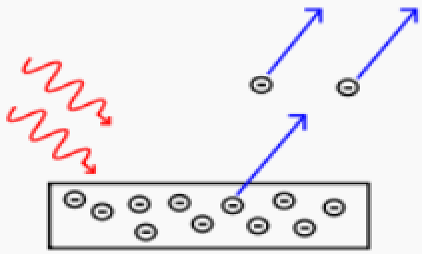
Chapters 8 and 9
Photovoltaic (PV): (8) Materials and Electrical Characteristics, and (9) Systems

Chapter 8 Photovoltaic (PV) Materials and Electrical Characteristics

Photo-Electricity

Light–matter interaction



Low-energy phenomena:
Photoelectric effect

Mid-energy phenomena:
Thomson scattering
Compton scattering

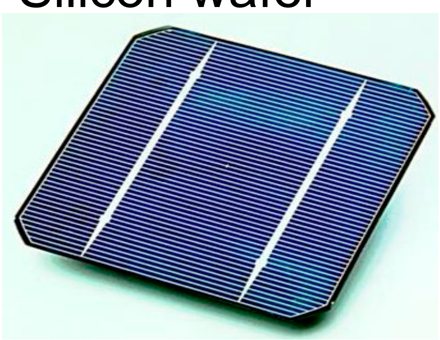
High-energy phenomena:
Pair production

V • T • E

In the **photoelectric effect**, electrons are emitted from solids, liquids or gases when they absorb energy from light. Electrons emitted in this manner may be called *photoelectrons*.[1][2]

In 1887, Heinrich Hertz[2][3] discovered that electrodes illuminated with ultraviolet light create electric sparks more easily. In 1905 Albert Einstein published a paper that explained experimental data from the photoelectric effect as being the result of light energy being carried in discrete quantized packets. This discovery led to the quantum revolution. Einstein was awarded the Nobel Prize in 1921 for "his discovery of the law of the photoelectric effect".[4]

- Silicon wafer



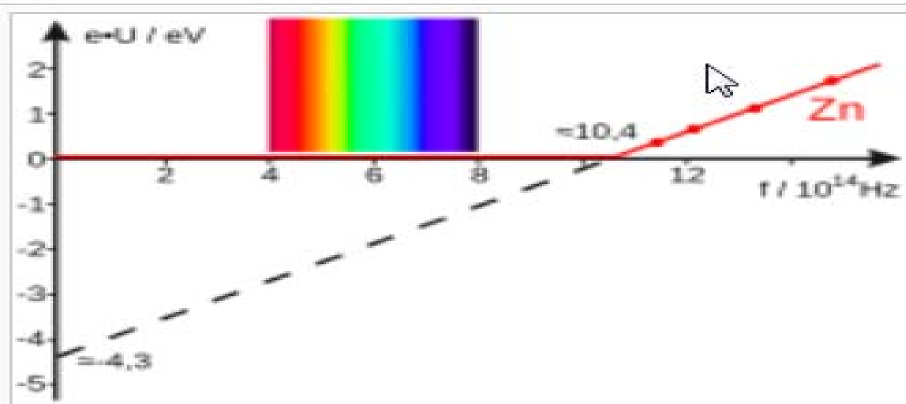
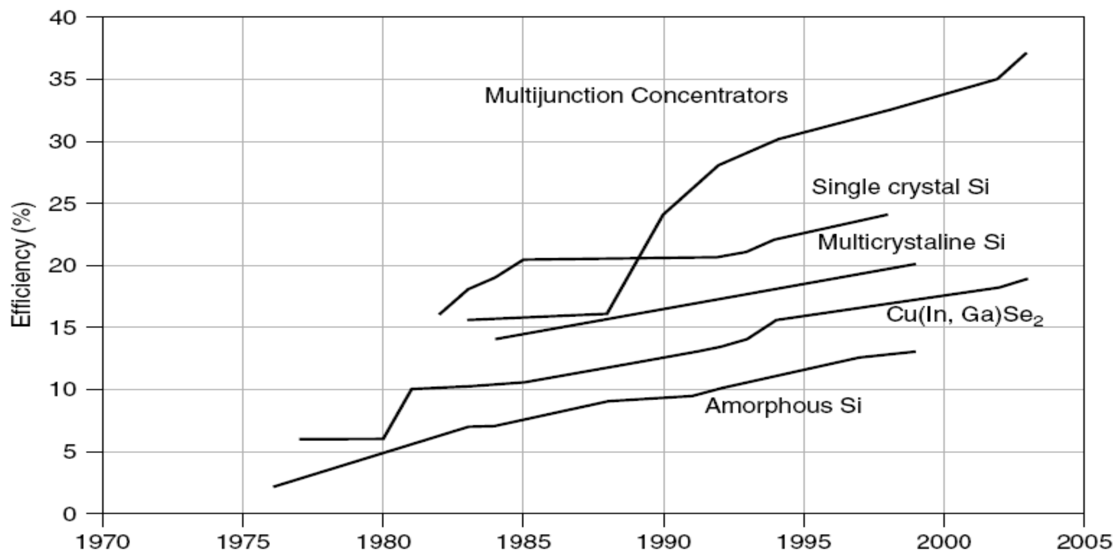
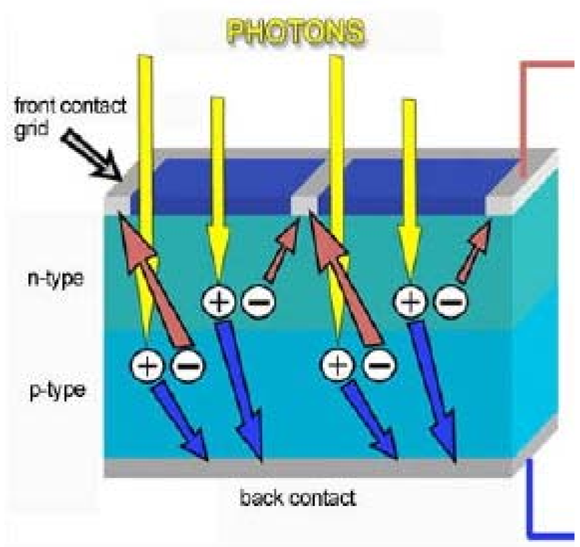


Diagram of the maximum kinetic energy as a function of the frequency of light on zinc

Photovoltaic Material and Electrical Characteristics

- Photovoltaic (PV): a device that is capable of converting the energy contained in photons of light into an electrical voltage or current
- A photon (short wavelength and high energy) breaks free electrons from the atoms in the photovoltaic material.
- **"The surface of the earth receives 6000 times as much solar energy as our total energy demand"**
- PV Cell Efficiency

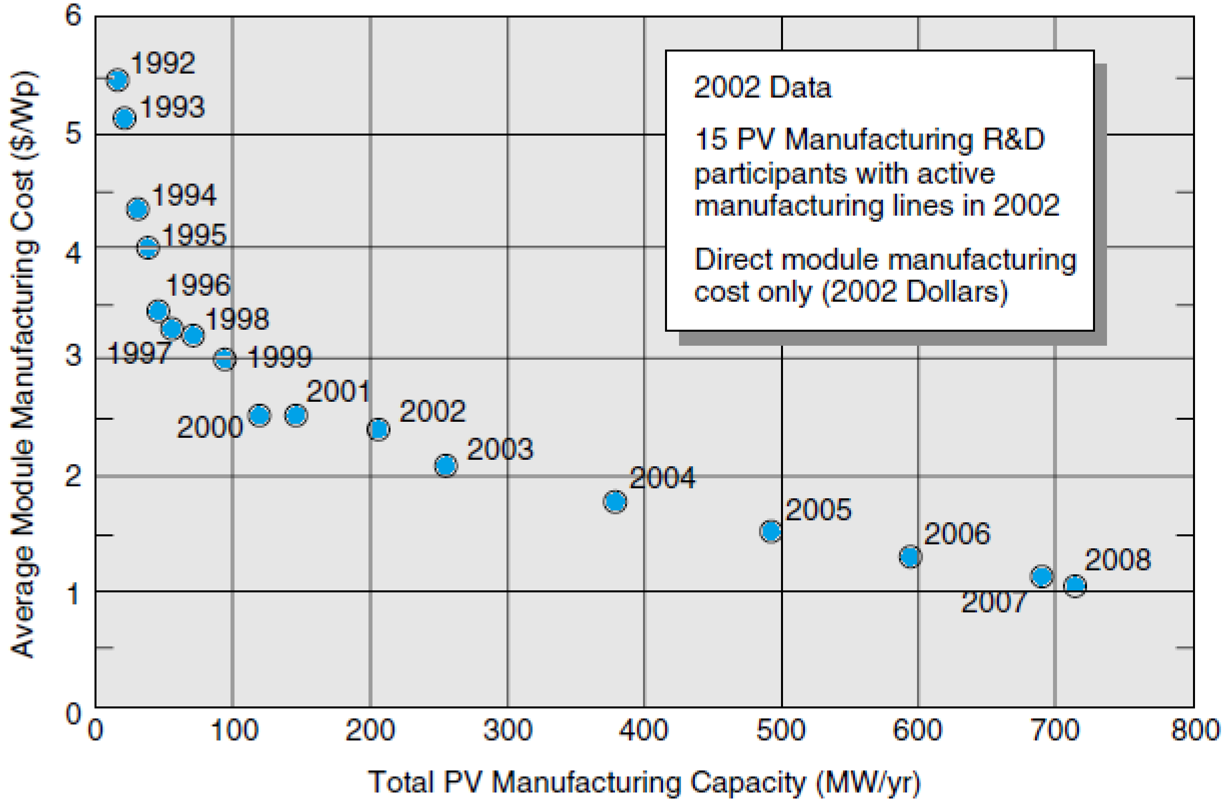




PV History

- 1829: Edmund Becquerel – voltage development on an metal electrode under illumination
- 1876: Adams and Day - PV effect on solid – built a cell made of Selenium with 1- 2 % efficiency
- 1904: Albert Einstein – Theoretical explanation of PV effect
- 1904: Czochralski (Polish Scientist) developed a method to grow perfect crystals of silicon →which later in 1940s and 1950s were adopted to make the first generation of single-crystal silicon PV cells, which continues to dominate the PV industry today
- Before 1958: Cost prohibitive
- 1958: Practical PV, used is space for Vanguard I satellite
- 1970s: Oil shock spurred the commercial PV development
- 1980s: High efficiency and low cost PV emerged
- 2002: Worldwide PV production
  - 600MW/year and increasing by 40% per year

PV Manufacturing Cost & Capacity



PV module manufacturing costs for DOE/US Industry Partners. Historical data through 2002, projections thereafter (www.nrel.gov/pvmat).

