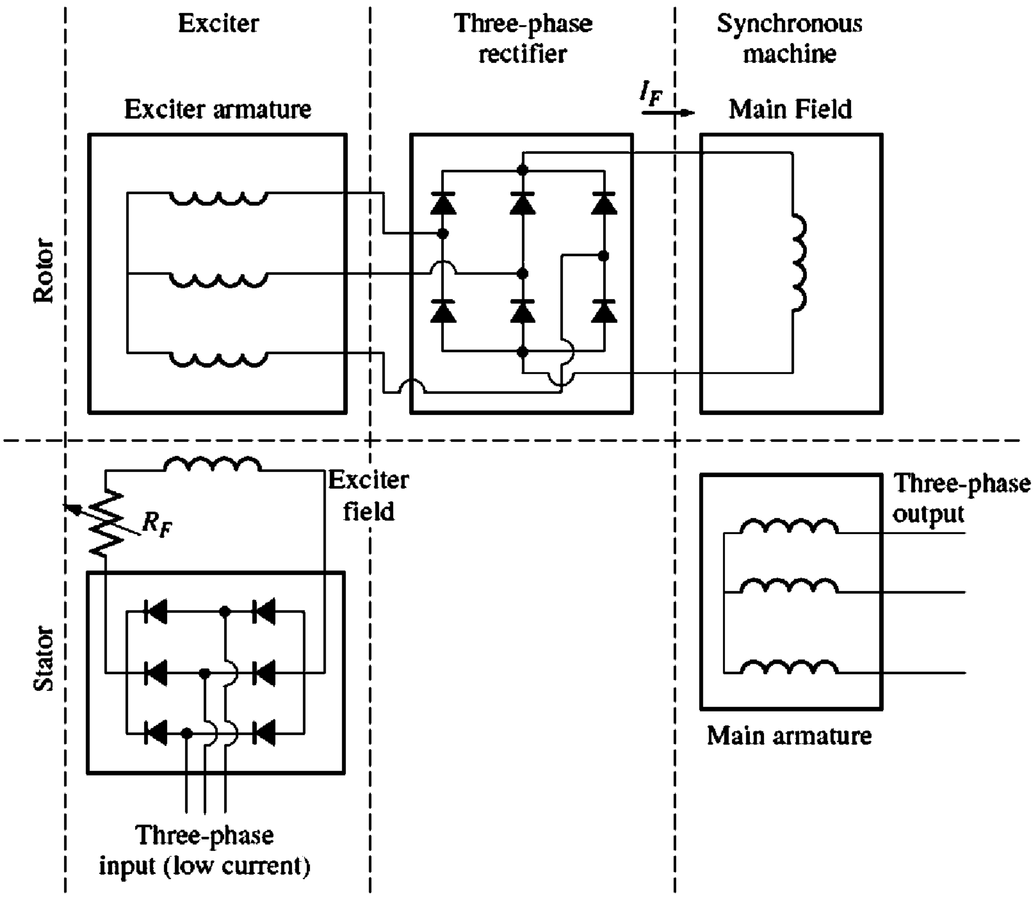


FIGURE 5–3
A brushless exciter circuit. A small three-phase current is rectified and used to supply the field circuit of the exciter, which is located on the stator. The output of the armature circuit of the exciter (on the rotor) is then rectified and used to supply the field current of the main machine.

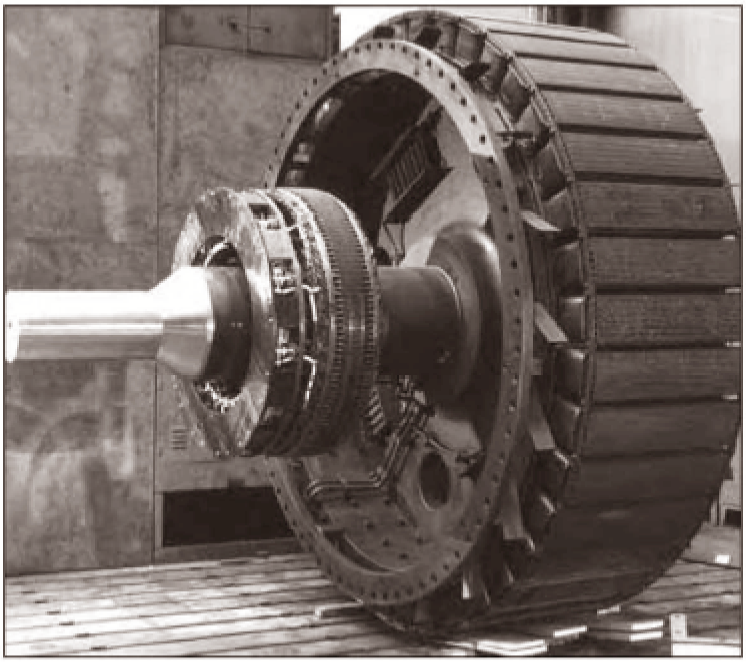


FIGURE 5–4
Photograph of a synchronous machine rotor with a brushless exciter mounted on the same shaft. Notice the rectifying electronics visible next to the armature of the exciter. (Courtesy of Westinghouse Electric Company.)

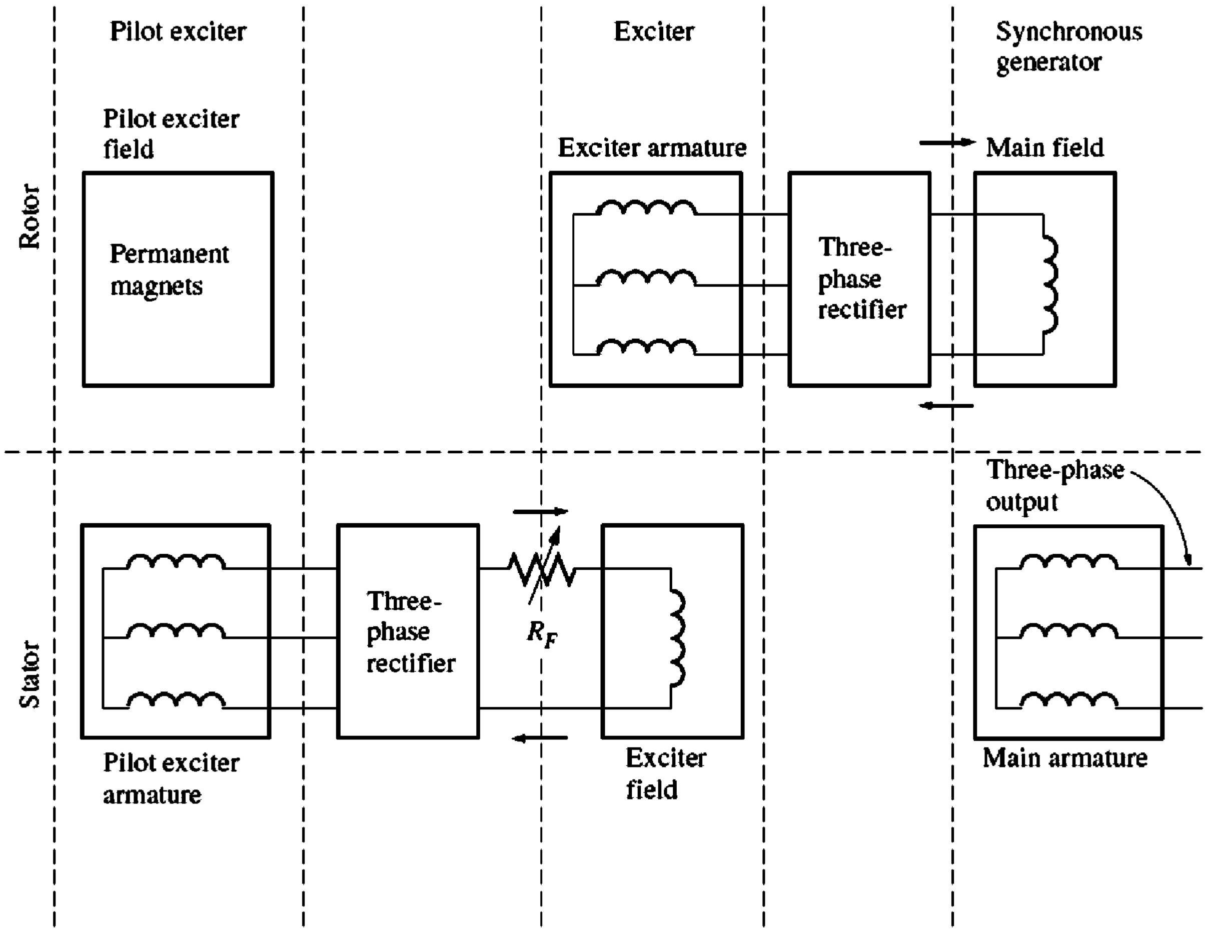


FIGURE 5–5
A brushless excitation scheme that includes a pilot exciter. The permanent magnets of the pilot exciter produce the field current of the exciter, which in turn produces the field current of the main machine.

To make the excitation of a generator *completely* independent of any external power sources, a small pilot exciter is often included in the system. A *pilot exciter* is a small ac generator with *permanent magnets* mounted on the rotor shaft and a three-phase winding on the stator. It produces the power for the field circuit of the exciter, which in turn controls the field circuit of the main machine. If a pilot exciter is included on the generator shaft, then *no external electric power* is required to run the generator (see Figure 5–5).

Many synchronous generators that include brushless exciters also have slip rings and brushes, so that an auxiliary source of dc field current is available in emergencies.

The stator of a synchronous generator has already been described in Chapter 4, and more details of stator construction are found in Appendix B. Synchronous generator stators are normally made of preformed stator coils in a double-layer winding. The winding itself is distributed and chorded in order to reduce the harmonic content of the output voltages and currents, as described in Appendix B.

A cutaway diagram of a complete large synchronous machine is shown in Figure 5–6. This drawing shows an eight-pole salient-pole rotor, a stator with distributed double-layer windings, and a brushless exciter.

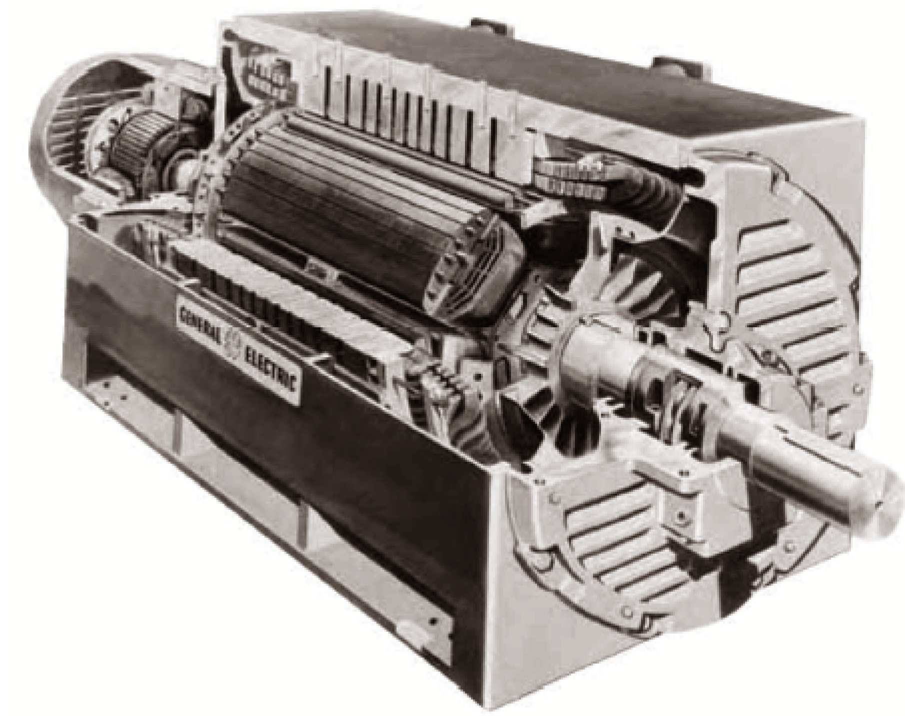


FIGURE 5–6
A cutaway diagram of a large synchronous machine. Note the salient-pole construction and the on-shaft exciter. (*Courtesy of General Electric Company.*)

5.2 THE SPEED OF ROTATION OF A SYNCHRONOUS GENERATOR

Synchronous generators are by definition *synchronous*, meaning that the electrical frequency produced is locked in or synchronized with the mechanical rate of rotation of the generator. A synchronous generator's rotor consists of an electromagnet to which direct current is supplied. The rotor's magnetic field points in whatever direction the rotor is turned. Now, the rate of rotation of the magnetic fields in the machine is related to the stator electrical frequency by Equation (4–34):

$$\boxed{f_e = \frac{n_m P}{120}} \tag{4–34}$$

where f_e = electrical frequency, in Hz

n_m = mechanical speed of magnetic field, in r/min (equals speed of rotor for synchronous machines)

P = number of poles

Since the rotor turns at the same speed as the magnetic field, *this equation relates the speed of rotor rotation to the resulting electrical frequency*. Electric power is generated at 50 or 60 Hz, so the generator must turn at a fixed speed depending on the number of poles on the machine. For example, to generate 60-Hz power in a two-pole machine, the rotor *must* turn at 3600 r/min. To generate 50-Hz power in a four-pole machine, the rotor *must* turn at 1500 r/min. The required rate of rotation for a given frequency can always be calculated from Equation (4–34).

