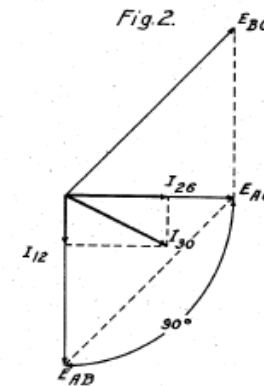
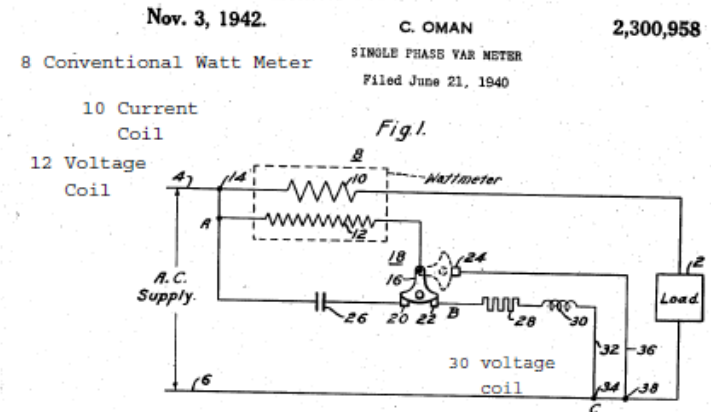
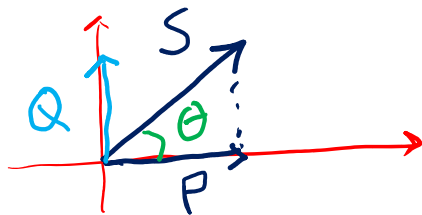


# Lab 3 – Real Power & Reactive Power

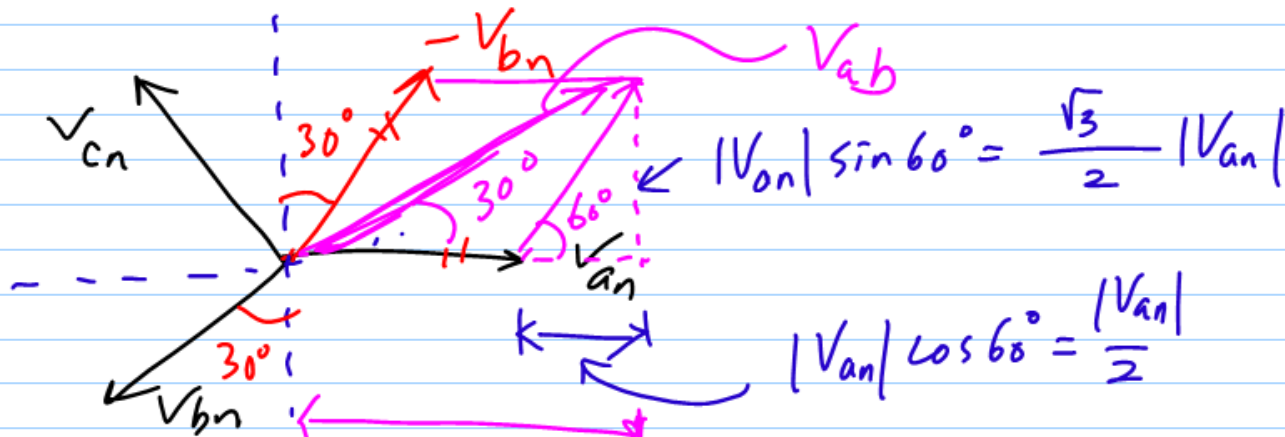
- Real Power (P) [Watt] [W]  $V |I| \cos \theta$
- Reactive Power (Q) [Var]  $V |I| \sin \theta$
- Complex Power (S) = P + jQ
- Apparent Power ( $|S|$ ) =  $\text{SQRT}(P^2 + Q^2)$