PV Semiconductor Physics

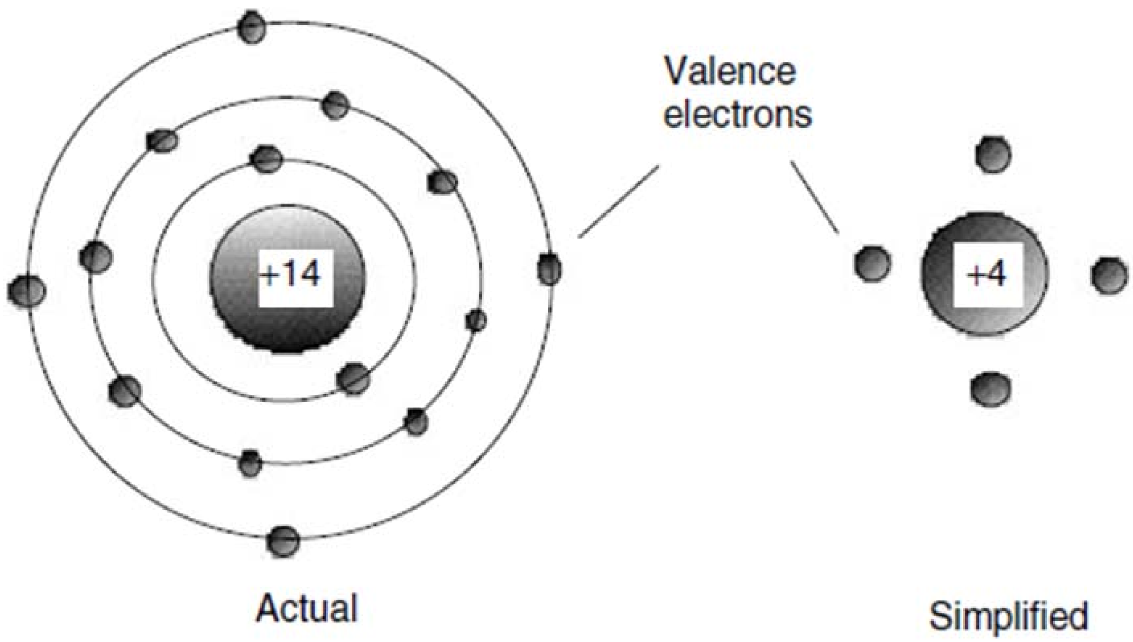
- Pure crystalline silicon (Si) Solar Cells - Group IV
- Other elements added to Silicon (Si) to make PVs (Groups III and V)
  - Boron (B)
  - Phosphorus (P)
- Other Solar Cells
  - **GaAs (Gallium Arsenide) Solar Cells** – Groups III (Gallium) and V (Arsenic)
  - **CdTe (Cadmium Telluride) Solar Cells** – Groups II (Cadmium) and VI (Tellurium)

The Portion of the Periodic Table of Greatest Importance for Photovoltaics Includes the Elements Silicon, Boron, Phosphorus, Gallium, Arsenic, Cadmium, and Tellurium

I	II	III	IV	V	VI
		5 B	6 C	7 N	8 O
		13 Al	**14 Si**	**15 P**	16 S
29 Cu	30 Zn	**31 Ga**	32 Ge	**33 As**	34 Se
47 Ag	**48 Cd**	49 In	50 Sn	51 Sb	**52 Te**

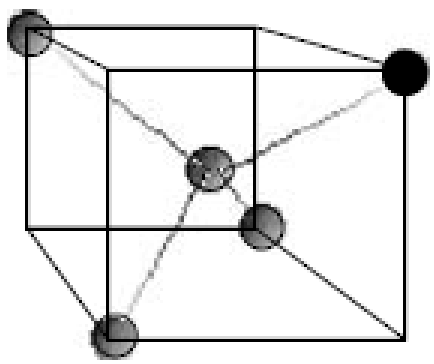
PV Semiconductor Physics

- Silicon (Si) – Group IV
- 14 Protons
- 4 outer valence electrons

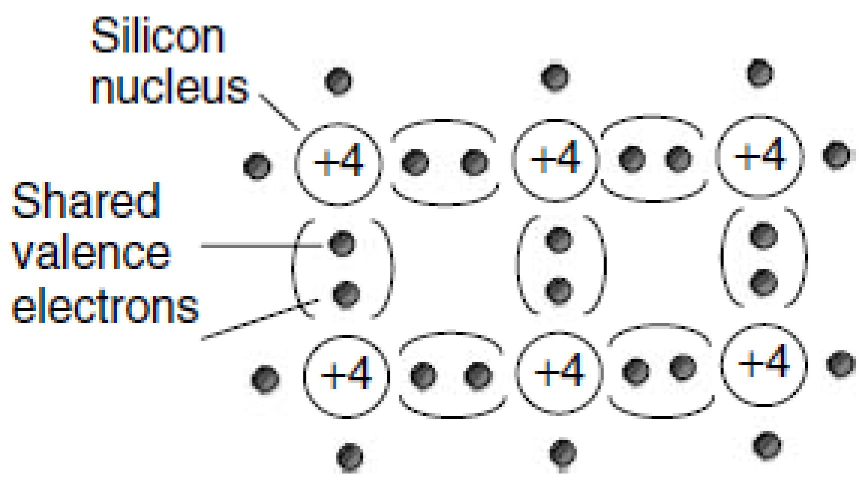


PV Semiconductor Physics

- Silicon (Si) – Group IV
- Pure Crystalline Silicon ---- Covalent bonds with four adjacent atoms



Tetrahedral



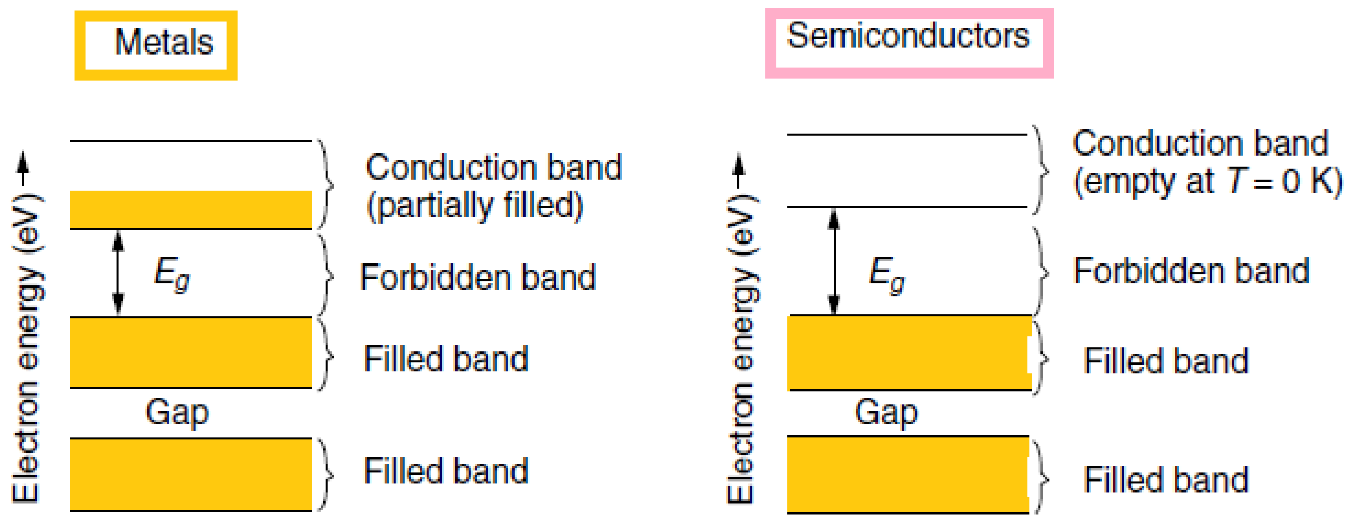
Two-dimensional version

Band Gap Energy

- Silicon (Si) – Group IV
  - At K=0 temperature, no free electrons to roam around → a perfect insulator
  - As Temp increases, some electrons will be given enough energy to free themselves from their nuclei → conductivity increases as temp increase (as opposed to the case in metals).
  - At normal temperature, conductivity is still very low → semi-conductor
  - Adding minute quantities of other materials ("contamination"), conductivity can be greatly increased
- Difference between conductors (metals) and semiconductors (Si)
  - Described by Quantum theory
  - Using energy-band diagram

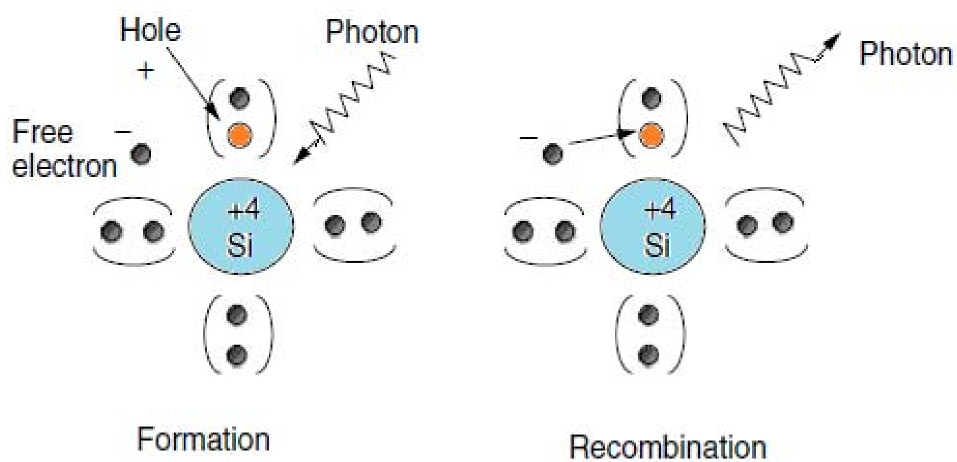
Band Gap Energy

- Band-Gap
  - Electrons have energies that must fit within certain allowable energy bands
  - Top energy band: (   ) → electrons within the band contribute to current flow
  - Conduction band for metals partially filled, while that for semiconductor is (almost) empty
  - Allowable bands & (   ) bands
  - (   ): band between the (   ) band and the highest filled band
  - **Band-Gap Energy (E_g)**: " (Definition:____)"



Band Gap Energy

- Band-Gap Energy (E_g)
  - Unit: electron-volts [eV]: energy that an electron acquires when its voltage is increased by 1 V. { 1 eV = 1.6 x 10^{-19} [J] }
  - Band Gap Energy for Silicon = 1.12 eV
- Energy Sources to jump into the conduction band
  - Thermal Energy
  - PV: Photons of Electromagnetic Energy from Sun
  - A photon with more than 1.12 eV can move 1 electron from solar cell, and then due to 1 fewer electrons → forms a "hole" (a net positive charge).
  - When the electron ( ), the energy (from the conduction band to the filled band) is released as a ( ) → light emitting diode (LED)



Band Gap Energy

- Photons with enough energy create hole-electron pairs in a semiconductor
- Energy of a photon

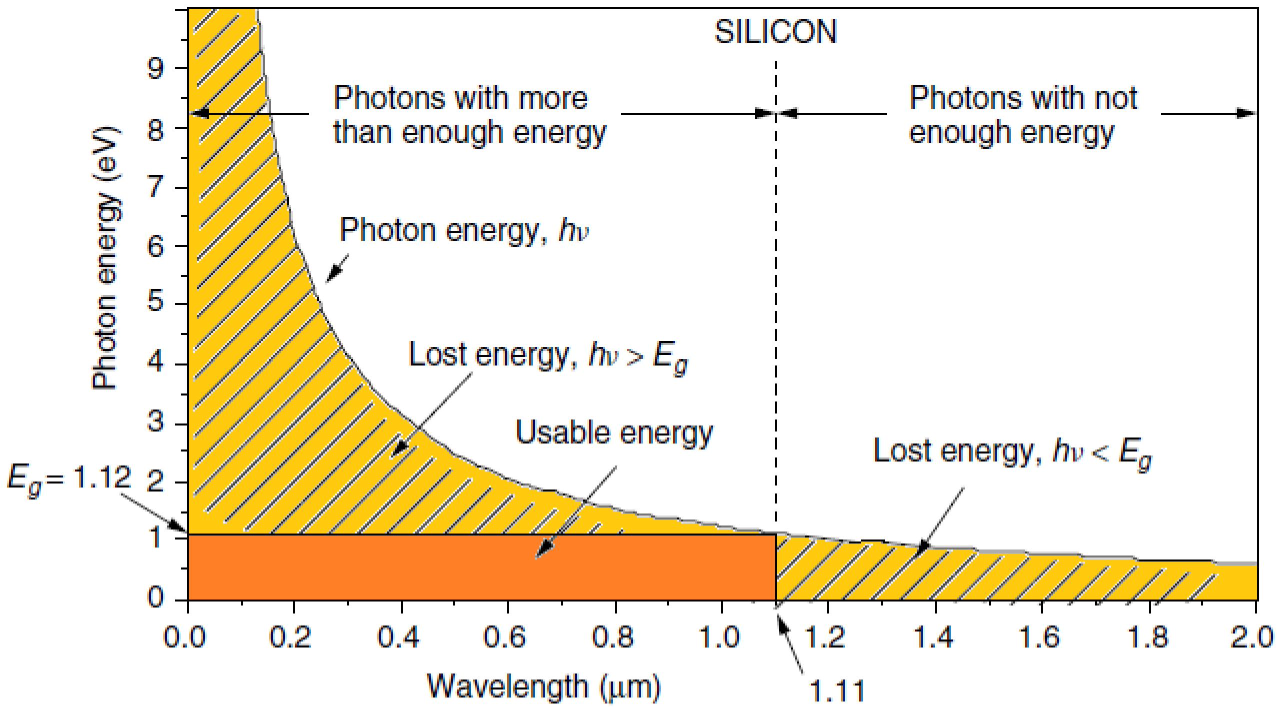
$$E = h\nu = \frac{hc}{\lambda}$$

E	energy of a photon (J)
c	speed of light (3×10^8 m/s)
ν	frequency (hertz),
h	Planck's constant (6.626×10^{-34} J-s)
λ	wavelength (m)