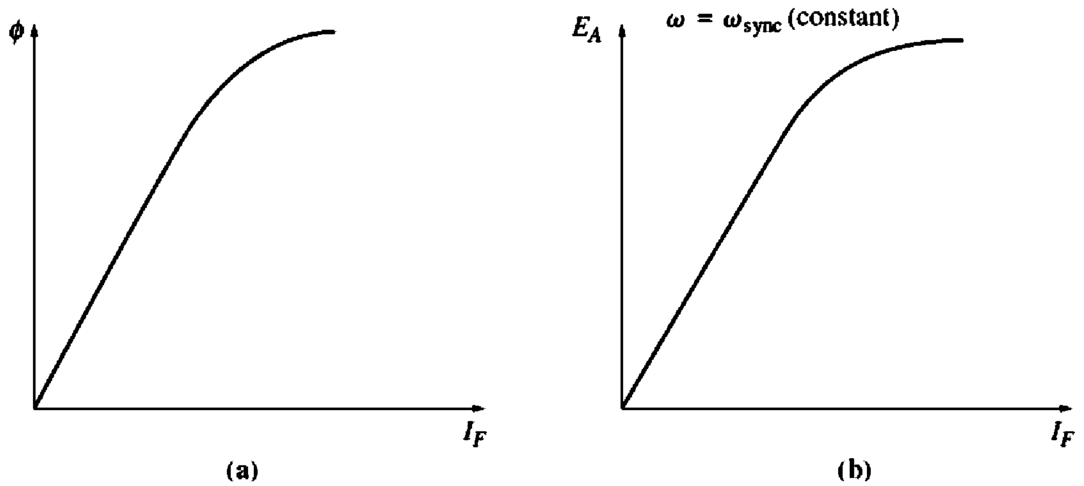


FIGURE 5–7
(a) Plot of flux versus field current for a synchronous generator. (b) The magnetization curve for the synchronous generator.

5.3 THE INTERNAL GENERATED VOLTAGE OF A SYNCHRONOUS GENERATOR

In Chapter 4, the magnitude of the voltage induced in a given stator phase was found to be

$$E_A = \sqrt{2}\pi N_C \phi f \tag{4–50}$$

This voltage depends on the flux ϕ in the machine, the frequency or speed of rotation, and the machine's construction. In solving problems with synchronous machines, this equation is sometimes rewritten in a simpler form that emphasizes the quantities that are variable during machine operation. This simpler form is

$$\boxed{E_A = K\phi\omega} \tag{5–1}$$

where K is a constant representing the construction of the machine. If ω is expressed in *electrical* radians per second, then

$$K = \frac{N_c}{\sqrt{2}} \tag{5–2}$$

while if ω is expressed in *mechanical* radians per second, then

$$K = \frac{N_c P}{\sqrt{2}} \tag{5–3}$$

The internal generated voltage E_A is directly proportional to the flux and to the speed, but the flux itself depends on the current flowing in the rotor field circuit. The field circuit I_F is related to the flux ϕ in the manner shown in Figure 5–7a. Since E_A is directly proportional to the flux, the internal generated voltage E_A is related to the field current as shown in Figure 5–7b. This plot is called the *magnetization curve* or the *open-circuit characteristic* of the machine.

5.4 THE EQUIVALENT CIRCUIT OF A SYNCHRONOUS GENERATOR

The voltage $\mathbf{E}_A$ is the internal generated voltage produced in one phase of a synchronous generator. However, this voltage $\mathbf{E}_A$ is *not* usually the voltage that appears at the terminals of the generator. In fact, the only time the internal voltage $\mathbf{E}_A$ is the same as the output voltage $\mathbf{V}_\phi$ of a phase is when there is no armature current flowing in the machine. Why is the output voltage $\mathbf{V}_\phi$ from a phase not equal to $\mathbf{E}_A$, and what is the relationship between the two voltages? The answer to these questions yields the model of a synchronous generator.

There are a number of factors that cause the difference between $\mathbf{E}_A$ and $\mathbf{V}_\phi$:

1. The distortion of the air-gap magnetic field by the current flowing in the stator, called *armature reaction.*
2. The self-inductance of the armature coils.
3. The resistance of the armature coils.
4. The effect of salient-pole rotor shapes.

We will explore the effects of the first three factors and derive a machine model from them. In this chapter, the effects of a salient-pole shape on the operation of a synchronous machine will be ignored; in other words, all the machines in this chapter are assumed to have nonsalient or cylindrical rotors. Making this assumption will cause the calculated answers to be slightly inaccurate if a machine does indeed have salient-pole rotors, but the errors are relatively minor. A discussion of the effects of rotor pole saliency is included in Appendix C.

The first effect mentioned, and normally the largest one, is armature reaction. When a synchronous generator's rotor is spun, a voltage $\mathbf{E}_A$ is induced in the generator's stator windings. If a load is attached to the terminals of the generator, a current flows. But a three-phase stator current flow will produce a magnetic field of its own in the machine. This *stator* magnetic field distorts the original rotor magnetic field, changing the resulting phase voltage. This effect is called *armature reaction* because the armature (stator) current affects the magnetic field which produced it in the first place.

To understand armature reaction, refer to Figure 5–8. Figure 5–8a shows a two-pole rotor spinning inside a three-phase stator. There is no load connected to the stator. The rotor magnetic field $\mathbf{B}_R$ produces an internal generated voltage $\mathbf{E}_A$ whose peak value coincides with the direction of $\mathbf{B}_R$. As was shown in the last chapter, the voltage will be positive out of the conductors at the top and negative into the conductors at the bottom of the figure. With no load on the generator, there is no armature current flow, and $\mathbf{E}_A$ will be equal to the phase voltage $\mathbf{V}_\phi$.

Now suppose that the generator is connected to a lagging load. Because the load is lagging, the peak current will occur at an angle *behind* the peak voltage. This effect is shown in Figure 5–8b.

The current flowing in the stator windings produces a magnetic field of its own. This stator magnetic field is called $\mathbf{B}_S$ and its direction is given by the right-

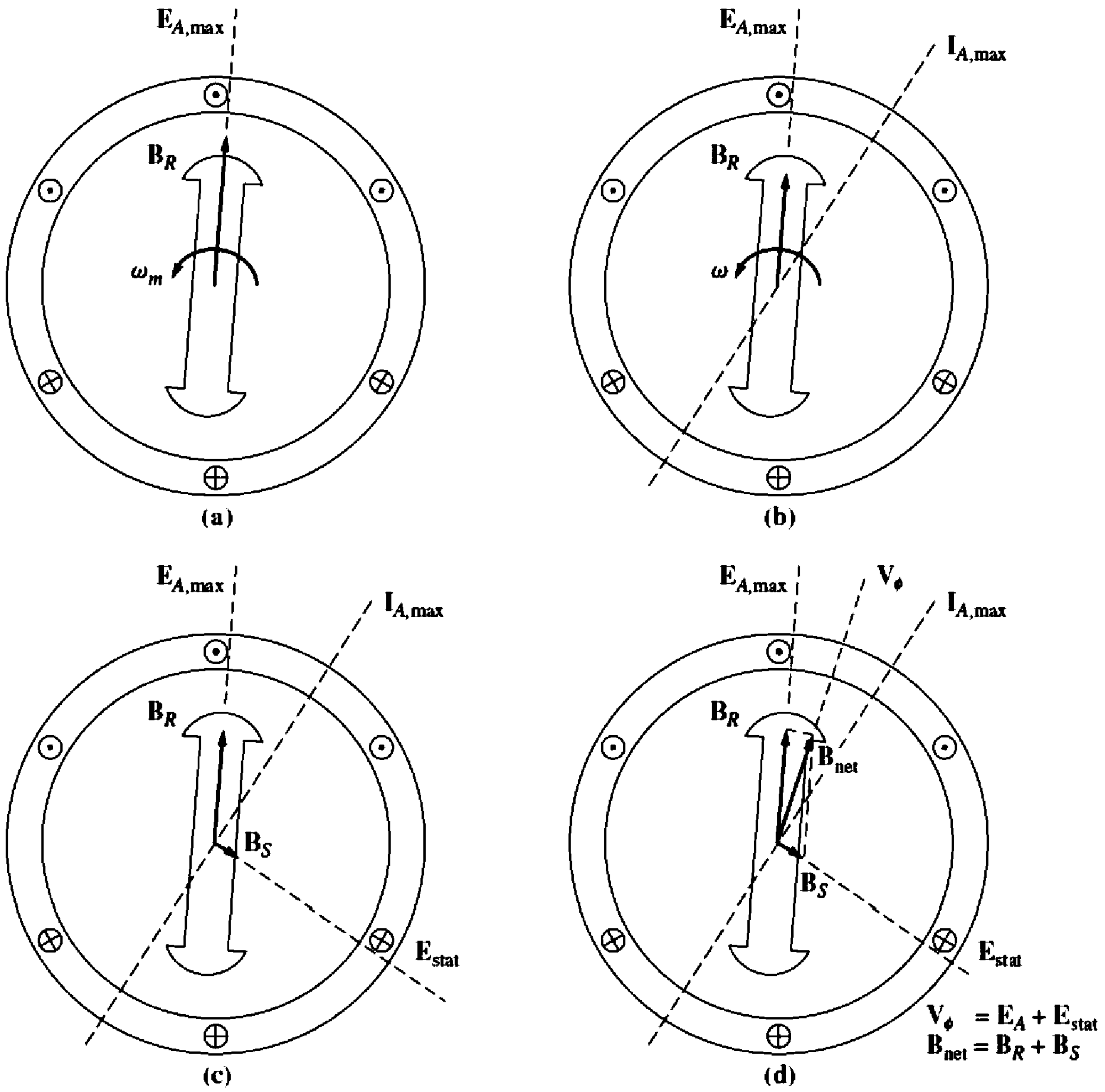


FIGURE 5–8
The development of a model for armature reaction: (a) A rotating magnetic field produces the internal generated voltage $\mathbf{E}_A$. (b) The resulting voltage produces a lagging *current flow* when connected to a lagging load. (c) The stator current produces its own magnetic field $\mathbf{B}_S$, which produces its own voltage $\mathbf{E}_{stat}$ in the stator windings of the machine. (d) The field $\mathbf{B}_S$ adds to $\mathbf{B}_R$, distorting it into $\mathbf{B}_{net}$. The voltage $\mathbf{E}_{stat}$ adds to $\mathbf{E}_A$, producing $\mathbf{V}_\phi$ at the output of the phase.

hand rule to be as shown in Figure 5–8c. The stator magnetic field $\mathbf{B}_S$ produces a voltage of its own in the stator, and this voltage is called $\mathbf{E}_{stat}$ on the figure.

With two voltages present in the stator windings, the total voltage in a phase is just the *sum* of the internal generated voltage $\mathbf{E}_A$ and the armature reaction voltage $\mathbf{E}_{stat}$:

$$\mathbf{V}_\phi = \mathbf{E}_A + \mathbf{E}_{stat} \tag{5–4}$$

The net magnetic field $\mathbf{B}_{net}$ is just the sum of the rotor and stator magnetic fields:

$$\mathbf{B}_{net} = \mathbf{B}_R + \mathbf{B}_S \tag{5–5}$$

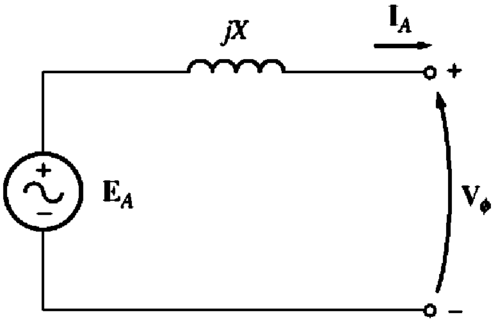


FIGURE 5–9
A simple circuit (see text).

Since the angles of $\mathbf{E}_A$ and $\mathbf{B}_R$ are the same and the angles of $\mathbf{E}_{\text{stat}}$ and $\mathbf{B}_S$, are the same, the resulting magnetic field $\mathbf{B}_{\text{net}}$ will coincide with the net voltage $\mathbf{V}_\phi$. The resulting voltages and currents are shown in Figure 5–8d.