## 3-phase power systems

Line-to-Line Voltage

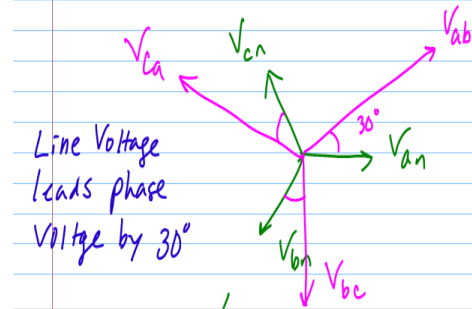
$$\vec{V}_{ab} = \vec{V}_{an} + \vec{V}_{nb} = \vec{V}_{an} - \vec{V}_{bn}$$



$$|V_{ab}| = \sqrt{\left(\frac{\sqrt{3}}{2} |V_{an}|\right)^2 + \left(\frac{1}{2} |V_{an}|\right)^2} = \sqrt{\frac{12}{4} |V_{an}|^2} = \sqrt{3} |V_{an}|$$

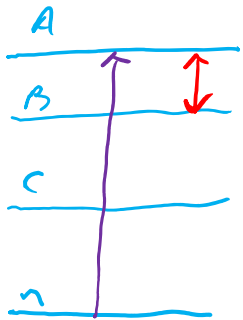
$$\Rightarrow \vec{V}_{ab} = \sqrt{3} \vec{V}_{an} \angle 30^\circ$$

$$\Rightarrow \vec{V}_{bc} = \sqrt{3} \vec{V}_{bn} \angle 30^\circ \quad \Rightarrow \vec{V}_{ca} = \sqrt{3} \vec{V}_{cn} \angle 30^\circ$$



## 3-phase power: P and Q

$V_{AB} = V_L$   
("Line voltage")



$$\Rightarrow P_{3\phi} = 3 V_p I_p \cos \theta_p$$

$V_L$ : Line-to-Line Voltage magnitude on 1  $\phi$  eqn

$$V_p = \frac{V_L}{\sqrt{3}} \quad \text{and} \quad I_p = I_L \quad (\text{Y-load case})$$

$$I_L = I_p \sqrt{3} \quad \text{current over a line}$$

$$\Rightarrow P_{3\phi} = 3 V_p I_p \cos \theta_p$$

$$= 3 \frac{V_L}{\sqrt{3}} I_L \cos \theta_p = \sqrt{3} V_L I_L \cos \theta_p$$

$$\& \times Q_{3\phi} = 3 V_p I_p \sin \theta_p = \sqrt{3} V_L I_L \sin \theta_p$$

$V_{An} \rightarrow V_p$   
"phase voltage"  
"line-to-neutral" voltage

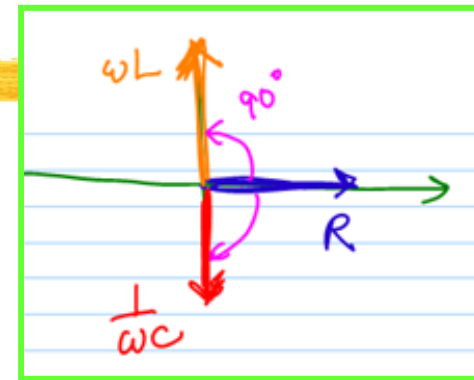
# P and Q & R and X

$\cos \theta = \frac{P}{\sqrt{P^2 + Q^2}} = \frac{P}{VI}$  ← power factor  
 $\text{pf} = \cos \theta = 1$       $\cos \theta > .95$

$Z = R + jX$

$Z = \sqrt{R^2 + X^2}$   
 $R = Z \cos \theta$   
 $X = Z \sin \theta$

$P = VI \cos \theta$



$$Z = R + jX = R + j(X_L - X_C)$$

$$X_L = \omega L, \quad X_C = \frac{1}{\omega C}$$

$$\bar{V} = \bar{I} \bar{Z} \quad \bar{I} = \frac{\bar{V}}{Z} = \frac{V \angle 0^\circ}{Z \angle \theta} = \frac{V}{Z} \angle -\theta$$