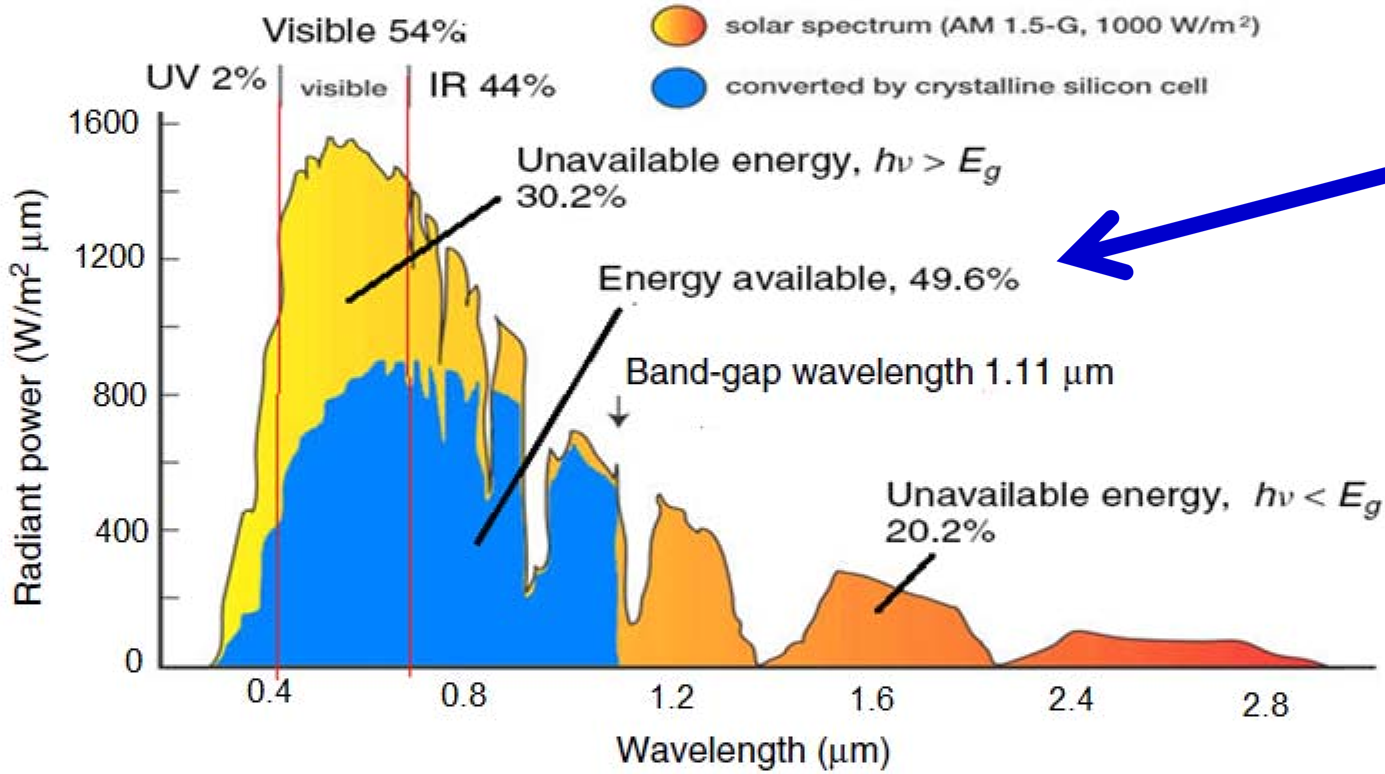
- **Sample Calculation**: Silicon has a band gap of 1.12 eV and 1 eV = 1.6 x 10^{-19} [J] (a) What maximum wavelength can a photon have to create hole-electron pairs in silicon? (b) What minimum frequency is that?

Band Gap Energy

- For Si, photons with wavelength above **1.11 μm** don't have the **1.12 eV** needed to excite an electron, and this energy is lost. Photons with shorter wavelengths have more than enough energy, but any energy above 1.12 eV is wasted any way – since one photon can excite only one electron – waste heat in the cell.



Solar Spectrum and Band-Gap Impact



- Maximum possible fraction of the sun's energy which may be collected by a solar cell: 50%

- AM (Air Mass) Ratio
  - AM0: Sun in space (no atmosphere) --> Average Radiant Flux of 1.377 kW/m².
  - AM1: Sun is directly overhead
  - AM1.5: Sun is 42 degrees above the horizon (standard condition) → 1 kW/m².

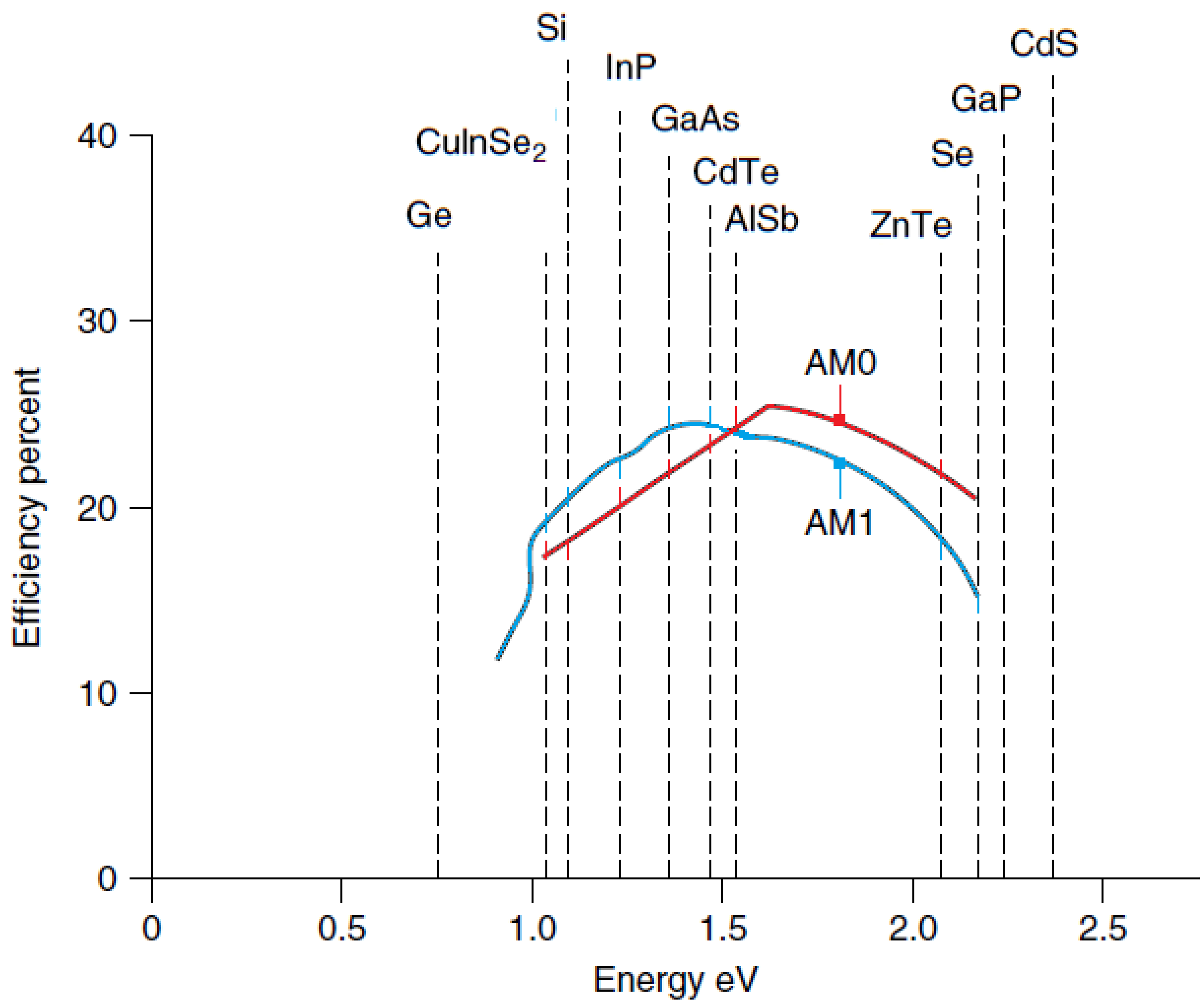
Band Gap Energy

- Band-Gap and Cut-Off Wavelength for Electron Excitation

PV Material	Silicon (Si)	Gallium Arsenide (GaAs)	Cadmium Telluride (CdTe)	Indium Phosphide (InP)
Band Gap [eV]	1.12	1.42	1.5	1.35
Cut-off wavelength [μm]	1.11	0.87	0.83	0.92

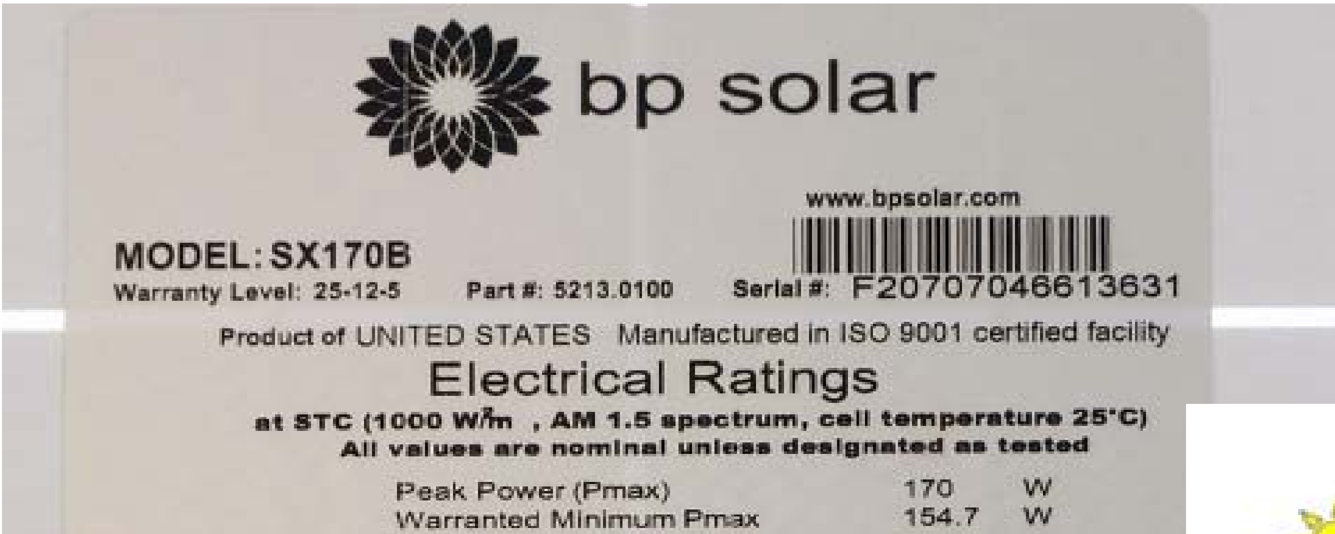
- Trade-off between choosing PV materials
- Smaller Band-Gap Material
  - More solar photons have the energy needed to excite electrons
  - More photons have surplus energy above the threshold → waste
- Higher Band-Gap Material
  - Fewer photons have energy to create current-carrying electrons & holes
  - Less left-over surplus energy