How can the effects of armature reaction on the phase voltage be modeled? First, note that the voltage $\mathbf{E}_{\text{stat}}$ lies at an angle of 90° behind the plane of maximum current $\mathbf{I}_A$. Second, the voltage $\mathbf{E}_{\text{stat}}$ is directly proportional to the current $\mathbf{I}_A$. If X is a constant of proportionality, then *the armature reaction voltage can be expressed as*

$$\mathbf{E}_{\text{stat}} = -jX\mathbf{I}_A \tag{5–6}$$

The voltage on a phase is thus

$$\boxed{\mathbf{V}_\phi = \mathbf{E}_A - jX\mathbf{I}_A} \tag{5–7}$$

Look at the circuit shown in Figure 5–9. The Kirchhoff's voltage law equation for this circuit is

$$\mathbf{V}_\phi = \mathbf{E}_A - jX\mathbf{I}_A \tag{5–8}$$

This is exactly the same equation as the one describing the armature reaction voltage. Therefore, the armature reaction voltage can be modeled as an inductor in series with the internal generated voltage.

In addition to the effects of armature reaction, the stator coils have a self-inductance and a resistance. If the stator self-inductance is called L_A (and its corresponding reactance is called X_A) while the stator resistance is called R_A, then the total difference between $\mathbf{E}_A$ and $\mathbf{V}_\phi$ is given by

$$\mathbf{V}_\phi = \mathbf{E}_A - jX\mathbf{I}_A - jX_A\mathbf{I}_A - R_A\mathbf{I}_A \tag{5–9}$$

The armature reaction effects and the self-inductance in the machine are both represented by reactances, and it is customary to combine them into a single reactance, called the *synchronous reactance* of the machine:

$$X_S = X + X_A \tag{5–10}$$

Therefore, the final equation describing $\mathbf{V}_\phi$ is

$$\boxed{\mathbf{V}_\phi = \mathbf{E}_A - jX_S\mathbf{I}_A - R_A\mathbf{I}_A} \tag{5–11}$$

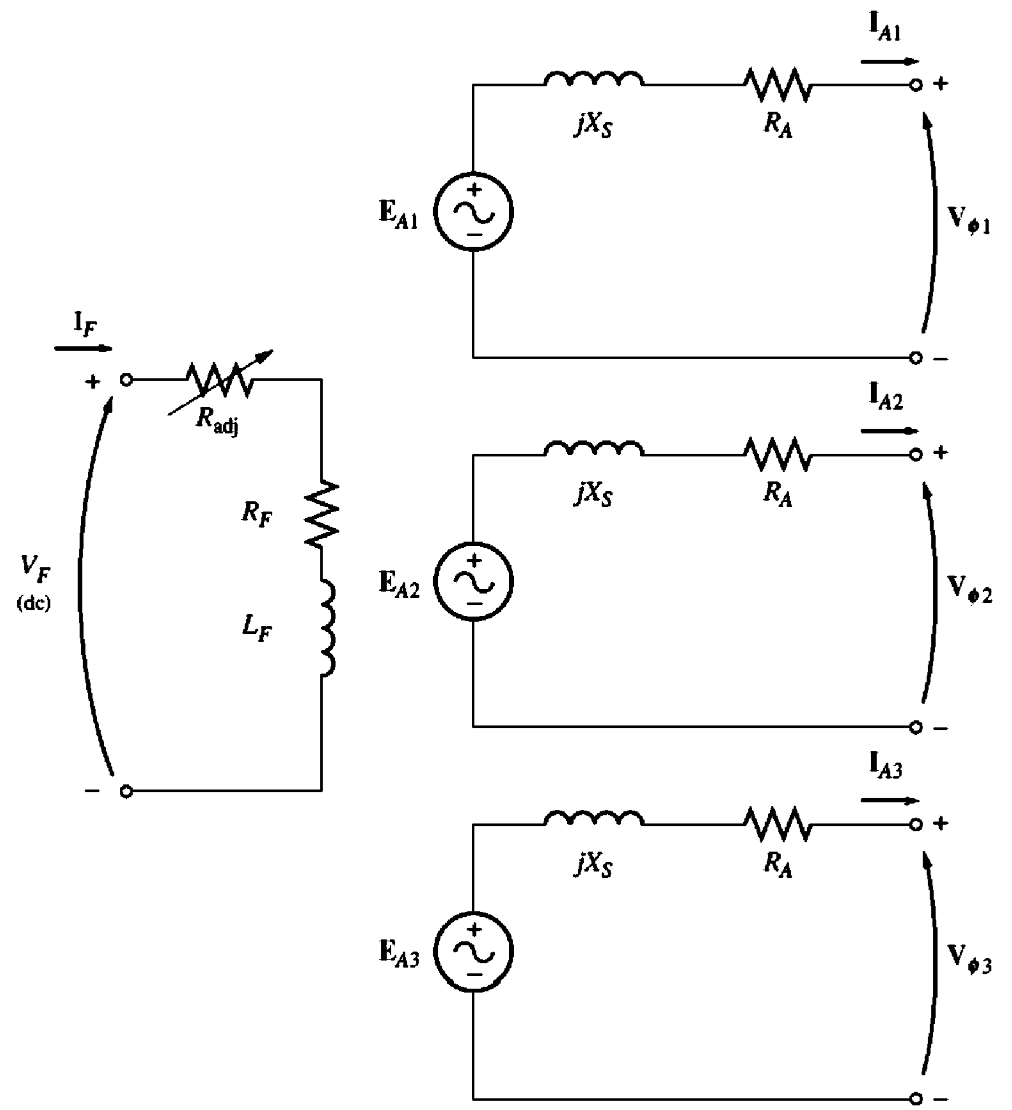


FIGURE 5-10
The full equivalent circuit of a three-phase synchronous generator.

It is now possible to sketch the equivalent circuit of a three-phase synchronous generator. The full equivalent circuit of such a generator is shown in Figure 5–10. This figure shows a dc power source supplying the rotor field circuit, which is modeled by the coil's inductance and resistance in series. In series with R_F is an adjustable resistor R_{adj} which controls the flow of field current. The rest of the equivalent circuit consists of the models for each phase. Each phase has an internal generated voltage with a series inductance X_S (consisting of the sum of the armature reactance and the coil's self-inductance) and a series resistance R_A. The voltages and currents of the three phases are 120° apart in angle, but otherwise the three phases are identical.

These three phases can be either Y- or Δ-connected as shown in Figure 5–11. If they are Y-connected, then the terminal voltage V_T is related to the phase voltage by

$$V_T = \sqrt{3}V_\phi \tag{5–12}$$

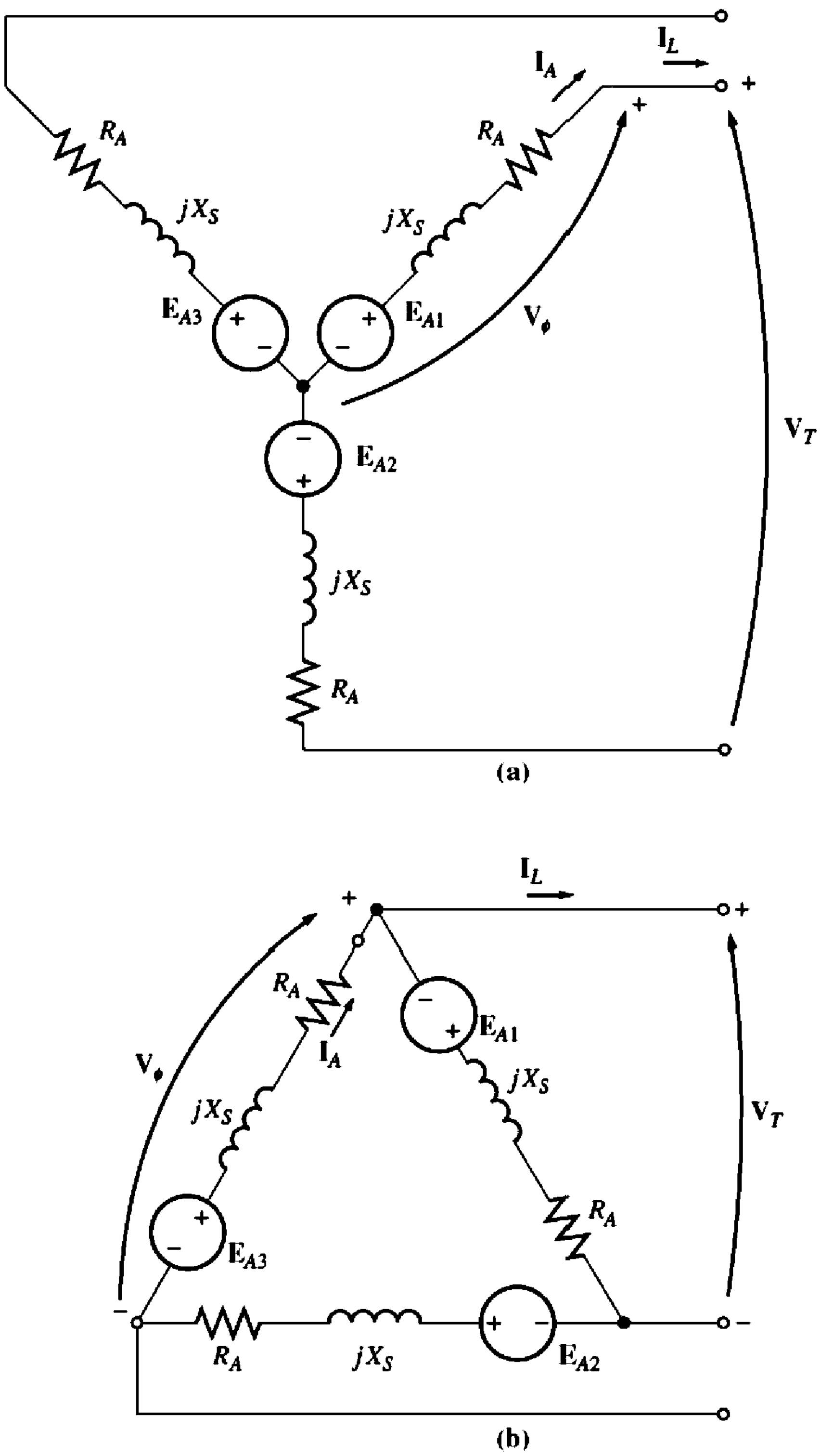


FIGURE 5–11
The generator equivalent circuit connected in (a) Y and (b) Δ.

If they are Δ-connected, then

$$V_T = V_\phi \qquad (5\text{–}13)$$

The fact that the three phases of a synchronous generator are identical in all respects except for phase angle normally leads to the use of a *per-phase equivalent circuit*. The per-phase equivalent circuit of this machine is shown in Fig-

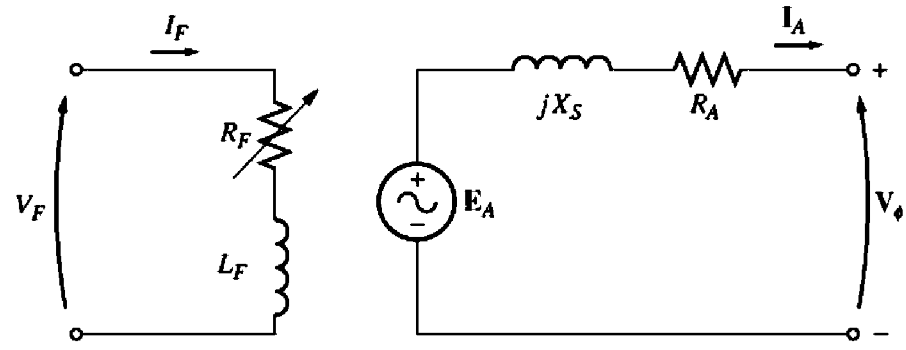


FIGURE 5–12
The per-phase equivalent circuit of a synchronous generator. The internal field circuit resistance and the external variable resistance have been combined into a single resistor R_F.

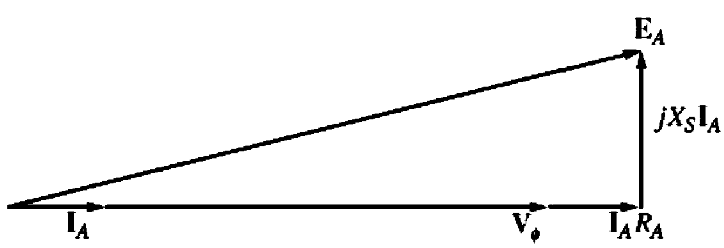


FIGURE 5–13
The phasor diagram of a synchronous generator at unity power factor.