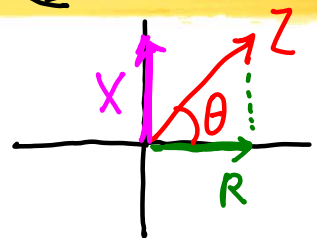
$$P = VI \cos \theta = I^2 Z \cos \theta = I^2 R$$

$$Q = VI \sin \theta = I^2 Z \sin \theta = I^2 X$$

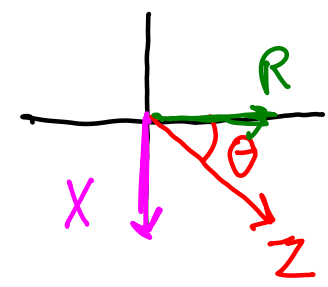
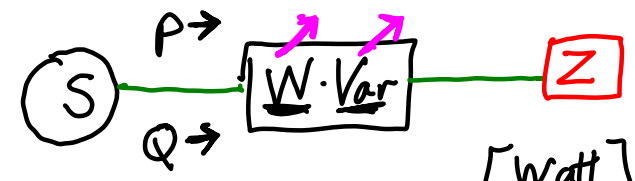
$\cos \theta = \frac{R}{Z}$  ← power factor

# P and Q with Z

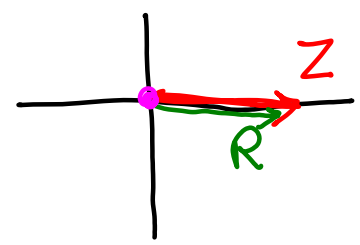
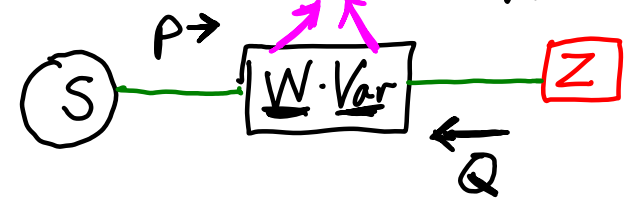
$$P = VI \cos \theta \quad Q = VI \sin \theta$$



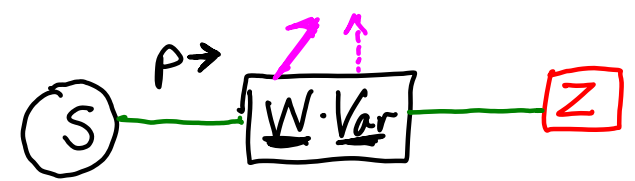
$R = 5$      $X = 5$      $\theta = 45^\circ$      $P > 0$      $Q > 0$