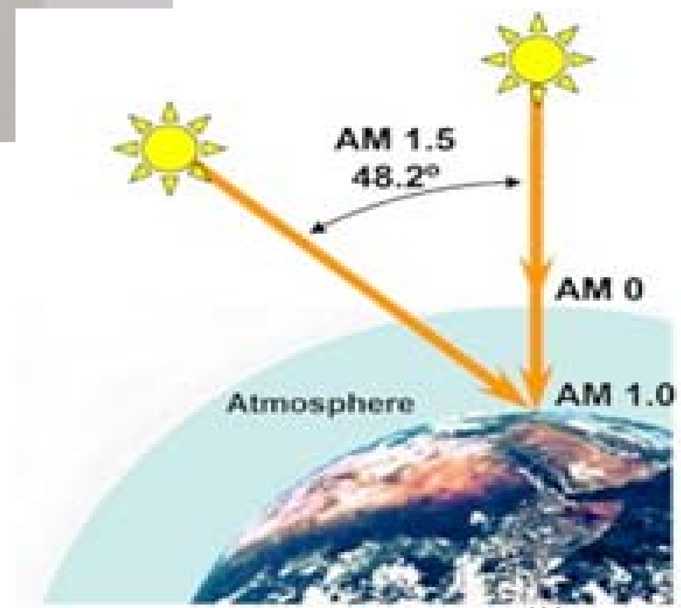
Band Gap Energy



- Maximum Efficiency of PVs as a function of their Band Gap

AM Ratio and PV plate



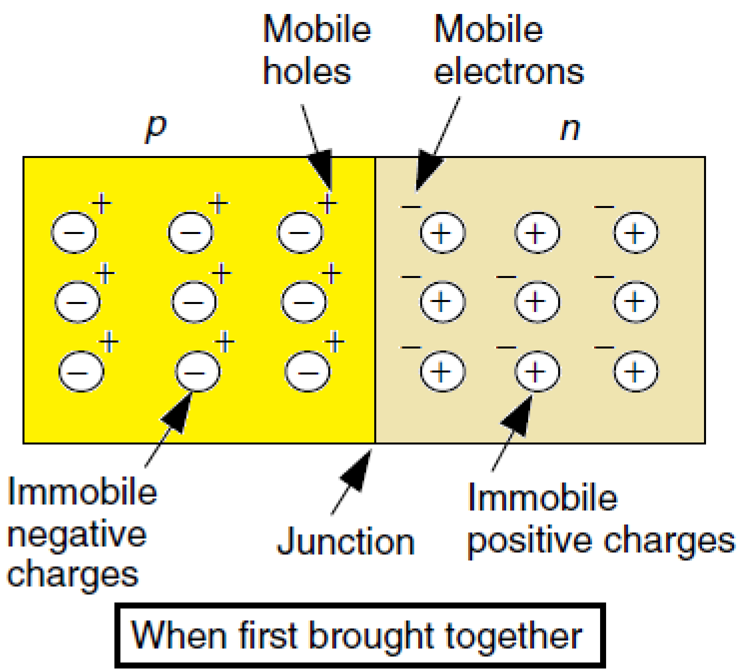


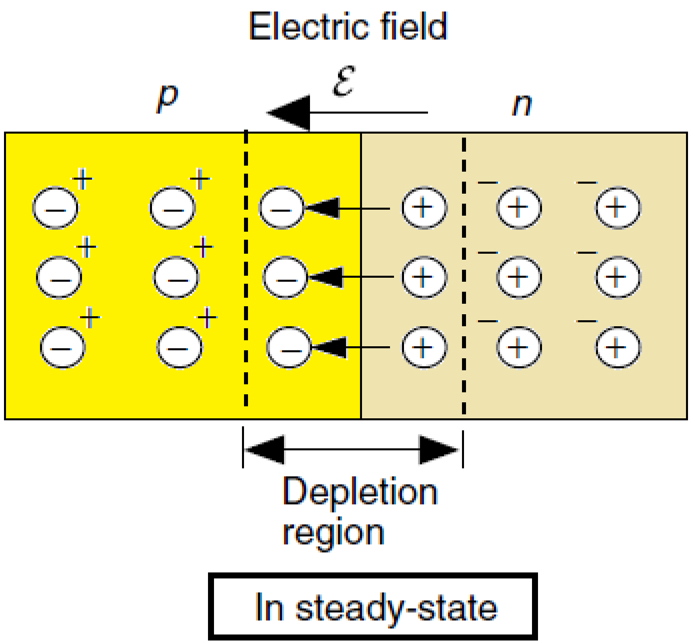
p-n Junction

- Problem for the created hole-electron pairs
  - The free electrons can fall right back in to a hole (Recombination) – both charge carriers disappear
- How to avoid the recombination: The electrons in the conduction band must continuously “swept” away from holes → how?
- Built-in Electric Field within the semiconductor itself to push electrons in one direction and holes in the other
- How to build the Electric Field: Two regions are to be established within the crystal
  - One side: Contaminated (“Doped’) with a trivalent element of Group III (Boron B)→ p-type
  - The other side: Doped with pentavalent element of Group V (Phosphorus P) → n-type
- Electric Field builds up between p-n junction

p-n Junction

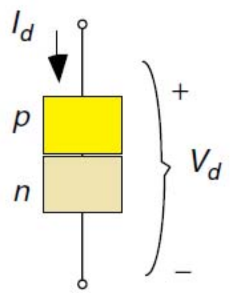
- Electric Field builds up between p-n junction



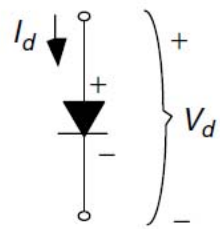


p-n Junction Diode

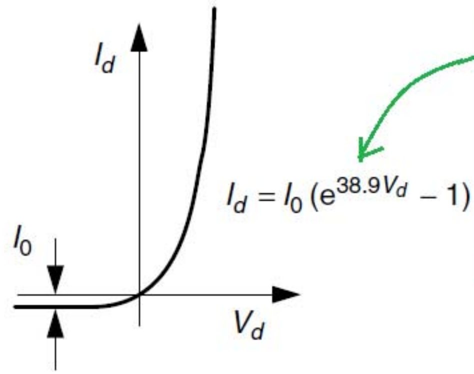
Shockley diode equation: $I_d = I_0(e^{qV_d/kT} - 1)$



p–n junction diode



Symbol for real diode



Diode characteristic curve

$$\frac{q}{kT} = 38.9$$

at junction temperature of 25°C → standard.

- I_0 reverse saturation current (A)
  reverse saturation current is the result of thermally generated carriers with the holes being swept into the p-side and the electrons into the n-side.
- I_d the diode current in the direction of the arrow (A)
- V_d the voltage across the diode terminals from the p-side to the n-side (V).
- q the electron charge (1.602×10^{-19}C)
- k Boltzmann's constant (1.381×10^{-23} J/K)
- T the junction temperature (K).

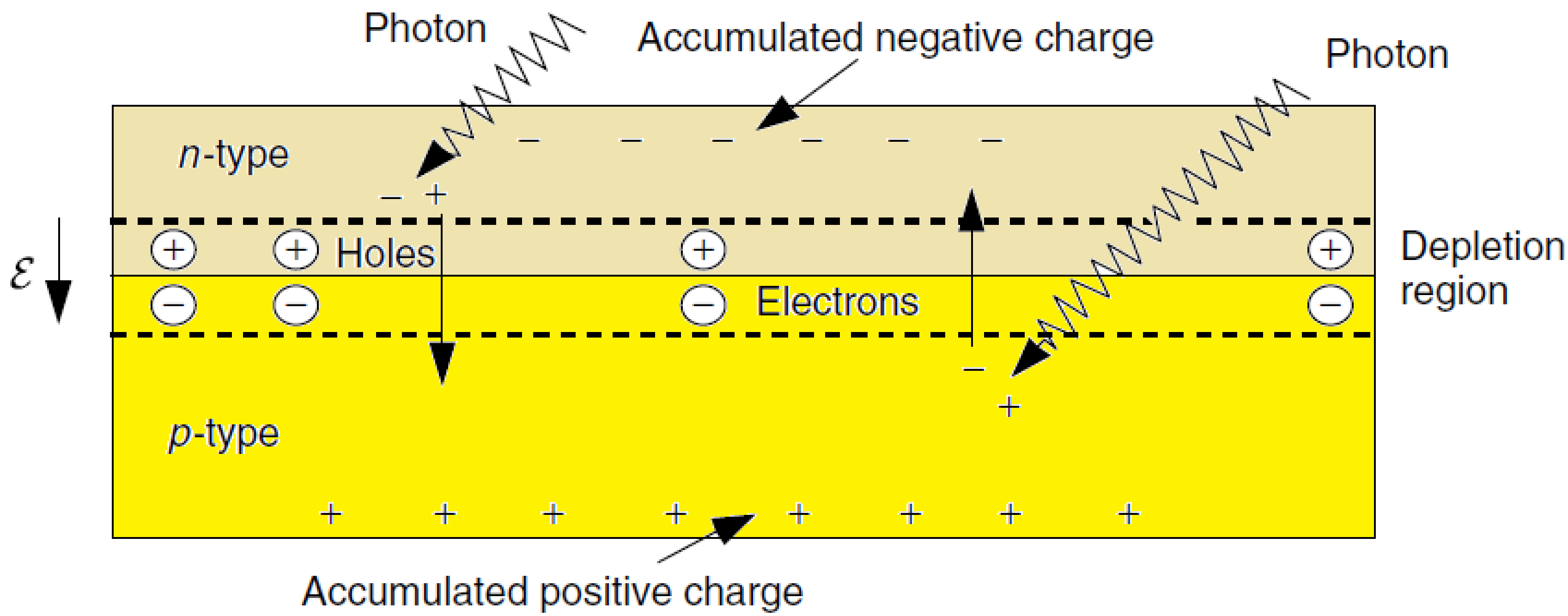
p-n Junction Diode

- Diode Voltage Drop (0.6 V) – Example Calculation
- Question: Consider a p-n junction diode at 25 °C with a reverse saturation current of 10^{-9} A. Find the voltage drop across the diode when it is carrying the following diode currents: (a) no current (open-circuit voltage); (b) 1 A; and (c) 10 A.

