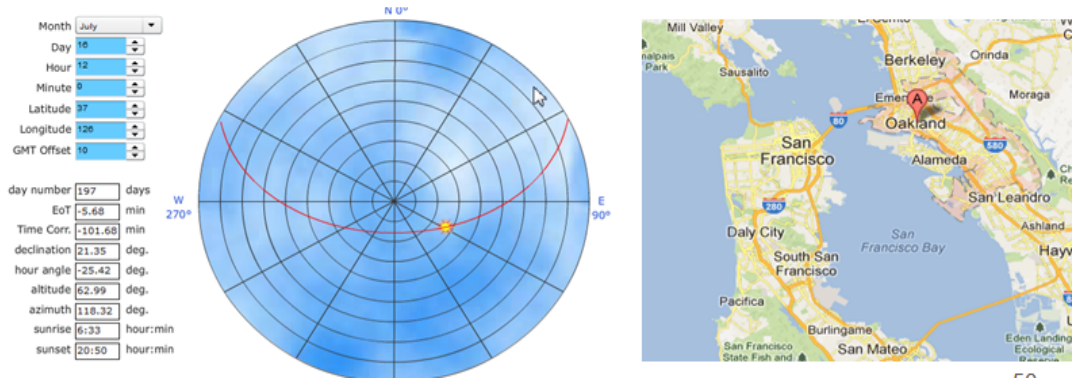


7 - 3 Average daily Insolation **Daily**

- ✚ Average ~~Monthly~~ Insolation on a Tilted Collector
- ✚ Average horizontal insolation (I_H) in Oakland, California (latitude 37.73°N) in July 16 is $7.32 \text{ kWh/m}^2\text{-day}$. Assume ground reflectivity of 0.2.
- ✚ **Question:** Estimate the insolation on a south-facing collector at a tilt angle of 30° with respect to the horizontal.



✚ 0. Target

- ✚ 1. Sun declination (δ) for July 16 ($n=197$)
- ✚ 2. Sunrise Hour Angle (H_{SR}) using $L=37.73^\circ$
- ✚ 3. Extraterrestrial Insolation (I_0) (with $SC=1.37 \text{ kW/m}^2$)
- ✚ 4. Clearness Index (K_T)
- ✚ 5. Horizontal Diffuse Radiation (I_{DH})
- ✚ 6. Diffuse Radiation on the Collector (I_{DC})
- ✚ 7. Reflected Radiation on the Collector (I_{RC})
- ✚ 8. Horizontal Beam Radiation (I_{BH})
- ✚ 9. Sunrise Hour Angle on the Collector (H_{SRC})
- ✚ 10. Beam Tilt Factor (R_B)
- ✚ 11. Beam Radiation on the Collator (I_{BC})
- ✚ 12. Total Insolation on the Collector (I_C)

$I_H := 7.32$ kWh /m²-day **average horizontal insolation**

Collector tilt angle $\Sigma := 30$ deg

$SC := 1.37$

$L := 37.73$ Day number of May 21 $n := 197$

1 Sun Declination (δ)

Solar Declination Angle (δ)

$$\delta := 23.45 \cdot \sin\left(\frac{360}{365} \cdot (n - 81) \cdot \frac{\pi}{180}\right) = 21.3537 \text{ deg}$$

2 Sun Rise Hour Angle (HSR)

$$HSR := \arccos\left(-\tan\left(L \cdot \frac{\pi}{180}\right) \cdot \tan\left(\delta \cdot \frac{\pi}{180}\right)\right) = 1.8781 \text{ rad}$$

$$H_{SR} = \cos^{-1}(-\tan L \tan \delta)$$

3 **Average** Extraterrestrial Insolation (I_0) with $SC = 1.37$ kW/m²

$$\bar{I}_0 = \left(\frac{24}{\pi}\right) SC \left[1 + 0.034 \cos\left(\frac{360n}{365}\right)\right] (\cos L \cos \delta \sin H_{SR} + H_{SR} \sin L \sin \delta)$$

$$I_0 := \left(\frac{24}{\pi}\right) \cdot SC \cdot \left[1 + 0.034 \cdot \cos\left(\frac{360 \cdot n}{365} \cdot \frac{\pi}{180}\right)\right] \cdot \left[\cos\left(L \cdot \frac{\pi}{180}\right) \cdot \cos\left(\delta \cdot \frac{\pi}{180}\right) \cdot \sin(HSR) + HSR \cdot \sin\left(L \cdot \frac{\pi}{180}\right) \cdot \sin\left(\delta \cdot \frac{\pi}{180}\right)\right] \frac{kW}{m^2}$$