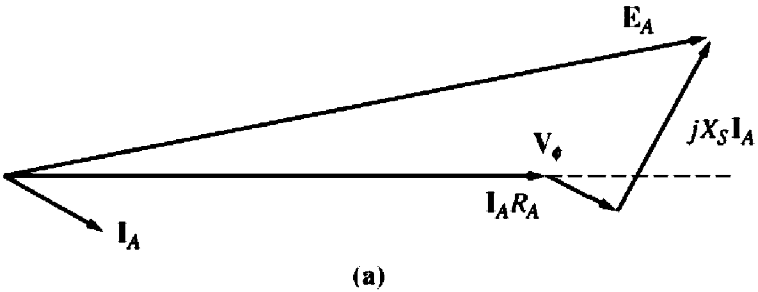
ure 5–12. One important fact must be kept in mind when the per-phase equivalent circuit is used: The three phases have the same voltages and currents *only* when the loads attached to them are *balanced*. If the generator's loads are not balanced, more sophisticated techniques of analysis are required. These techniques are beyond the scope of this book.

5.5 THE PHASOR DIAGRAM OF A SYNCHRONOUS GENERATOR

Because the voltages in a synchronous generator are ac voltages, they are usually expressed as phasors. Since phasors have both a magnitude and an angle, the relationship between them must be expressed by a two-dimensional plot. When the voltages within a phase ($\mathbf{E}_A$, $\mathbf{V}_\phi$, $jX_S\mathbf{I}_A$, and $R_A\mathbf{I}_A$) and the current $\mathbf{I}_A$ in the phase are plotted in such a fashion as to show the relationships among them, the resulting plot is called a *phasor diagram*.

For example, Figure 5–13 shows these relationships when the generator is supplying a load at unity power factor (a purely resistive load). From Equation (5–11), the total voltage $\mathbf{E}_A$ differs from the terminal voltage of the phase $\mathbf{V}_\phi$ by the resistive and inductive voltage drops. All voltages and currents are referenced to $\mathbf{V}_\phi$, which is arbitrarily assumed to be at an angle of 0°.

This phasor diagram can be compared to the phasor diagrams of generators operating at lagging and leading power factors. These phasor diagrams are shown



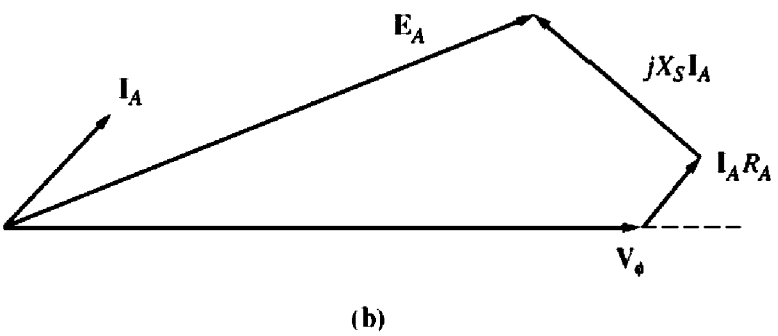


FIGURE 5–14
The phasor diagram of a synchronous generator at (a) lagging and (b) leading power factor.

in Figure 5–14. Notice that, *for a given phase voltage and armature current,* a larger internal generated voltage $\mathbf{E}_A$ is needed for lagging loads than for leading loads. Therefore, a larger field current is needed with lagging loads to get the same terminal voltage, because

$$E_A = K\phi\omega \tag{5–1}$$

and ω must be constant to keep a constant frequency.

Alternatively, *for a given field current and magnitude of load current, the terminal voltage is lower for lagging loads and higher for leading loads.*

In real synchronous machines, the synchronous reactance is normally much larger than the winding resistance R_A, so R_A is often neglected in the *qualitative* study of voltage variations. For accurate numerical results, R_A must of course be considered.

5.6 POWER AND TORQUE IN SYNCHRONOUS GENERATORS

A synchronous generator is a synchronous machine used as a generator. It converts mechanical power to three-phase electrical power. The source of mechanical power, the *prime mover,* may be a diesel engine, a steam turbine, a water turbine, or any similar device. Whatever the source, it must have the basic property that its speed is almost constant regardless of the power demand. If that were not so, then the resulting power system's frequency would wander.

Not all the mechanical power going into a synchronous generator becomes electrical power out of the machine. The difference between input power and output power represents the losses of the machine. A power-flow diagram for a synchro-

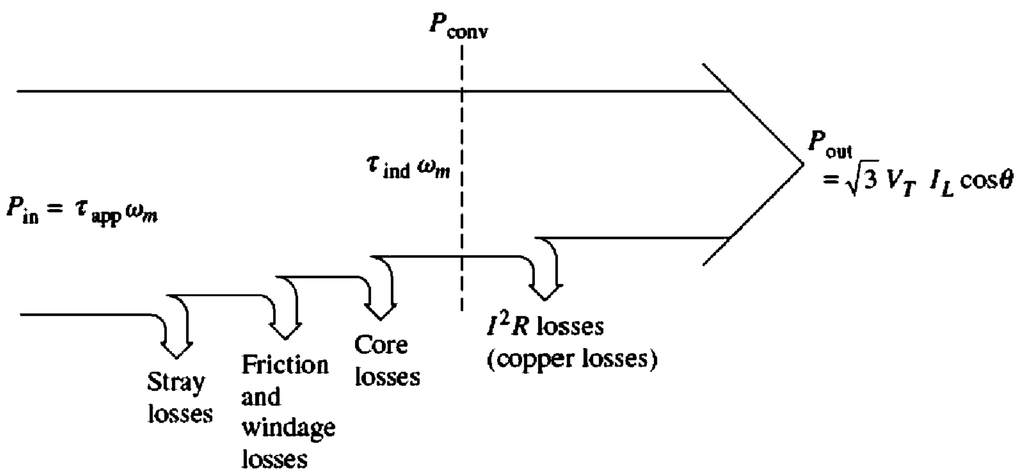


FIGURE 5–15
The power-flow diagram of a synchronous generator.

nous generator is shown in Figure 5–15. The input mechanical power is the shaft power in the generator $P_{in} = \tau_{app}\omega_m$, while the power converted from mechanical to electrical form internally is given by

$$P_{conv} = \tau_{ind}\omega_m \tag{5–14}$$

$$= 3E_A I_A \cos\gamma \tag{5–15}$$

where γ is the angle between $\mathbf{E}_A$ and $\mathbf{I}_A$. The difference between the input power to the generator and the power converted in the generator represents the mechanical, core, and stray losses of the machine.

The real electrical output power of the synchronous generator can be expressed in line quantities as

$$P_{out} = \sqrt{3}V_T I_L \cos\theta \tag{5–16}$$

and in phase quantities as

$$P_{out} = 3V_\phi I_A \cos\theta \tag{5–17}$$

The reactive power output can be expressed in line quantities as

$$Q_{out} = \sqrt{3}V_T I_L \sin\theta \tag{5–18}$$

or in phase quantities as

$$Q_{out} = 3V_\phi I_A \sin\theta \tag{5–19}$$

If the armature resistance R_A is ignored (since $X_S >> R_A$), then a very useful equation can be derived to approximate the output power of the generator. To derive this equation, examine the phasor diagram in Figure 5–16. Figure 5–16 shows a simplified phasor diagram of a generator with the stator resistance ignored. Notice that the vertical segment bc can be expressed as either $E_A \sin\delta$ or $X_S I_A \cos\theta$. Therefore,

$$I_A \cos\theta = \frac{E_A \sin\delta}{X_S}$$

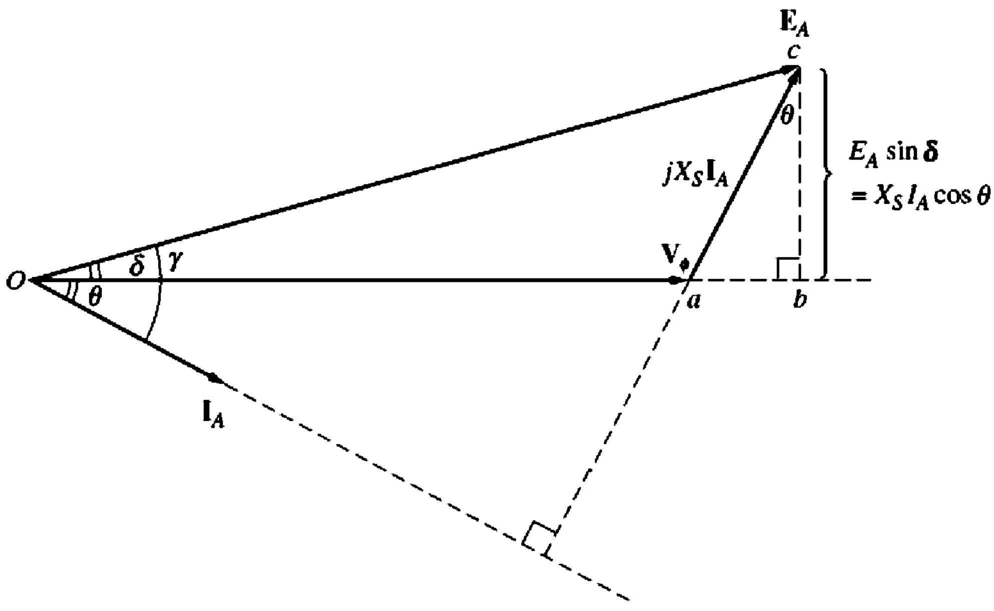


FIGURE 5–16
Simplified phasor diagram with armature resistance ignored.

and substituting this expression into Equation (5–17) gives

$$\boxed{P = \frac{3V_\phi E_A \sin\delta}{X_S}} \tag{5–20}$$

Since the resistances are assumed to be zero in Equation (5–20), there are no electrical losses in this generator, and this equation is both P_{conv} and P_{out}.

Equation (5–20) shows that the power produced by a synchronous generator depends on the angle δ between $\mathbf{V}_\phi$ and $\mathbf{E}_A$. The angle δ is known as the *torque angle* of the machine. Notice also that the maximum power that the generator can supply occurs when $\delta = 90°$. At $\delta = 90°$, $\sin\delta = 1$, and

$$P_{\max} = \frac{3V_\phi E_A}{X_S} \tag{5–21}$$

The maximum power indicated by this equation is called the *static stability limit* of the generator. Normally, real generators never even come close to that limit. Full-load torque angles of 15 to 20° are more typical of real machines.

Now take another look at Equations (5–17), (5–19), and (5–20). If V_ϕ is assumed constant, then the *real power output is directly proportional* to the quantities $I_A \cos\theta$ and $E_A \sin\delta$, and the reactive power output is directly proportional to the quantity $I_A \sin\theta$. These facts are useful in plotting phasor diagrams of synchronous generators as loads change.

From Chapter 4, the induced torque in this generator can be expressed as

$$\tau_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_S \tag{4–58}$$

or as

$$\tau_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_{\text{net}} \tag{4–60}$$

The magnitude of Equation (4–60) can be expressed as

$$\tau_{\text{ind}} = kB_R B_{\text{net}} \sin \delta \tag{4–61}$$

where δ is the angle between the rotor and net magnetic fields (the so-called *torque angle*). Since $\mathbf{B}_R$ produces the voltage $\mathbf{E}_A$ and $\mathbf{B}_{\text{net}}$ produces the voltage $\mathbf{V}_\phi$, the angle δ between $\mathbf{E}_A$ *and* $\mathbf{V}_\phi$ is the same as the angle δ between $\mathbf{B}_R$ and $\mathbf{B}_{\text{net}}$.

An alternative expression for the induced torque in a synchronous generator can be derived from Equation (5–20). Because $P_{\text{conv}} = \tau_{\text{ind}}\omega_m$, the induced torque can be expressed as

$$\boxed{\tau_{\text{ind}} = \frac{3V_\phi E_A \sin \delta}{\omega_m X_S}} \tag{5–22}$$

This expression describes the induced torque in terms of electrical quantities, whereas Equation (4–60) gives the same information in terms of magnetic quantities.

5.7 MEASURING SYNCHRONOUS GENERATOR MODEL PARAMETERS

The equivalent circuit of a synchronous generator that has been derived contains three quantities that must be determined in order to completely describe the behavior of a real synchronous generator:

1. The relationship between field current and flux (and therefore between the field current and E_A)
2. The synchronous reactance
3. The armature resistance

This section describes a simple technique for determining these quantities in a synchronous generator.