$$I_d = I_0\,(e^{38.9 V_d} - 1)$$

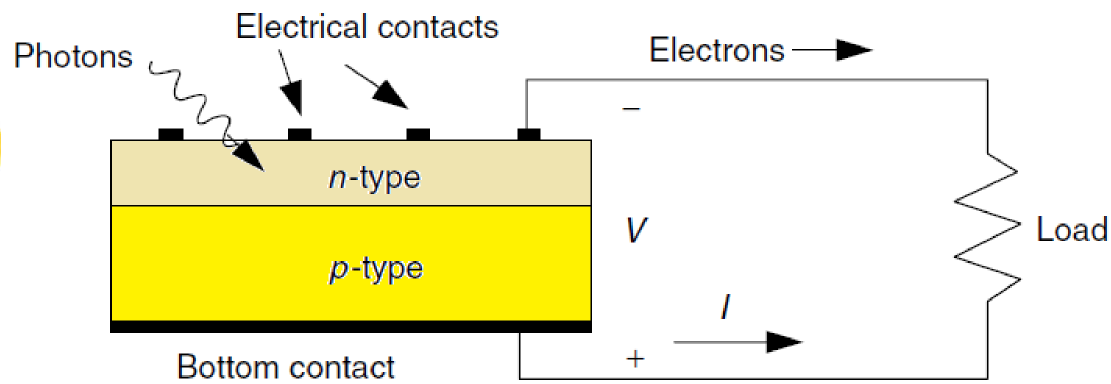
PV Cell

- p-n junction exposed to sunlight

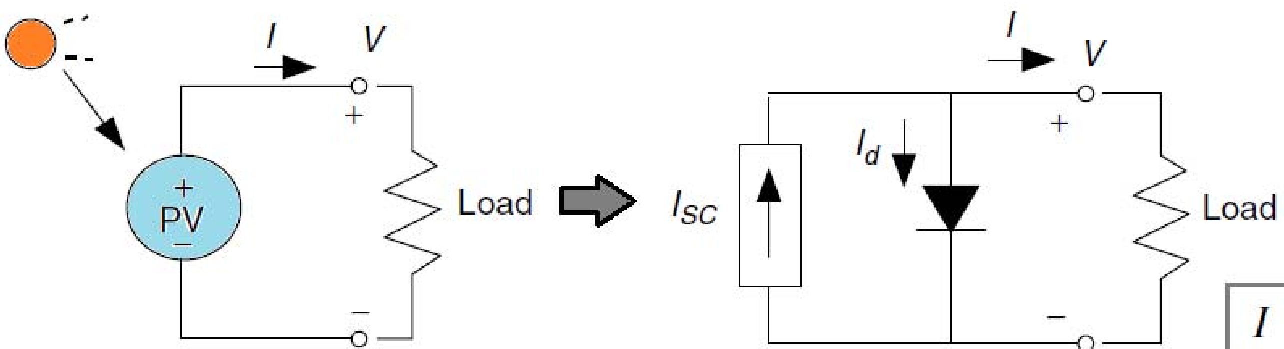


PV Cell Equivalent Circuit

- Electron flow



- Equivalent Circuit: Ideal Current Source in parallel with a real diode

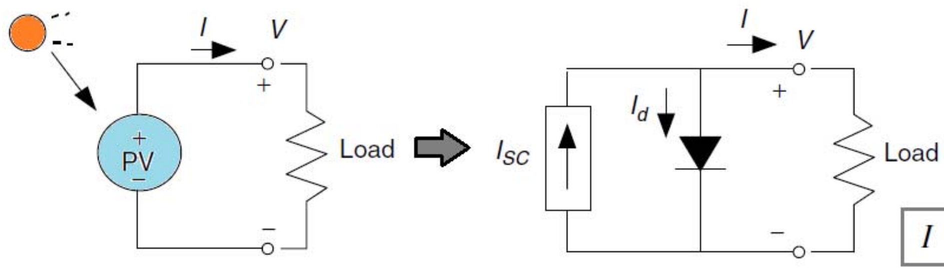


$$I = I_{SC} - I_d$$

I_{SC} short-circuit current

PV Cell Equivalent Circuit

- Equivalent Circuit: Ideal Current Source in parallel with a real diode



$$I_d = I_0(e^{qV_d/kT} - 1)$$

$$I = I_{SC} - I_d$$

I_{SC} short-circuit current

- Open-Circuit Voltage (V_{OC}) & Short-Circuit Current (I_{SC})

$$I = I_{SC} - I_0\left(e^{qV/kT} - 1\right)$$

$$V_d = \frac{kT}{q}\ln\left(\frac{I_d}{I_0} + 1\right)$$

$$I = I_{SC} - I_0(e^{38.9\ V} - 1)$$

at 25°C

$$V_{OC} = \frac{kT}{q}\ln\left(\frac{I_{SC}}{I_0} + 1\right)$$

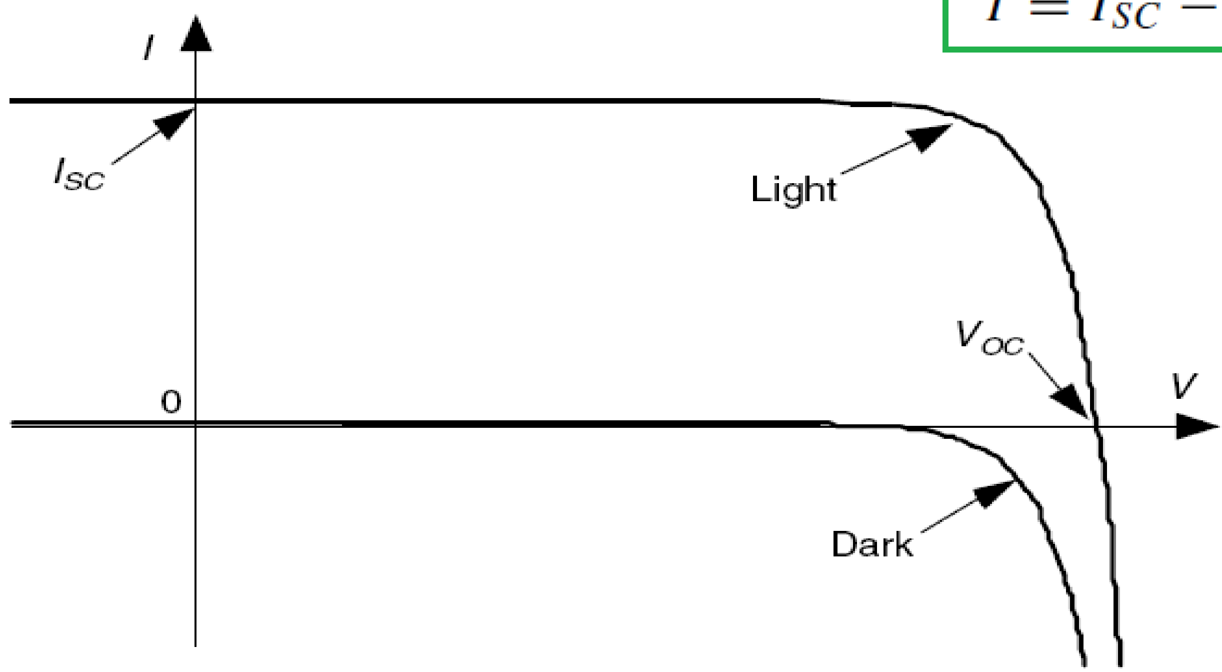
open-circuit voltage $I = 0$

$$V_{OC} = 0.0257\ln\left(\frac{I_{SC}}{I_0} + 1\right)$$

I-V Curve

- Isc (Short Circuit Current) is directly proportional to solar insolation

$$I = I_{SC} - I_0(e^{38.9\ V} - 1)$$



I-V Curve Example

- Example: Consider a 100 cm^2 PV cell with reverse saturation current 10^{-12} A/cm^2. In the full sun ("peak sun"), it produces a short-circuit current of 40 mA/cm^2 at 25 °C. Find the open-circuit voltage at full sun and again for 50% sunlight. Plot I-V curve.

I-V Curve Example - Plot

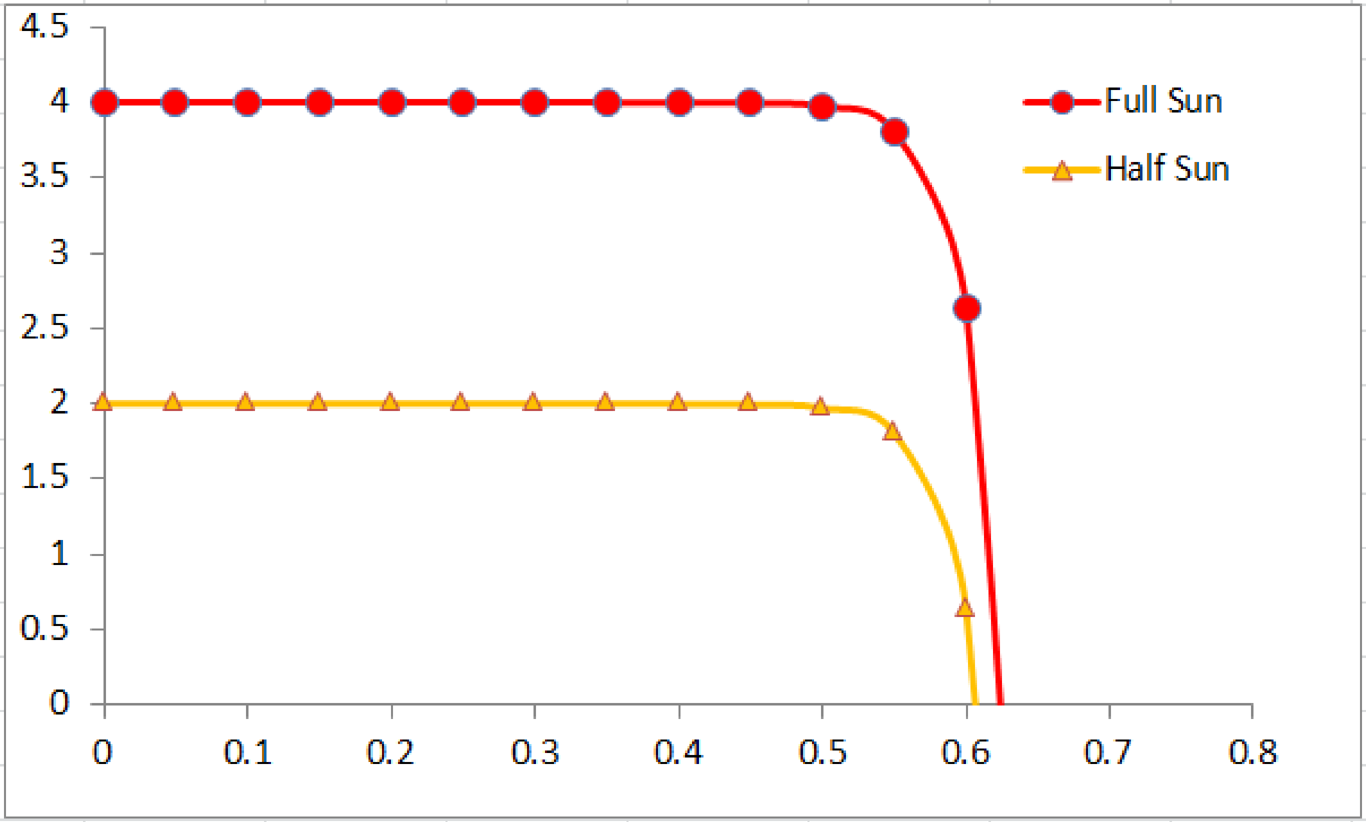
$$I_d = I_0(e^{qV_d/kT} - 1)$$

$$I = I_{SC} - I_0(e^{38.9\ V} - 1)$$

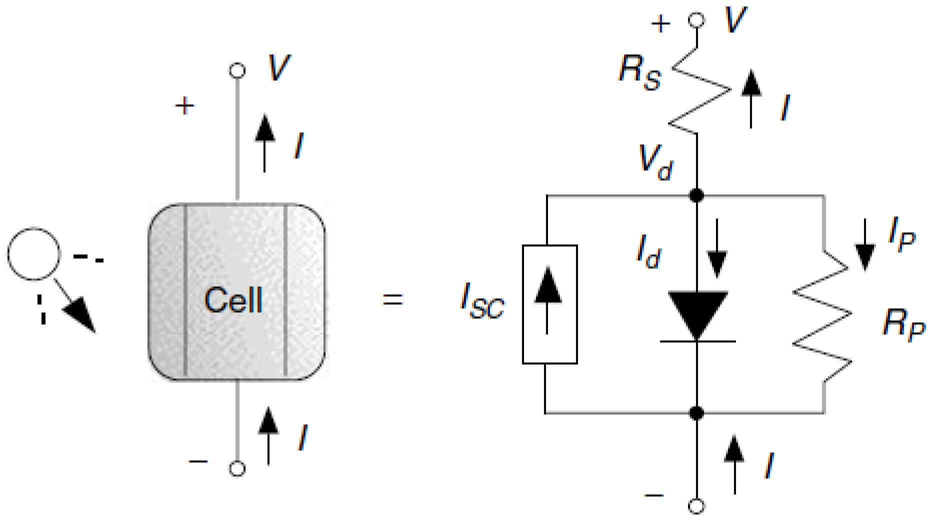
Ex8.3.xlsx

	A	B	C	D	E	F	G
1	Io	Vd	Id	IscFull	Ifull	IscHalf	Ihalf
2	1E-10	0	0	4	4	2	2
3	1E-10	0.05	5.99E-10	4	4	2	2
4	1E-10	0.1	4.79E-09	4	4	2	2
5	1E-10	0.15	3.41E-08	4	4	2	2
6	1E-10	0.2	2.39E-07	4	4	2	2
7	1E-10	0.25	1.67E-06	4	3.999998	2	1.999998
8	1E-10	0.3	1.17E-05	4	3.999988	2	1.999988
9	1E-10	0.35	8.18E-05	4	3.999918	2	1.999918
10	1E-10	0.4	0.000572	4	3.999428	2	1.999428
11	1E-10	0.45	0.004002	4	3.995998	2	1.995998
12	1E-10	0.5	0.027992	4	3.972008	2	1.972008
13	1E-10	0.55	0.195763	4	3.804237	2	1.804237
14	1E-10	0.6	1.369094	4	2.630906	2	0.630906
15	1E-10	0.65	9.574938	4	-5.57494	2	-7.57494
16	1E-10	0.7	66.96359	4	-62.9636	2	-64.9636
17	1E-10	0.75	468.3187	4	-464.319	2	-466.319