4 Clearness Index (K_T)

$$K_T := \frac{I_H}{I_0} = 0.6454$$

$$K_T = \frac{\bar{I}_H}{\bar{I}_0}$$

$$\frac{\bar{I}_{DH}}{\bar{I}_H} = 1.390 - 4.027 K_T + 5.531 K_T^2 - 3.108 K_T^3$$

5 **Average** Horizontal Diffuse Radiation (I_{DH})

$$I_{DH} := I_H \cdot \left(1.390 - 4.027 \cdot K_T + 5.531 \cdot K_T^2 - 3.108 \cdot K_T^3\right) = 1.8982 \text{ kWh /m}^2\text{-day}$$

6 **Average** Diffuse Radiation on the Collector (IDC)

$$IDC := IDH \cdot \left(\frac{1 + \cos \left(\Sigma \cdot \frac{\pi}{180} \right)}{2} \right) = 1.771$$

$$\bar{I}_{DC} = \bar{I}_{DH} \left(\frac{1 + \cos \Sigma}{2} \right)$$

kWh /m²-day

7 **Average** Reflected Radiation on the Collector (IRC)

$$\rho := 0.2$$

$$IRC := \rho \cdot IH \cdot \left(\frac{1 - \cos \left(\Sigma \cdot \frac{\pi}{180} \right)}{2} \right) = 0.0981$$

$$\bar{I}_{RC} = \rho \bar{I}_H \left(\frac{1 - \cos \Sigma}{2} \right)$$

kWh /m²-day

8 **Average** Horizontal Beam (only) Radiation (IBH)

$$IBH := IH - IDH = 5.4218$$

kWh /m²-day

$$\bar{I}_{BH} = \bar{I}_H - \bar{I}_{DH}$$

9 Sunrise Hour Angle on the Collector (HSRC)

$$H_{SRC} = \min\{\cos^{-1}(-\tan L \tan \delta), \cos^{-1}[-\tan(L - \Sigma) \tan \delta]\}$$

$$HSRC1 := \arccos \left(-\tan \left(L \cdot \frac{\pi}{180} \right) \cdot \tan \left(\delta \cdot \frac{\pi}{180} \right) \right) = 1.8781$$

$$A := \begin{bmatrix} 2 & 5 \end{bmatrix}$$

$$\min(A) = 2$$

$$HSRC2 := \arccos \left(-\tan \left((L - \Sigma) \cdot \frac{\pi}{180} \right) \cdot \tan \left(\delta \cdot \frac{\pi}{180} \right) \right) = 1.6239$$

$$HSRC := \min([HSRC1 \quad HSRC2]) = 1.6239$$

10 **Average** Beam Tilt Factor (RB)

$$\overline{R}_B = \frac{\cos(L - \Sigma) \cos \delta \sin H_{SRC} + H_{SRC} \sin(L - \Sigma) \sin \delta}{\cos L \cos \delta \sin H_{SR} + H_{SR} \sin L \sin \delta}$$

$$RB := \frac{\cos\left((L - \Sigma) \cdot \frac{\pi}{180}\right) \cdot \cos\left(\delta \cdot \frac{\pi}{180}\right) \cdot \sin(H_{SRC}) + H_{SRC} \cdot \sin\left((L - \Sigma) \cdot \frac{\pi}{180}\right) \cdot \sin\left(\delta \cdot \frac{\pi}{180}\right)}{\cos\left(L \cdot \frac{\pi}{180}\right) \cdot \cos\left(\delta \cdot \frac{\pi}{180}\right) \cdot \sin(H_{SR}) + H_{SR} \cdot \sin\left(L \cdot \frac{\pi}{180}\right) \cdot \sin\left(\delta \cdot \frac{\pi}{180}\right)}$$

$$RB = 0.8934$$

$$\overline{I}_{BC} = \overline{I}_{BH} \overline{R}_B$$

11 **Average** Beam Radiation on the Collector (IBC)

$$IBC := IBH \cdot RB = 4.8438 \quad \text{kWh /m}^2\text{-day}$$

12 **Total Average** Insolation on the Collector

$$\overline{I}_C = \overline{I}_{BC} + \overline{I}_{DC} + \overline{I}_{RC}$$

$$IC := IBC + IDC + IRC = 6.7129 \quad \text{kWh /m}^2\text{-day}$$