The first step in the process is to perform the *open-circuit test* on the generator. To perform this test, the generator is turned at the rated speed, the terminals are disconnected from all loads, and the field current is set to zero. Then the field current is gradually increased in steps, and the terminal voltage is measured at each step along the way. With the terminals open, $I_A = 0$, so E_A is equal to V_ϕ. It is thus possible to construct a plot of E_A or V_T versus I_F from this information. This plot is the so-called *open-circuit characteristic* (OCC) of a generator. With this characteristic, it is possible to find the internal generated voltage of the generator for any given field current. A typical open-circuit characteristic is shown in Figure 5–17a. Notice that at first the curve is almost perfectly linear, until some saturation is observed at high field currents. The unsaturated iron in the frame of the synchronous machine has a reluctance several thousand times lower than the air-gap reluctance, so at first almost *all* the magnetomotive force is across the air gap, and the resulting flux increase is linear. When the iron finally saturates, the reluctance of the iron

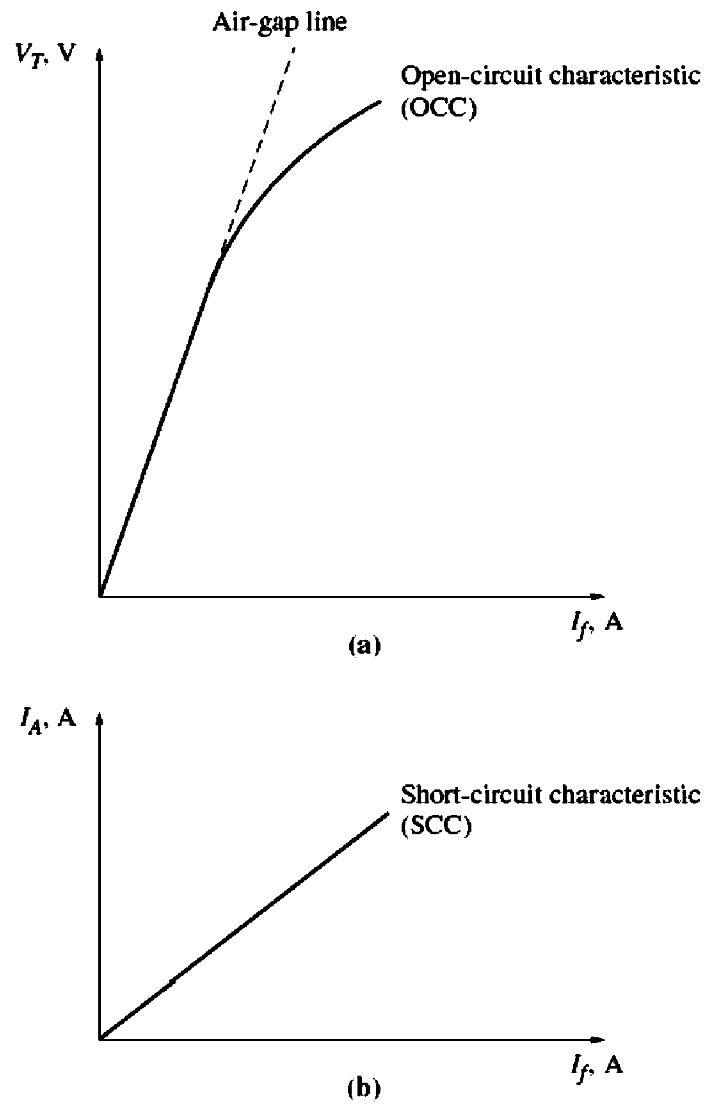


FIGURE 5–17
(a) The open-circuit characteristic (OCC) of a synchronous generator. (b) The short-circuit characteristic (SCC) of a synchronous generator.

increases dramatically, and the flux increases much more slowly with an increase in magnetomotive force. The linear portion of an OCC is called the *air-gap line* of the characteristic.

The second step in the process is to conduct the *short-circuit test*. To perform the short-circuit test, adjust the field current to zero again and short-circuit the terminals of the generator through a set of ammeters. Then the armature current I_A or the line current I_L is measured as the field current is increased. Such a plot is called a *short-circuit characteristic* (SCC) and is shown in Figure 5–17b. It is essentially a straight line. To understand why this characteristic is a straight line, look at the equivalent circuit in Figure 5–12 when the terminals of the machine are short-circuited. Such a circuit is shown in Figure 5–18a. Notice that when the terminals are short-circuited, the armature current $\mathbf{I}_A$ is given by

$$\mathbf{I}_A = \frac{\mathbf{E}_A}{R_A + jX_S} \tag{5–23}$$

and its magnitude is just given by

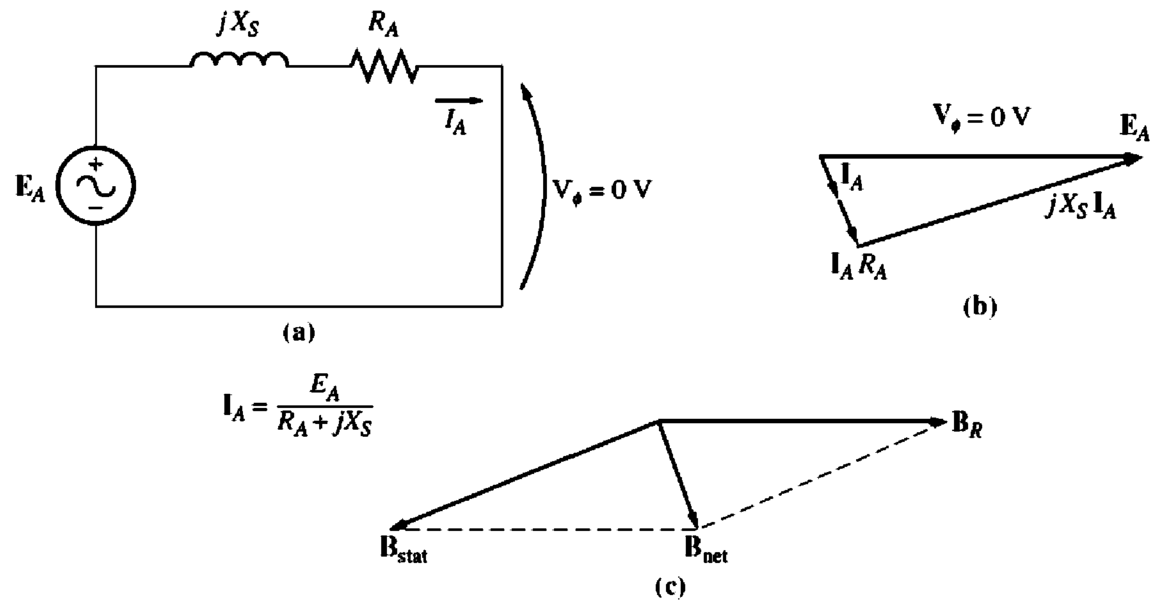


FIGURE 5–18
(a) The equivalent circuit of a synchronous generator during the short-circuit test. (b) The resulting phasor diagram. (c) The magnetic fields during the short-circuit test.

$$I_A = \frac{E_A}{\sqrt{R_A^2 + X_S^2}} \tag{5–24}$$

The resulting phasor diagram is shown in Figure 5–18b, and the corresponding magnetic fields are shown in Figure 5–18c. Since $\mathbf{B}_S$ almost cancels $\mathbf{B}_R$, the net magnetic field $\mathbf{B}_{net}$ is *very* small (corresponding to internal resistive and inductive drops only). Since the net magnetic field in the machine is so small, the machine is unsaturated and the SCC is linear.

To understand what information these two characteristics yield, notice that, with $\mathbf{V}_\phi$ equal to zero in Figure 5–18, the *internal machine impedance* is given by

$$Z_S = \sqrt{R_A^2 + X_S^2} = \frac{E_A}{I_A} \tag{5–25}$$

Since $X_S >> R_A$, this equation reduces to

$$X_S \approx \frac{E_A}{I_A} = \frac{V_{\phi,oc}}{I_A} \tag{5–26}$$

If E_A and I_A are known for a given situation, then the synchronous reactance X_S can be found.

Therefore, an *approximate* method for determining the synchronous reactance X_S at a given field current is

1. Get the internal generated voltage E_A from the OCC at that field current.
2. Get the short-circuit current flow $I_{A,SC}$ at that field current from the SCC.
3. Find X_S by applying Equation (5–26).

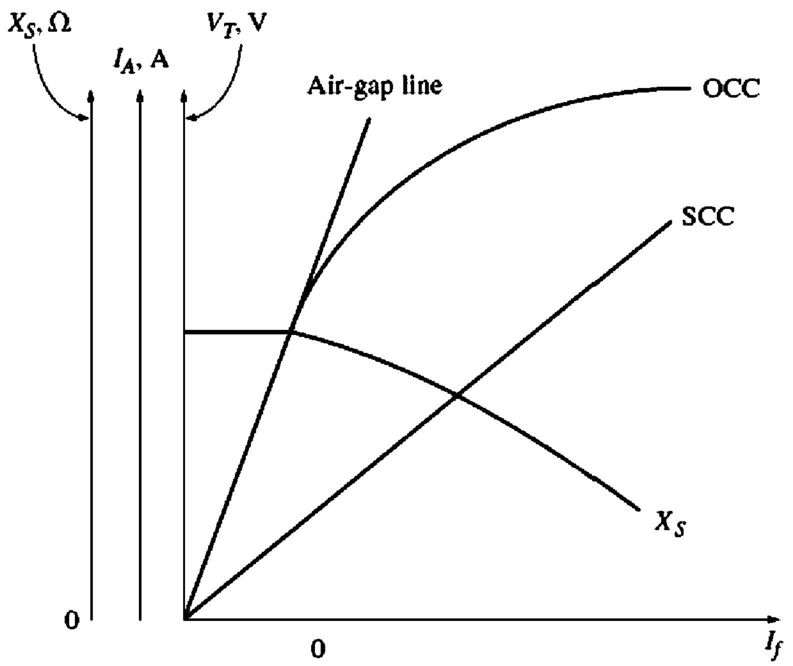


FIGURE 5–19
A sketch of the approximate synchronous reactance of a synchronous generator as a function of the field current in the machine. The constant value of reactance found at low values of field current is the *unsaturated* synchronous reactance of the machine.

There is a problem with this approach, however. The internal generated voltage E_A comes from the OCC, where the machine is partially *saturated* for large field currents, while I_A is taken from the SCC, where the machine is *unsaturated* at all field currents. Therefore, at higher field currents, the E_A taken from the OCC at a given field current is *not* the same as the E_A at the same field current under short-circuit conditions, and this difference makes the resulting value of X_S only approximate.

However, the answer given by this approach *is* accurate up to the point of saturation, so the *unsaturated synchronous reactance* $X_{S,u}$ of the machine can be found simply by applying Equation (5–26) at any field current in the linear portion (on the air-gap line) of the OCC curve.

The approximate value of synchronous reactance varies with the degree of saturation of the OCC, so the value of the synchronous reactance to be used in a given problem should be one calculated at the approximate load on the machine. A plot of approximate synchronous reactance as a function of field current is shown in Figure 5–19.

To get a more accurate estimation of the saturated synchronous reactance, refer to Section 5–3 of Reference 2.

If it is important to know a winding's resistance as well as its synchronous reactance, the resistance can be approximated by applying a dc voltage to the windings while the machine is stationary and measuring the resulting current flow. The use of dc voltage means that the reactance of the windings will be zero during the measurement process.

This technique is not perfectly accurate, since the ac resistance will be slightly larger than the dc resistance (as a result of the skin effect at higher frequencies). The measured value of the resistance can even be plugged into Equation (5–26) to improve the estimate of X_S, if desired. (Such an improvement is not much help in the approximate approach—saturation causes a much larger error in the X_S calculation than ignoring R_A does.)

The Short-Circuit Ratio

Another parameter used to describe synchronous generators is the short-circuit ratio. The *short-circuit ratio* of a generator is defined as the ratio of the *field current required for the rated voltage at open circuit* to the *field current required for the rated armature current at short circuit.* It can be shown that this quantity is just the reciprocal of the per-unit value of the approximate saturated synchronous reactance calculated by Equation (5–26).

Although the short-circuit ratio adds no new information about the generator that is not already known from the saturated synchronous reactance, it is important to know what it is, since the term is occasionally encountered in industry.

Example 5–1. A 200-kVA, 480-V, 50-Hz, Y-connected synchronous generator with a rated field current of 5 A was tested, and the following data were taken:

1. $V_{T,OC}$ at the rated I_F was measured to be 540 V.
2. $I_{L,SC}$ at the rated I_F was found to be 300 A.
3. When a dc voltage of 10 V was applied to two of the terminals, a current of 25 A was measured.

Find the values of the armature resistance and the approximate synchronous reactance in ohms that would be used in the generator model at the rated conditions.