I-V Curve Example - Plot



More Complex Equivalent Circuit



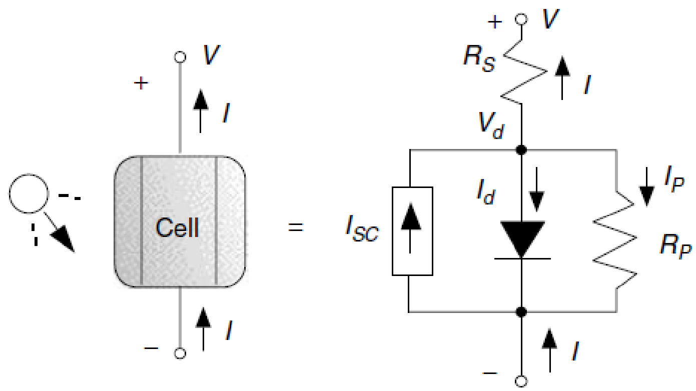
$$I_{SC} = I + I_d + I_P$$

$$I = I_{SC} - I_0\left\{\exp\left[\frac{q(V + I \cdot R_S)}{kT}\right] - 1\right\} - \left(\frac{V + I \cdot R_S}{R_P}\right)$$

$$I = I_{SC} - I_0\left[e^{38.9(V+IR_S)} - 1\right] - \frac{1}{R_P}(V + IR_S) \qquad \text{at } 25^\circ\text{C}$$

$$I = I_{SC} - I_0(e^{38.9V_d} - 1) - \frac{V_d}{R_P} \qquad V = V_d - IR_S$$

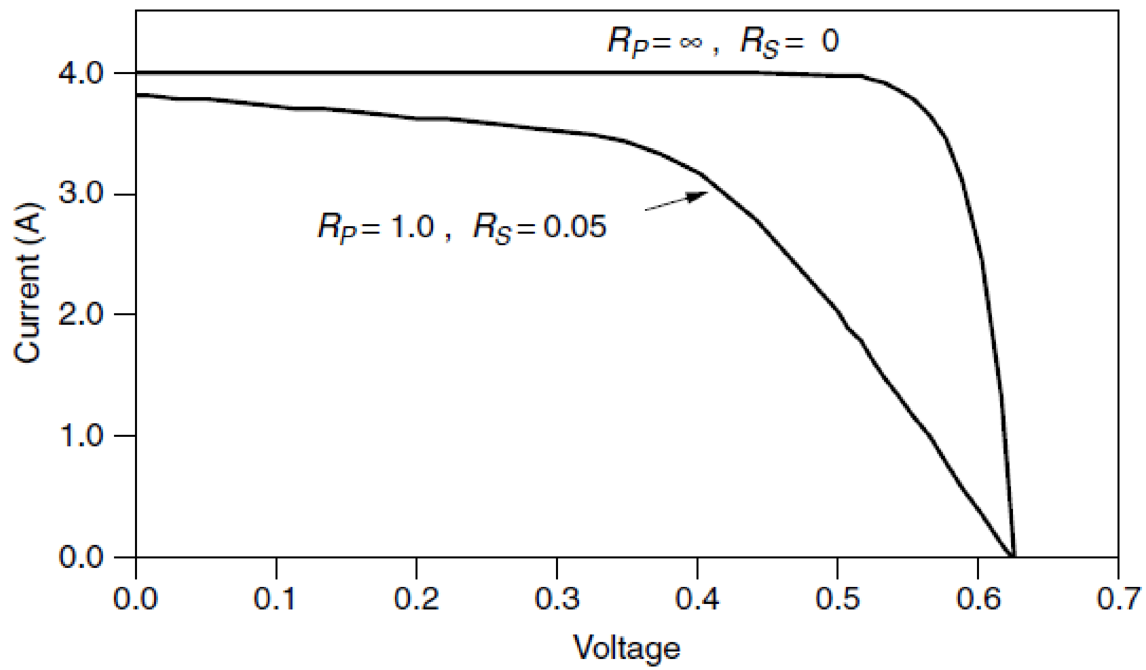
More Complex Equivalent Circuit



$$I = I_{SC} - I_0\left[e^{38.9(V+IR_S)} - 1\right] - \frac{1}{R_P}(V + IR_S)$$

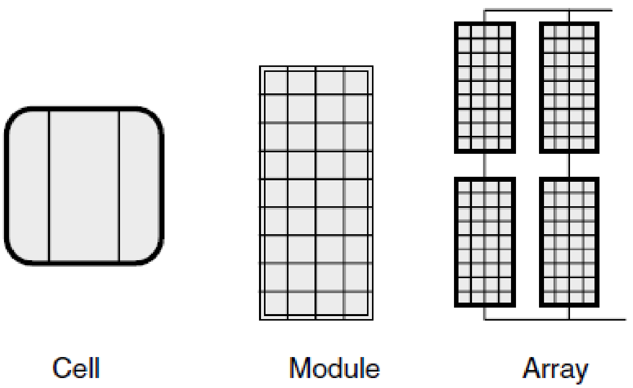
$$I = I_{SC} - \mathrm{I}_0(e^{38.9V_d} - 1) - \frac{V_d}{R_P}$$

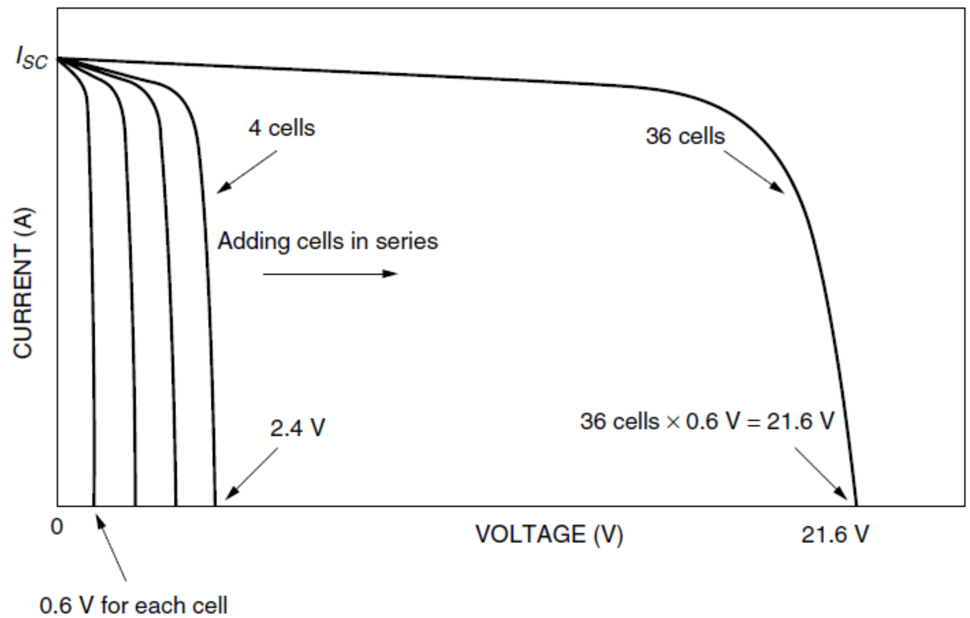
$$V = V_d - IR_S$$



Cell to Module

- A typical module has 36 cells.





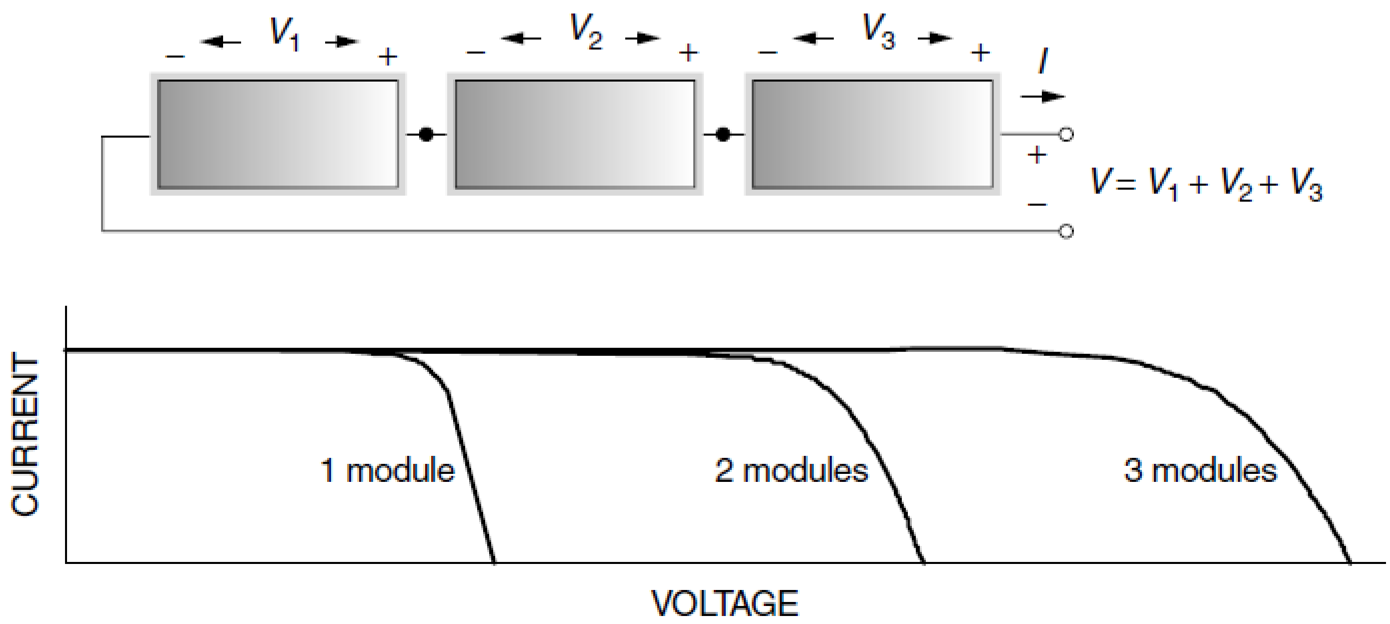
$$V_{\text{module}} = n(V_d - IR_S)$$

Class Activity 8

- **Voltage and Current from a PV Module.** A PV module is made up of 36 identical cells, all wired in series. With 1-sun insolation (1 kW/m2), each cell has short-circuit current $I_{SC} = 3.4$ A and at 25°C its reverse saturation current is $I_0 = 6 \times 10{-}10$ A. Parallel resistance $R_P = 6.6\ \Omega$ and series resistance $R_S = 0.005\ \Omega$.
- Question1: Find the voltage (Vmodule), current (I), and power delivered (P=Vmodule*I) when the junction voltage of each cell (Vd) is
  - (a) 0.50 V.
  - (b) 0.49 V
  - (c) 0.51 V
  - (d) 0.55V
- Question 2: What is the maximum power point in terms of current and module voltage