$R = 5$      $X = -5$      $\theta = -45^\circ$      $P > 0$      $Q < 0$

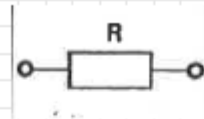


$R = 5$      $X = 0$      $\theta = 0$      $P > 0$      $Q = 0$



# Example

## LAB 3 -- EXAMPLE CALCULATION



$$P_{3\phi} = 3 V_p I_p \cos \theta_p$$

$$= 3 \frac{V_L}{\sqrt{3}} I_L \cos \theta_p = \sqrt{3} V_L I_L \cos \theta_p$$

$$Q_{3\phi} = 3 V_p I_p \sin \theta_p = \sqrt{3} V_L I_L \sin \theta_p$$

$$V_p = 120$$

$$V_L = V_p \cdot \sqrt{3} = 207.846097$$

$$R = 300$$

$$Z = R \quad |Z| = 300$$

$$\theta = \arg(Z) = 0$$

$$I_L = \frac{V_p}{Z} = 0.4 \quad \text{We measure this in the lab}$$

$$|I_L| = 0.4$$

$$P = 3 \cdot |V_p| \cdot |I_L| \cdot \cos(\theta) = 144$$

$$P = \sqrt{3} \cdot |V_L| \cdot |I_L| \cdot \cos(\theta) = 144 \quad \text{W}$$

$$Q = \sqrt{3} \cdot |V_L| \cdot |I_L| \cdot \sin(\theta) = 0 \quad \text{Var}$$

$\rightarrow$   
P

Q = 0

## Example



$$P_{3\phi} = 3 V_p I_p \cos \theta_p$$

$$= 3 \frac{V_L}{\sqrt{3}} I_L \cos \theta_p = \sqrt{3} V_L I_L \cos \theta_p$$

$$Q_{3\phi} = 3 V_p I_p \sin \theta_p = \sqrt{3} V_L I_L \sin \theta_p$$

$$V_p = 120$$

$$V_L = V_p \cdot \sqrt{3} = 207.846097$$

$$R = 0 \quad X_L = 300$$

$$Z = R + i \cdot X_L \quad |Z| = 300$$

$$\theta = \arg(Z) = 1.570796$$

$$\theta_{deg} = \theta \cdot \frac{180}{\pi} = 90$$

$$I_L = \frac{V_p}{Z} = -0.4 \cdot i$$

We measure this in the lab

$$|I_L| = 0.4$$

$$P = \sqrt{3} \cdot |V_L| \cdot |I_L| \cdot \cos(\theta) = -5.027736 \cdot 10^{-13} \quad \text{W}$$

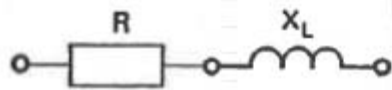
$$Q = \sqrt{3} \cdot |V_L| \cdot |I_L| \cdot \sin(\theta) = -144 \quad \text{Var}$$

$$P = 0$$

$$Q \rightarrow$$

+

## Example



$$P_{3\phi} = 3V_p I_p \cos \theta_p$$

$$= 3 \frac{V_L}{\sqrt{3}} I_L \cos \theta_p = \sqrt{3} V_L I_L \cos \theta_p$$

$$Q_{3\phi} = 3V_p I_p \sin \theta_p = \sqrt{3} V_L I_L \sin \theta_p$$

$$V_P = 120$$

$$V_L = V_P \cdot \sqrt{3} = 207.846097$$

$$R = 300$$

$$X_L = 300$$

$$Z = R + i \cdot X_L$$

$$|Z| = 424.264069$$

$$\theta = \arg[Z] = 0.785398$$

$$\theta_{deg} = \theta \cdot \frac{180}{\pi} = 45$$

$$I_L = \frac{V_P}{Z} = 0.2 - 0.2 \cdot i$$

We measure this in the lab

$$|I_L| = 0.282843$$

$$P = \sqrt{3} \cdot |V_L| \cdot |I_L| \cdot \cos(\theta) = 72 \quad \text{W}$$

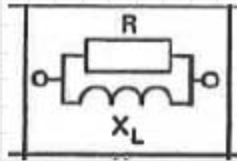
$P \rightarrow$

$$Q = \sqrt{3} \cdot |V_L| \cdot |I_L| \cdot \sin(\theta) = 72 \quad \text{Var}$$

$Q \rightarrow$



# Example



$$P_{3\phi} = 3V_p I_p \cos \theta_p$$

$$= 3 \frac{V_L}{\sqrt{3}} I_L \cos \theta_p = \sqrt{3} V_L I_L \cos \theta_p$$

$$Q_{3\phi} = 3V_p I_p \sin \theta_p = \sqrt{3} V_L I_L \sin \theta_p$$

$$V_p = 120$$

$$V_L = V_p \sqrt{3} = 207.846097$$

$$R = 300 \quad X_L = 300 \quad X = i \cdot X_L$$

$$Z = \frac{R \cdot X}{R + X} = 150 + 150 \cdot i \quad |Z| = 212.132034$$

$$\theta = \arg(Z) = 0.785398$$

$$I_L = \frac{V_p}{Z} = 0.4 - 0.4 \cdot i$$

$$\theta_{deg} = \theta \cdot \frac{180}{\pi} = 45$$

We measure this in the lab

$$|I_L| = 0.565685$$

$$P = 3 \cdot |V_p| \cdot |I_L| \cdot \cos(\theta) = 144$$

$$P = \sqrt{3} \cdot |V_L| \cdot |I_L| \cdot \cos(\theta) = 144$$

W

$$Q = \sqrt{3} \cdot |V_L| \cdot |I_L| \cdot \sin(\theta) = 144$$

Var

P →

Q →