Solution

The generator described above is Y-connected, so the direct current in the resistance test flows through two windings. Therefore, the resistance is given by

$$2R_A = \frac{V_{DC}}{I_{DC}}$$

$$R_A = \frac{V_{DC}}{2I_{DC}} = \frac{10\text{ V}}{(2)(25\text{ A})} = 0.2\ \Omega$$

The internal generated voltage at the rated field current is equal to

$$E_A = V_{\phi,OC} = \frac{V_T}{\sqrt{3}}$$

$$= \frac{540\text{ V}}{\sqrt{3}} = 311.8\text{ V}$$

The short-circuit current I_A is just equal to the line current, since the generator is Y-connected:

$$I_{A,SC} = I_{L,SC} = 300\text{ A}$$

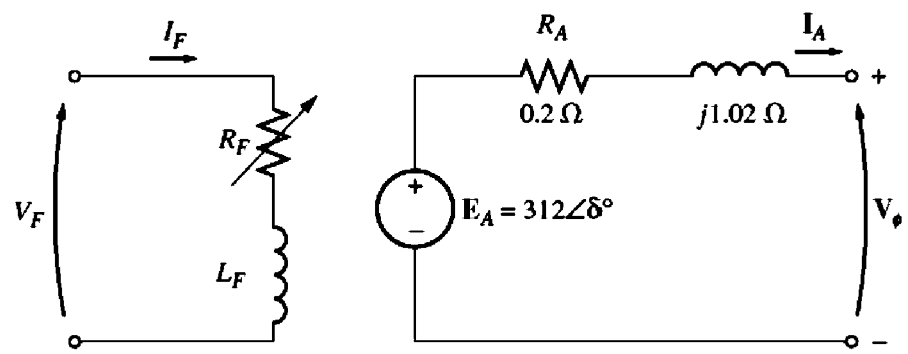


FIGURE 5–20
The per-phase equivalent circuit of the generator in Example 5–1.

Therefore, the synchronous reactance at the rated field current can be calculated from Equation (5–25):

$$\sqrt{R_A^2 + X_S^2} = \frac{E_A}{I_A} \tag{5–25}$$

$$\sqrt{(0.2\ \Omega)^2 + X_S^2} = \frac{311.8\ \text{V}}{300\ \text{A}}$$

$$\sqrt{(0.2\ \Omega)^2 + X_S^2} = 1.039\ \Omega$$

$$0.04 + X_S^2 = 1.08$$

$$X_S^2 = 1.04$$

$$X_S = 1.02\ \Omega$$

How much effect did the inclusion of R_A have on the estimate of X_S? Not much. If X_S is evaluated by Equation (5–26), the result is

$$X_S = \frac{E_A}{I_A} = \frac{311.8\ \text{V}}{300\ \text{A}} = 1.04\ \Omega$$

Since the error in X_S due to ignoring R_A is much less than the error due to saturation effects, approximate calculations are normally done with Equation (5–26).

The resulting per-phase equivalent circuit is shown in Figure 5–20.

5.8 THE SYNCHRONOUS GENERATOR OPERATING ALONE

The behavior of a synchronous generator under load varies greatly depending on the power factor of the load and on whether the generator is operating alone or in parallel with other synchronous generators. In this section, we will study the behavior of synchronous generators operating alone. We will study the behavior of synchronous generators operating in parallel in Section 5.9.

Throughout this section, concepts will be illustrated with simplified phasor diagrams ignoring the effect of R_A. In some of the numerical examples the resistance R_A will be included.

Unless otherwise stated in this section, the speed of the generators will be assumed constant, and all terminal characteristics are drawn assuming constant

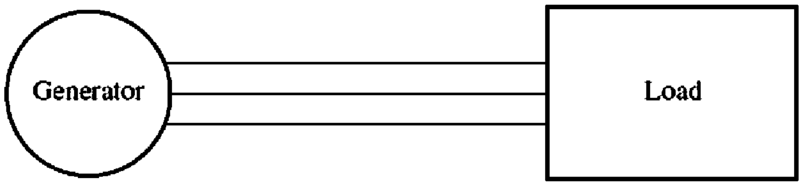


FIGURE 5–21
A single generator supplying a load.

speed. Also, the rotor flux in the generators is assumed constant unless their field current is explicitly changed.

The Effect of Load Changes on a Synchronous Generator Operating Alone

To understand the operating characteristics of a synchronous generator operating alone, examine a generator supplying a load. A diagram of a single generator supplying a load is shown in Figure 5–21. What happens when we increase the load on this generator?

An increase in the load is an increase in the real and/or reactive power drawn from the generator. Such a load increase increases the load current drawn from the generator. Because the field resistor has not been changed, the field current is constant, and therefore the flux ϕ is constant. Since the prime mover also keeps a constant speed ω, the *magnitude of the internal generated voltage* $E_A = K\phi\omega$ *is constant.*

If E_A is constant, just what does vary with a changing load? The way to find out is to construct phasor diagrams showing an increase in the load, keeping the constraints on the generator in mind.

First, examine a generator operating at a lagging power factor. If more load is added at the *same power factor*, then $|\mathbf{I}_A|$ increases but remains at the same angle θ with respect to $\mathbf{V}_\phi$ as before. Therefore, the armature reaction voltage $jX_S\mathbf{I}_A$ is larger than before but at the same angle. Now since

$$\mathbf{E}_A = \mathbf{V}_\phi + jX_S\mathbf{I}_A$$

$jX_S\mathbf{I}_A$ must stretch between $\mathbf{V}_\phi$ at an angle of 0° and $\mathbf{E}_A$, which is constrained to be of the same magnitude as before the load increase. If these constraints are plotted on a phasor diagram, there is one and only one point at which the armature reaction voltage can be parallel to its original position while increasing in size. The resulting plot is shown in Figure 5–22a.

If the constraints are observed, then it is seen that as the load increases, the voltage $\mathbf{V}_\phi$ decreases rather sharply.

Now suppose the generator is loaded with unity-power-factor loads. What happens if new loads are added at the same power factor? With the same constraints as before, it can be seen that this time $\mathbf{V}_\phi$ decreases only slightly (see Figure 5–22b).

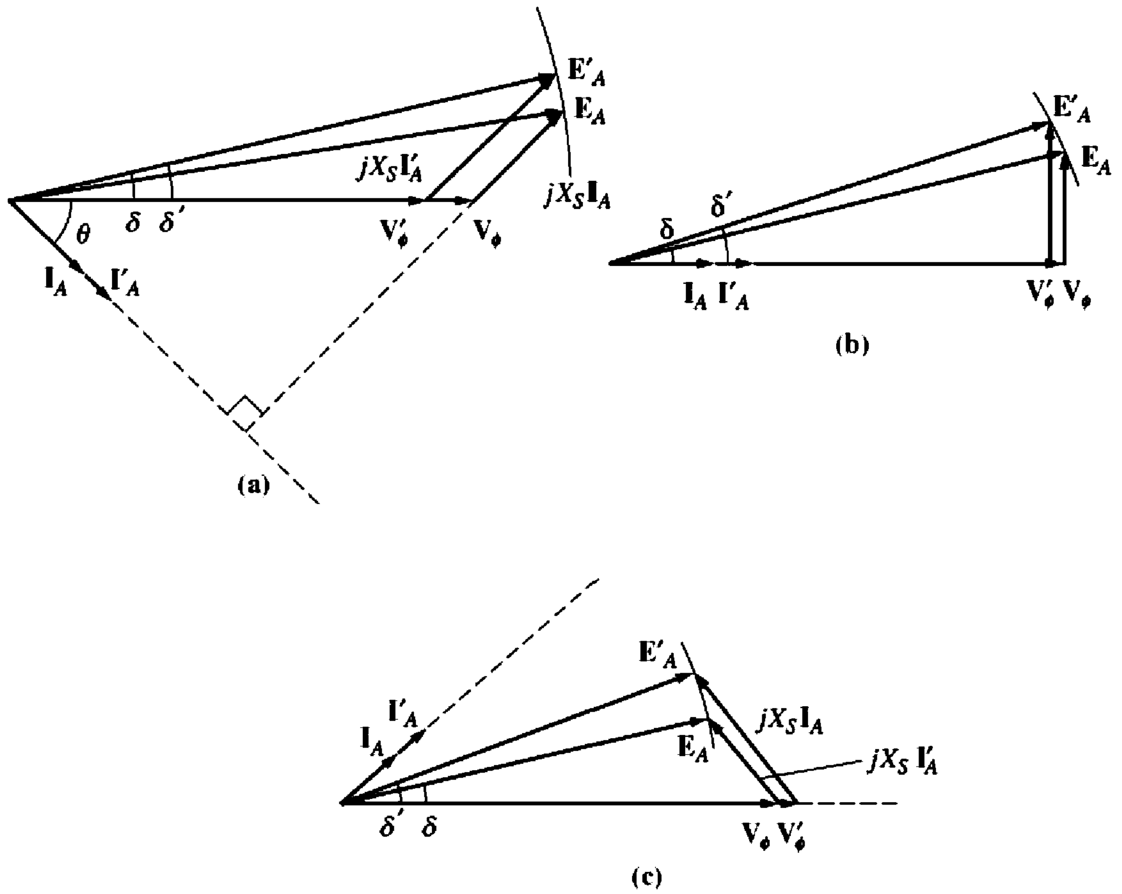


FIGURE 5–22
The effect of an increase in generator loads at constant power factor upon its terminal voltage. (a) Lagging power factor; (b) unity power factor; (c) leading power factor.

Finally, let the generator be loaded with leading-power-factor loads. If new loads are added at the same power factor this time, the armature reaction voltage lies outside its previous value, and $\mathbf{V}_\phi$ actually *rises* (see Figure 5–22c). In this last case, an increase in the load in the generator produced an increase in the terminal voltage. Such a result is not something one would expect on the basis of intuition alone.

General conclusions from this discussion of synchronous generator behavior are

1. If lagging loads ($+Q$ or inductive reactive power loads) are added to a generator, $\mathbf{V}_\phi$ and the terminal voltage V_T decrease significantly.
2. If unity-power-factor loads (no reactive power) are added to a generator, there is a slight decrease in $\mathbf{V}_\phi$ and the terminal voltage.
3. If leading loads ($-Q$ or capacitive reactive power loads) are added to a generator, $\mathbf{V}_\phi$ and the terminal voltage will rise.

A convenient way to compare the voltage behavior of two generators is by their *voltage regulation*. The voltage regulation (VR) of a generator is defined by the equation

$$\boxed{\mathrm{VR} = \frac{V_{\mathrm{nl}} - V_{\mathrm{fl}}}{V_{\mathrm{fl}}} \times 100\%} \tag{4–67}$$

where V_{nl} is the no-load voltage of the generator and V_{fl} is the full-load voltage of the generator. A synchronous generator operating at a lagging power factor has a fairly large positive voltage regulation, a synchronous generator operating at a unity power factor has a small positive voltage regulation, and a synchronous generator operating at a leading power factor often has a negative voltage regulation.

Normally, it is desirable to keep the voltage supplied to a load constant, even though the load itself varies. How can terminal voltage variations be corrected for? The obvious approach is to vary the magnitude of $\mathbf{E}_A$ to compensate for changes in the load. Recall that $E_A = K\phi\omega$. Since the frequency should not be changed in a normal system, E_A must be controlled by varying the flux in the machine.