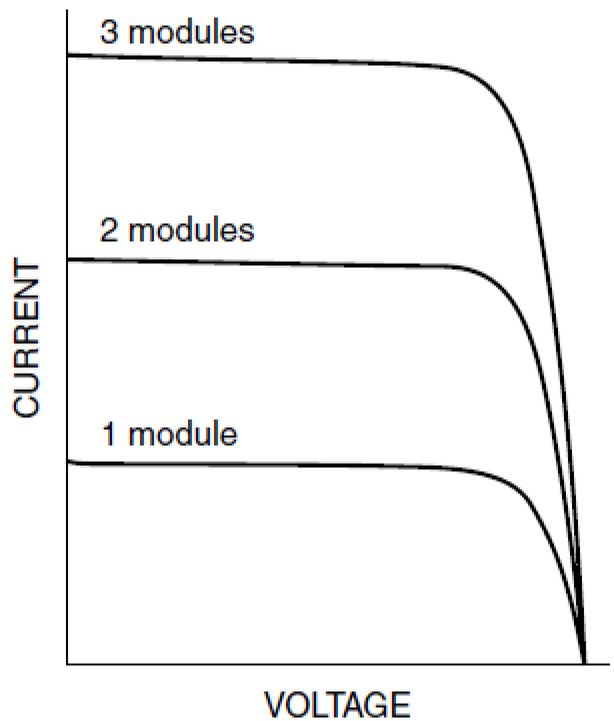
From Modules to Arrays

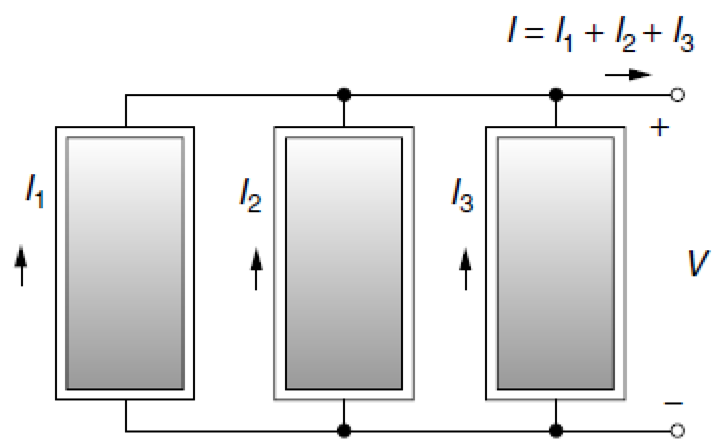
- PV Modules in Series



From Modules to Arrays

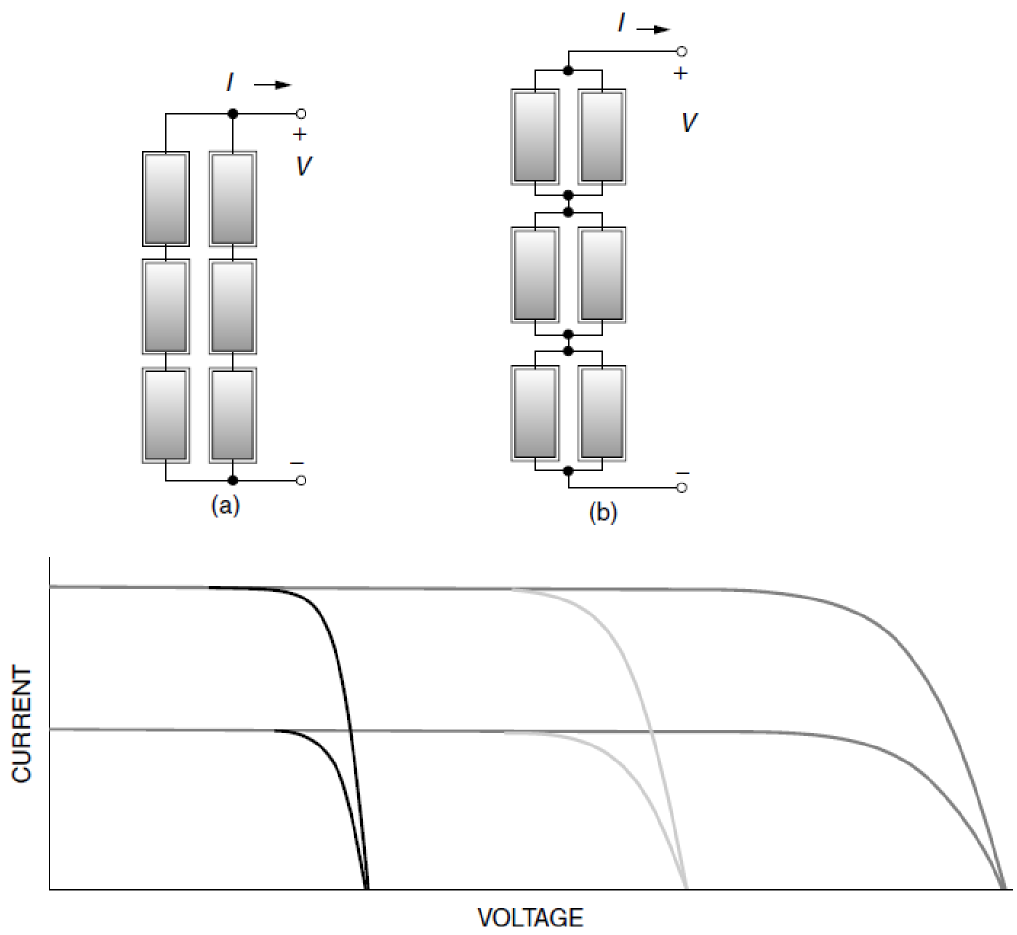
- PV Modules in Parallel





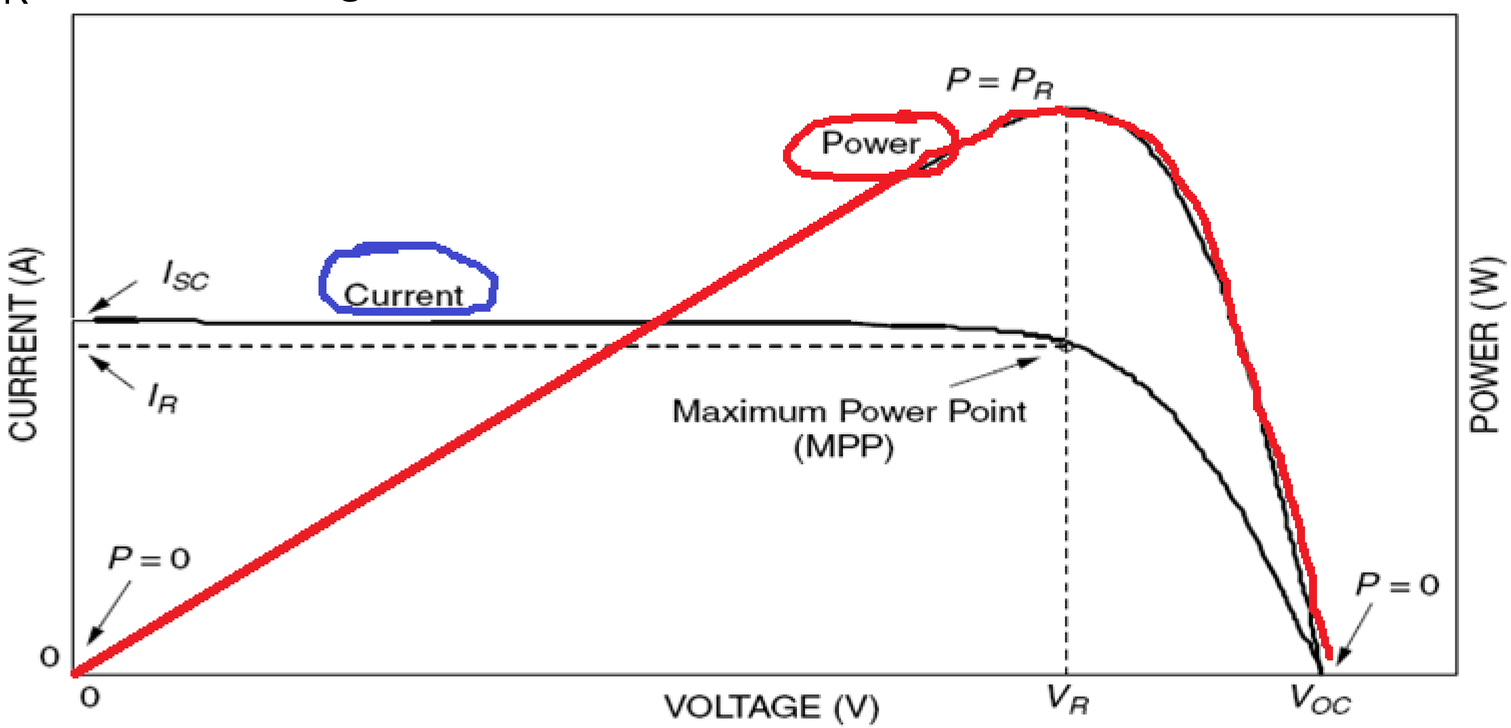
From Modules to Arrays

- PV Modules in Parallel and Series
- **Which array is better?**



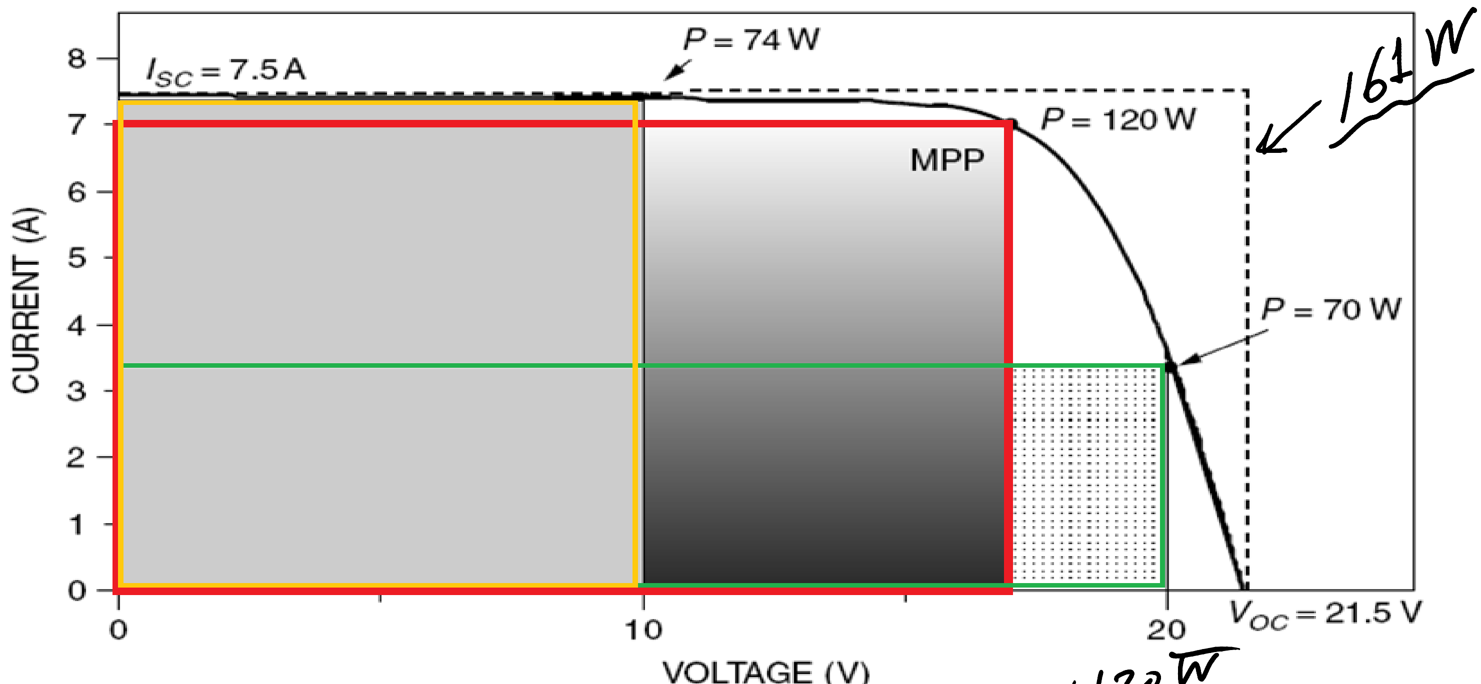
I-V Curve and Power Output

- Maximum Power Point (MPP)
- I_R: Rated Current
- V_R: Rated Voltage



MPP and FF (Form Factor)