For example, suppose that a lagging load is added to a generator. Then the terminal voltage will fall, as was previously shown. To restore it to its previous level, decrease the field resistor R_F. If R_F decreases, the field current will increase. An increase in I_F increases the flux, which in turn increases E_A, and an increase in E_A increases the phase and terminal voltage. This idea can be summarized as follows:

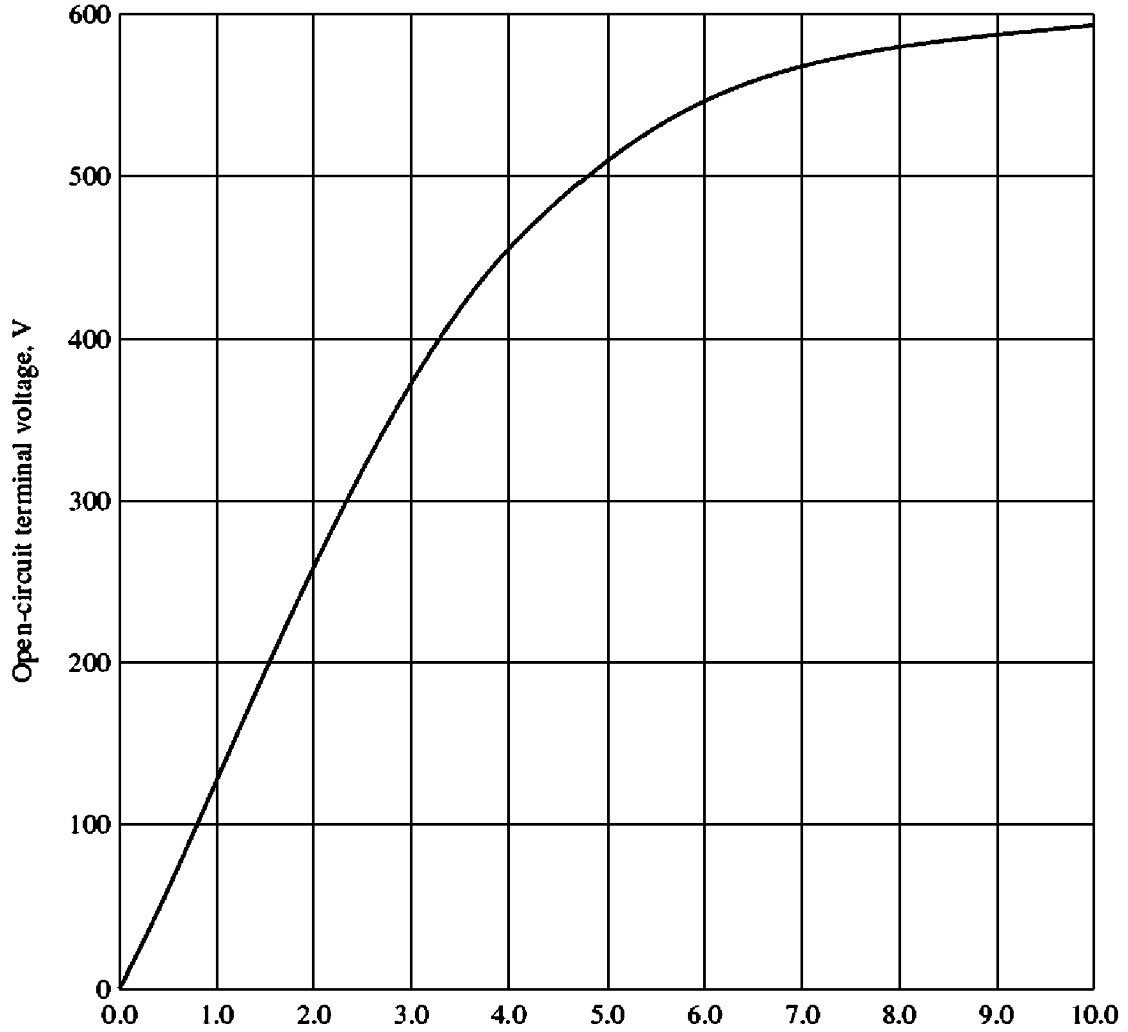
1. Decreasing the field resistance in the generator increases its field current.
2. An increase in the field current increases the flux in the machine.
3. An increase in the flux increases the internal generated voltage $E_A = K\phi\omega$.
4. An increase in E_A increases V_ϕ and the terminal voltage of the generator.

The process can be reversed to decrease the terminal voltage. It is possible to regulate the terminal voltage of a generator throughout a series of load changes simply by adjusting the field current.

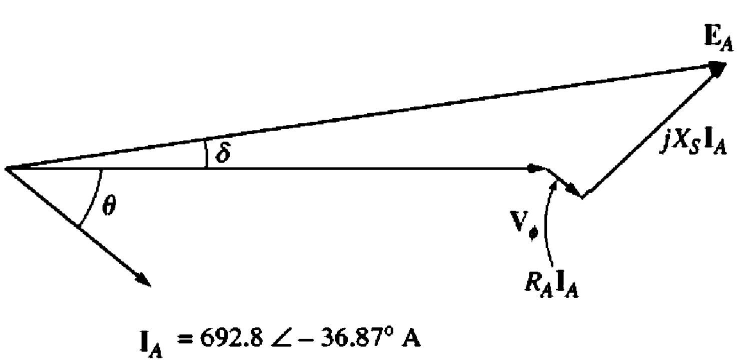
Example Problems

The following three problems illustrate simple calculations involving voltages, currents, and power flows in synchronous generators. The first problem is an example that includes the armature resistance in its calculations, while the next two ignore R_A. Part of the first example problem addresses the question: *How must a generator's field current be adjusted to keep V_T constant as the load changes?* On the other hand, part of the second example problem asks the question: *If the load changes and the field is left alone, what happens to the terminal voltage?* You should compare the calculated behavior of the generators in these two problems to see if it agrees with the qualitative arguments of this section. Finally, the third example illustrates the use of a MATLAB program to derive the terminal characteristics of synchronous generator.

Example 5–2. **A 480-V, 60-Hz, Δ-connected, four-pole synchronous generator has the OCC shown in Figure 5–23a. This generator has a synchronous reactance of 0.1 Ω and**



(a)



(b)

FIGURE 5–23
(a) Open-circuit characteristic of the generator in Example 5–2. (b) Phasor diagram of the generator in Example 5–2.

an armature resistance of 0.015 Ω. At full load, the machine supplies 1200 A at 0.8 PF lagging. Under full-load conditions, the friction and windage losses are 40 kW, and the core losses are 30 kW. Ignore any field circuit losses.

(a) What is the speed of rotation of this generator?
(b) How much field current must be supplied to the generator to make the terminal voltage 480 V at no load?
(c) If the generator is now connected to a load and the load draws 1200 A at 0.8 PF lagging, how much field current will be required to keep the terminal voltage equal to 480 V?
(d) How much power is the generator now supplying? How much power is supplied to the generator by the prime mover? What is this machine's overall efficiency?
(e) If the generator's load were suddenly disconnected from the line, what would happen to its terminal voltage?
(f) Finally, suppose that the generator is connected to a load drawing 1200 A at 0.8 PF *leading*. How much field current would be required to keep V_T at 480 V?

Solution

This synchronous generator is Δ-connected, so its phase voltage is equal to its line voltage $V_\phi = V_T$, while its phase current is related to its line current by the equation $I_L = \sqrt{3} I_\phi$.

(a) The relationship between the electrical frequency produced by a synchronous generator and the mechanical rate of shaft rotation is given by Equation (4–34):

$$f_e = \frac{n_m P}{120} \tag{4–34}$$

Therefore,

$$n_m = \frac{120 f_e}{P}$$

$$= \frac{120(60\text{ Hz})}{4\text{ poles}} = 1800\text{ r/min}$$

(b) In this machine, $V_T = V_\phi$. Since the generator is at no load, $\mathbf{I}_A = 0$ and $\mathbf{E}_A = \mathbf{V}_\phi$. Therefore, $V_T = V_\phi = E_A = 480$ V, and from the open-circuit characteristic, $I_F = 4.5$ A.

(c) If the generator is supplying 1200 A, then the armature current in the machine is

$$I_A = \frac{1200\text{ A}}{\sqrt{3}} = 692.8\text{ A}$$

The phasor diagram for this generator is shown in Figure 5–23b. If the terminal voltage is adjusted to be 480 V, the size of the internal generated voltage $\mathbf{E}_A$ is given by

$$\begin{aligned}
\mathbf{E}_A &= \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A \\
&= 480\angle 0^\circ\text{ V} + (0.015\ \Omega)(692.8\angle -36.87^\circ\text{ A}) + (j0.1\ \Omega)(692.8\angle -36.87^\circ\text{ A}) \\
&= 480\angle 0^\circ\text{ V} + 10.39\angle -36.87^\circ\text{ V} + 69.28\angle 53.13^\circ\text{ V} \\
&= 529.9 + j49.2\text{ V} = 532\angle 5.3^\circ\text{ V}
\end{aligned}$$

To keep the terminal voltage at 480 V, $\mathbf{E}_A$ must be adjusted to 532 V. From Figure 5–23, the required field current is 5.7 A.

(d) The power that the generator is now supplying can be found from Equation (5–16):

$$P_{\text{out}} = \sqrt{3} V_T I_L \cos\theta \tag{5–16}$$

$$= \sqrt{3}(480\text{ V})(1200\text{ A})\cos 36.87^\circ$$
$$= 798\text{ kW}$$

To determine the power input to the generator, use the power-flow diagram (Figure 5–15). From the power-flow diagram, the mechanical input power is given by

$$P_{\text{in}} = P_{\text{out}} + P_{\text{elec loss}} + P_{\text{core loss}} + P_{\text{mech loss}} + P_{\text{stray loss}}$$

The stray losses were not specified here, so they will be ignored. In this generator, the electrical losses are

$$P_{\text{elec loss}} = 3I_A^2R_A$$
$$= 3(692.8\text{ A})^2(0.015\ \Omega) = 21.6\text{ kW}$$

The core losses are 30 kW, and the friction and windage losses are 40 kW, so the total input power to the generator is

$$P_{\text{in}} = 798\text{ kW} + 21.6\text{ kW} + 30\text{ kW} + 40\text{ kW} = 889.6\text{ kW}$$

Therefore, the machine's overall efficiency is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{798\text{ kW}}{889.6\text{ kW}} \times 100\% = 89.75\%$$

(e) If the generator's load were suddenly disconnected from the line, the current $\mathbf{I}_A$ would drop to zero, making $\mathbf{E}_A = \mathbf{V}_\phi$. Since the field current has not changed, $|\mathbf{E}_A|$ has not changed and $\mathbf{V}_\phi$ and V_T must rise to equal $\mathbf{E}_A$. Therefore, if the load were suddenly dropped, the terminal voltage of the generator would rise to 532 V.

(f) If the generator were loaded down with 1200 A at 0.8 PF leading while the terminal voltage was 480 V, then the internal generated voltage would have to be

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$= 480\angle 0^\circ\text{ V} + (0.015\ \Omega)(692.8\angle 36.87^\circ\text{ A}) + (j0.1\ \Omega)(692.8\angle 36.87^\circ\text{ A})$$
$$= 480\angle 0^\circ\text{ V} + 10.39\angle 36.87^\circ\text{ V} + 69.28\angle 126.87^\circ\text{ V}$$
$$= 446.7 + j61.7\text{ V} = 451\angle 7.1^\circ\text{ V}$$

Therefore, the internal generated voltage E_A must be adjusted to provide 451 V if V_T is to remain 480 V. Using the open-circuit characteristic, the field current would have to be adjusted to 4.1 A.

Which type of load (leading or lagging) needed a larger field current to maintain the rated voltage? Which type of load (leading or lagging) placed more thermal stress on the generator? Why?

Example 5–3. A 480-V, 50-Hz, Y-connected, six-pole synchronous generator has a per-phase synchronous reactance of 1.0 Ω. Its full-load armature current is 60 A at 0.8 PF lagging. This generator has friction and windage losses of 1.5 kW and core losses of 1.0 kW at 60 Hz at full load. Since the armature resistance is being ignored, assume that the I^2R losses are negligible. The field current has been adjusted so that the terminal voltage is 480 V at no load.

(a) What is the speed of rotation of this generator?

(b) What is the terminal voltage of this generator if the following are true?

1. It is loaded with the rated current at 0.8 PF lagging.
2. It is loaded with the rated current at 1.0 PF.
3. It is loaded with the rated current at 0.8 PF leading.

(c) What is the efficiency of this generator (ignoring the unknown electrical losses) when it is operating at the rated current and 0.8 PF lagging?

(d) How much shaft torque must be applied by the prime mover at full load? How large is the induced countertorque?

(e) What is the voltage regulation of this generator at 0.8 PF lagging? At 1.0 PF? At 0.8 PF leading?

Solution

This generator is Y-connected, so its phase voltage is given by $V_\phi = V_T/\sqrt{3}$. That means that when V_T is adjusted to 480 V, $V_\phi = 277$ V. The field current has been adjusted so that $V_{T,\text{nl}} = 480$ V, so $V_\phi = 277$ V. At *no load*, the armature current is zero, so the armature reaction voltage and the $I_A R_A$ drops are zero. Since $\mathbf{I}_A = 0$, the internal generated voltage $E_A = V_\phi = 277$ V. The internal generated voltage $E_A (= K\phi\omega)$ varies only when the field current changes. Since the problem states that the field current is adjusted initially and then left alone, the magnitude of the internal generated voltage is $E_A = 277$ V and will not change in this example.

(a) The speed of rotation of a synchronous generator in revolutions per minute is given by Equation (4–34):

$$f_e = \frac{n_m P}{120} \tag{4–34}$$

Therefore,

$$n_m = \frac{120 f_e}{P}$$

$$= \frac{120(50\text{ Hz})}{6\text{ poles}} = 1000\text{ r/min}$$

Alternatively, the speed expressed in radians per second is

$$\omega_m = (1000\text{ r/min})\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{r}}\right)$$

$$= 104.7\text{ rad/s}$$

(b) 1. If the generator is loaded down with rated current at 0.8 PF lagging, the resulting phasor diagram looks like the one shown in Figure 5–24a. In this phasor diagram, we know that $\mathbf{V}_\phi$ is at an angle of 0°, that the magnitude of $\mathbf{E}_A$ is 277 V, and that the quantity $jX_S\mathbf{I}_A$ is

$$jX_S\mathbf{I}_A = j(1.0\ \Omega)(60\angle -36.87°\text{ A}) = 60\angle 53.13°\text{ V}$$

The two quantities not known on the voltage diagram are the magnitude of $\mathbf{V}_\phi$ and the angle δ of $\mathbf{E}_A$. To find these values, the easiest approach is to construct a right triangle on the phasor diagram, as shown in the figure. From Figure 5–24a, the right triangle gives

$$E_A^2 = (V_\phi + X_S I_A \sin\theta)^2 + (X_S I_A \cos\theta)^2$$

Therefore, the phase voltage at the rated load and 0.8 PF lagging is

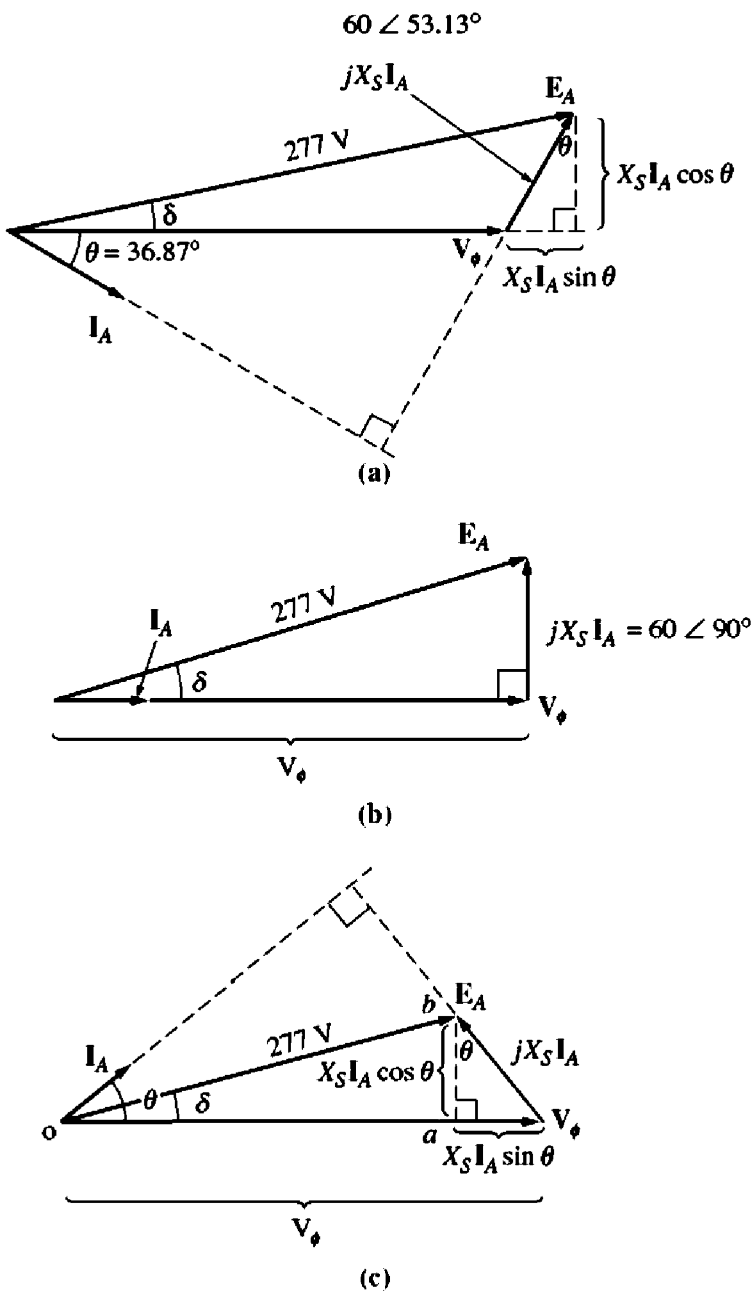


FIGURE 5–24
Generator phasor diagrams for Example 5–3. (a) Lagging power factor; (b) unity power factor; (c) leading power factor.

$$(277\text{ V})^2 = [V_\phi + (1.0\ \Omega)(60\text{ A})\sin 36.87°]^2 + [(1.0\ \Omega)(60\text{ A})\cos 36.87°]^2$$

$$76{,}729 = (V_\phi + 36)^2 + 2304$$

$$74{,}425 = (V_\phi + 36)^2$$

$$272.8 = V_\phi + 36$$

$$V_\phi = 236.8\text{ V}$$

Since the generator is Y-connected, $V_T = \sqrt{3}V_\phi = 410$ V.

2. If the generator is loaded with the rated current at unity power factor, then the phasor diagram will look like Figure 5–24b. To find $\mathbf{V}_\phi$ here the right triangle is

$$E_A^2 = V_\phi^2 + (X_S I_A)^2$$
$$(277\ \text{V})^2 = V_\phi^2 + [(1.0\ \Omega)(60\ \text{A})]^2$$
$$76{,}729 = V_\phi^2 + 3600$$
$$V_\phi^2 = 73{,}129$$
$$V_\phi = 270.4\ \text{V}$$

Therefore, $V_T = \sqrt{3}V_\phi = 468.4$ V.

3. When the generator is loaded with the rated current at 0.8 PF leading, the resulting phasor diagram is the one shown in Figure 5–24c. To find $\mathbf{V}_\phi$ in this situation, we construct the triangle OAB shown in the figure. The resulting equation is

$$E_A^2 = (V_\phi - X_S I_A)^2 + (X_S I_A \cos\theta)^2$$

Therefore, the phase voltage at the rated load and 0.8 PF leading is

$$(277\ \text{V})^2 = [V_\phi - (1.0\ \Omega)(60\ \text{A})\sin 36.87°]^2 + [(1.0\ \Omega)(60\ \text{A})\cos 36.87°]^2$$
$$76{,}729 = (V_\phi - 36)^2 + 2304$$
$$74{,}425 = (V_\phi - 36)^2$$
$$272.8 = V_\phi - 36$$
$$V_\phi = 308.8\ \text{V}$$

Since the generator is Y-connected, $V_T = \sqrt{3}V_\phi = 535$ V.

(c) The output power of this generator at 60 A and 0.8 PF lagging is

$$P_{\text{out}} = 3V_\phi I_A \cos\theta$$
$$= 3(236.8\ \text{V})(60\ \text{A})(0.8) = 34.1\ \text{kW}$$

The mechanical input power is given by

$$P_{\text{in}} = P_{\text{out}} + P_{\text{elec loss}} + P_{\text{core loss}} + P_{\text{mech loss}}$$
$$= 34.1\ \text{kW} + 0 + 1.0\ \text{kW} + 1.5\ \text{kW} = 36.6\ \text{kW}$$

The efficiency of the generator is thus

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{34.1\ \text{kW}}{36.6\ \text{kW}} \times 100\% = 93.2\%$$

(d) The input torque to this generator is given by the equation

$$P_{\text{in}} = \tau_{\text{app}}\omega_{\text{m}}$$

so $$\tau_{\text{app}} = \frac{P_{\text{in}}}{\omega_m} = \frac{36.6\ \text{kW}}{125.7\ \text{rad/s}} = 291.2\ \text{N}\cdot\text{m}$$

The induced countertorque is given by

$$P_{\text{conv}} = \tau_{\text{app}}\omega_{\text{m}}$$

so $$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_V} = \frac{34.1\ \text{kW}}{125.7\ \text{rad/s}} = 271.3\ \text{N}\cdot\text{m}$$

(e) The voltage regulation of a generator is defined as

$$\mathrm{VR} = \frac{V_{\mathrm{nl}} - V_{\mathrm{fl}}}{V_{\mathrm{fl}}} \times 100\% \tag{4–67}$$

By this definition, the voltage regulation for the lagging, unity, and leading power-factor cases are

1. Lagging case: $\mathrm{VR} = \dfrac{480\ \mathrm{V} - 410\ \mathrm{V}}{410\ \mathrm{V}} \times 100\% = 17.1\%$
2. Unity case: $\mathrm{VR} = \dfrac{480\ \mathrm{V} - 468\ \mathrm{V}}{468\ \mathrm{V}} \times 100\% = 2.6\%$
3. Leading case: $\mathrm{VR} = \dfrac{480\ \mathrm{V} - 535\ \mathrm{V}}{535\ \mathrm{V}} \times 100\% = -10.3\%$

In Example 5–3, lagging loads resulted in a drop in terminal voltage, unity-power-factor loads caused little effect on V_T, and leading loads resulted in an increase in terminal voltage.

Example 5–4. Assume that the generator of Example 5–3 is operating at no load with a terminal voltage of 480 V. Plot the terminal characteristic (terminal voltage versus line current) of this generator as its armature current varies from no-load to full load at a power factor of (*a*) 0.8 lagging and (*b*) 0.8 leading. Assume that the field current remains constant at all times.

Solution

The terminal characteristic of a generator is a plot of its terminal voltage versus line current. Since this generator is Y-connected, its phase voltage is given by $V_\phi = V_T/\sqrt{3}$. If V_T is adjusted to 480 V at no-load conditions, then $V_\phi = E_A = 277$ V. Because the field current remains constant, E_A will remain 277 V at all times. The output current I_L from this generator will be the same as its armature current I_A because it is Y-connected.

(*a*) If the generator is loaded with a 0.8 PF lagging current, the resulting phasor diagram looks like the one shown in Figure 5–24a. In this phasor diagram, we know that $\mathbf{V}_\phi$ is at an angle of 0°, that the magnitude of $\mathbf{E}_A$ is 277 V, and that the quantity $jX_S\mathbf{I}_A$ stretches between $\mathbf{V}_\phi$ and $\mathbf{E}_A$ as shown. The two quantities not known on the phasor diagram are the magnitude of $\mathbf{V}_\phi$ and the angle δ of $\mathbf{E}_A$. To find V_ϕ, the easiest approach is to construct a right triangle on the phasor diagram, as shown in the figure. From Figure 5–24a, the right triangle gives

$$E_A^2 = (V_\phi + X_SI_A \sin\theta)^2 + (X_SI_A\cos\theta)^2$$

This equation can be used to solve for V_ϕ as a function of the current I_A:

$$V_\phi = \sqrt{E_A^2 - (X_SI_A\cos\theta)^2} - X_SI_A\sin\theta$$

A simple MATLAB M-file can be used to calculate V_ϕ (and hence V_T) as a function of current. Such an M-file is shown below:

```
% M-file: term_char_a.m
% M-file to plot the terminal characteristics of the
%   generator of Example 5-4 with an 0.8 PF lagging load.

% First, initialize the current amplitudes (21 values
% in the range 0-60 A)
i_a = (0:1:20) * 3;
```

```
% Now initialize all other values
v_phase = zeros(1,21);
e_a = 277.0;
x_s = 1.0;
theta = 36.87 * (pi/180);  % Converted to radians

% Now calculate v_phase for each current level
for ii = 1:21
 v_phase(ii) = sqrt(e_a^2 - (x_s * i_a(ii) * cos(theta))^2) ...
                          - (x_s * i_a(ii) * sin(theta));
end

% Calculate terminal voltage from the phase voltage
v_t = v_phase * sqrt(3);

% Plot the terminal characteristic, remembering the
% the line current is the same as i_a
plot(i_a,v_t,'Color','k','Linewidth',2.0);
xlabel('Line Current (A)','Fontweight','Bold');
ylabel('Terminal Voltage (V)','Fontweight','Bold');
title ('Terminal Characteristic for 0.8 PF lagging load', ...
    'Fontweight','Bold');
grid on;
axis([0 60 400 550]);
```

The plot resulting when this M-file is executed is shown in Figure 5–25a.

(*b*) If the generator is loaded with a 0.8 PF leading current, the resulting phasor diagram looks like the one shown in Figure 5–24c. To find V_ϕ, the easiest approach is to construct a right triangle on the phasor diagram, as shown in the figure. From Figure 5–24c, the right triangle gives

$$E_A^2 = (V_\phi - X_S I_A \sin\theta)^2 + (X_S I_A \cos\theta)^2$$

This equation can be used to solve for V_ϕ as a function of the current I_A:

$$V_\phi = \sqrt{E_A^2 - (X_S I_A \cos\theta)^2} + X_S I_A \sin\theta$$

This equation can be used to calculate and plot the terminal characteristic in a manner similar to that in part *a* above. The resulting terminal characteristic is shown in Figure 5–25b.

5.9 PARALLEL OPERATION OF AC GENERATORS

In today's world, an isolated synchronous generator supplying its own load independently of other generators is very rare. Such a situation is found in only a few out-of-the-way applications such as emergency generators. For all usual generator applications, there is more than one generator operating in parallel to supply the power demanded by the loads. An extreme example of this situation is the U.S. power grid, in which literally thousands of generators share the load on the system.