- The biggest possible rectangle – the area is power
- Fill Factor (FF): performance measure: ratio of the power at MPP to the product of V_{oc} and I_{sc}. (solid_rectangle/dotted_rectangle)



$$\text{Fill factor (FF)} = \frac{\text{Power at the maximum power point}}{V_{OC}\ I_{SC}} = \frac{V_R I_R}{V_{OC} I_{SC}}$$

120 W

161 W

FF = .74

PV Module Data

Examples of PV Module Performance Data Under Standard Test Conditions (1 kW/m², AM 1.5, 25°C Cell Temperature) (STC)

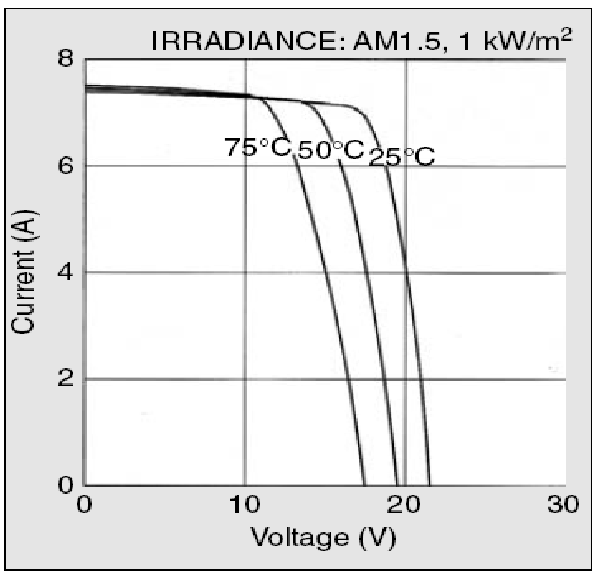
Manufacturer	Kyocera	Sharp	BP	Uni-Solar	Shell
Model	KC-120-1	NE-Q5E2U	2150S	US-64	ST40
Material	Multicrystal	Polycrystal	Monocrystal	Triple junction a-Si	CIS-thin film
Number of cells n	36	72	72		42
Rated Power $P_{DC,STC}$ (W)	120	165	150	64	40
Voltage at max power (V)	16.9	34.6	34	16.5	16.6
Current at rated power (A)	7.1	4.77	4.45	3.88	2.41
Open-circuit voltage V_{OC} (V)	21.5	43.1	42.8	23.8	23.3
Short-circuit current I_{SC} (A)	7.45	5.46	4.75	4.80	2.68
Length (mm/in.)	1425/56.1	1575/62.05	1587/62.5	1366/53.78	1293/50.9
Width (mm/in.)	652/25.7	826/32.44	790/31.1	741/29.18	329/12.9
Depth (mm/in.)	52/2.0	46/1.81	50/1.97	31.8/1.25	54/2.1
Weight (kg/lb)	11.9/26.3	17/37.5	15.4/34	9.2/20.2	14.8/32.6
Module efficiency	12.9%	12.7%	12.0%	6.3%	9.4%

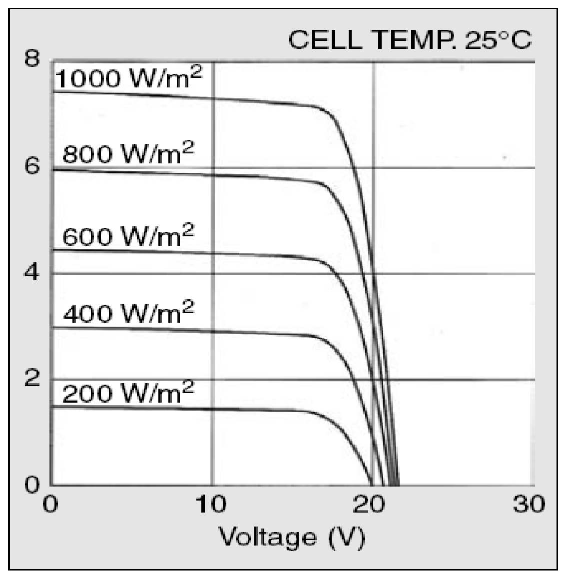
Insolation and Temperature Effect

- Decrease in insolation, decrease in short-circuit current
- Increase in cell temperature, **substantial decrease** in open-circuit voltage, and **slight increase** in short-circuit current
- **For Si cells, Voc drops by 0.37% for each degree Celsius increase in temperature, and ISC increases by 0.05% → Net result: Decrease in maximum power about 0.5%**

relatively too small to consider

- Kyocera 120-W multicrystal-Si module example





Temperature and Insolation Effect

- NOCT (Nominal Operating Cell Temperature) °C
- S: Solar Insolation (kW/m^2)
- Cell temperature (T_{cell})
- Ambient Temperature (T_{amb})

in 1 sun of insolation, cells tend to be 25–35°C hotter than their environment.

$$T_{cell} = T_{amb} + \left(\frac{NOCT - 20^\circ}{0.8}\right) \cdot S$$

$$T_{cell} = T_{amb} + \gamma \left(\frac{\text{Insolation}}{1\ kW/m^2}\right)$$

γ proportionality factor
depends somewhat on windspeed
how well ventilated
between 25°C and 35°C

$\Delta T = T_{cell} - T_{test}$

- **For Si cells, Voc drops by 0.37% for each degree Celsius increase in temperature, and I_{SC} increases by 0.05%. → Net result: Decrease in maximum power about 0.5%**

$$V_{oc_{New}} = V_{oc_{Test}} \cdot (1 - \Delta T \cdot (0.0037))$$

Temperature and Insolation Effect - Example

- **Impact of Cell Temperature on Power for a PV Module.** Estimate cell temperature, open-circuit voltage, and maximum power output for the 150-W BP2150S module under conditions of 1-sun insolation and ambient temperature 30°C. The module has a NOCT of 47°C.

$$V_{OC_{New}} = V_{OC_{Test}} \cdot (1 - \Delta T \cdot (0.0037))$$

Standard Test Conditions (1 kW/m², AM 1.5, 25°C Cell Temperature)

Manufacturer	BP
Model	2150S
Material	Monocrystal
Number of cells n	72
Rated Power $P_{DC,STC}$ (W)	150
Voltage at max power (V)	34
Current at rated power (A)	4.45
Open-circuit voltage V_{OC} (V)	42.8
Short-circuit current I_{SC} (A)	4.75
Length (mm/in.)	1587/62.5
Width (mm/in.)	790/31.1
Depth (mm/in.)	50/1.97
Weight (kg/lb)	15.4/34
Module efficiency	12.0%

Temperature and Insolation Effect - Example

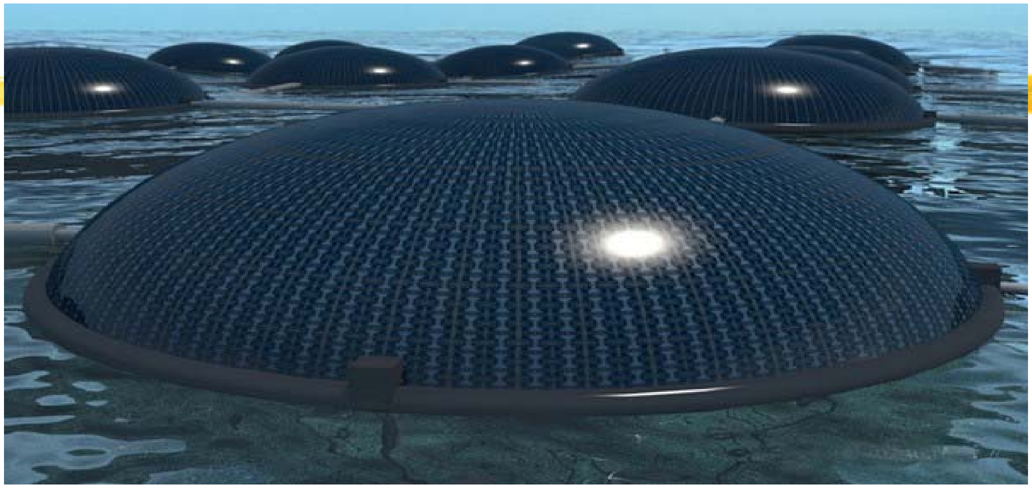
$V_{oc_{new}} = V_{oc_{OLD}} \cdot (1 - 0.0037\Delta T)$

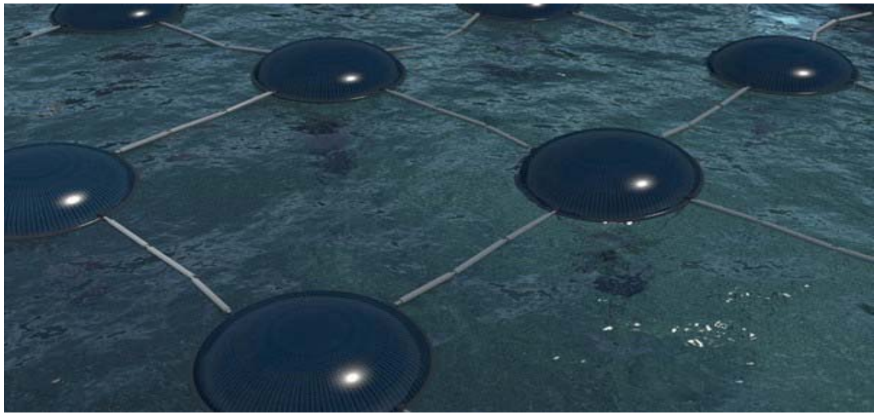
$\Delta T = T_{cell} - T_{test\ condition}$

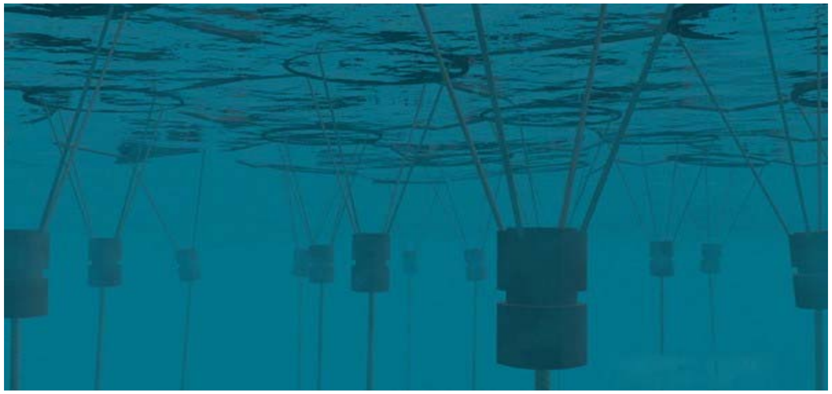
Standard Test Conditions (1 kW/m^2, AM 1.5, 25°C Cell Temperature)

Manufacturer	BP
Model	2150S
Material	Monocrystal
Number of cells n	72
Rated Power $P_{DC,STC}$ (W)	150
Voltage at max power (V)	34
Current at rated power (A)	4.45
Open-circuit voltage V_{OC} (V)	42.8
Short-circuit current I_{SC} (A)	4.75
Length (mm/in.)	1587/62.5
Width (mm/in.)	790/31.1
Depth (mm/in.)	50/1.97
Weight (kg/lb)	15.4/34
Module efficiency	12.0%

Solar Cell Cooling

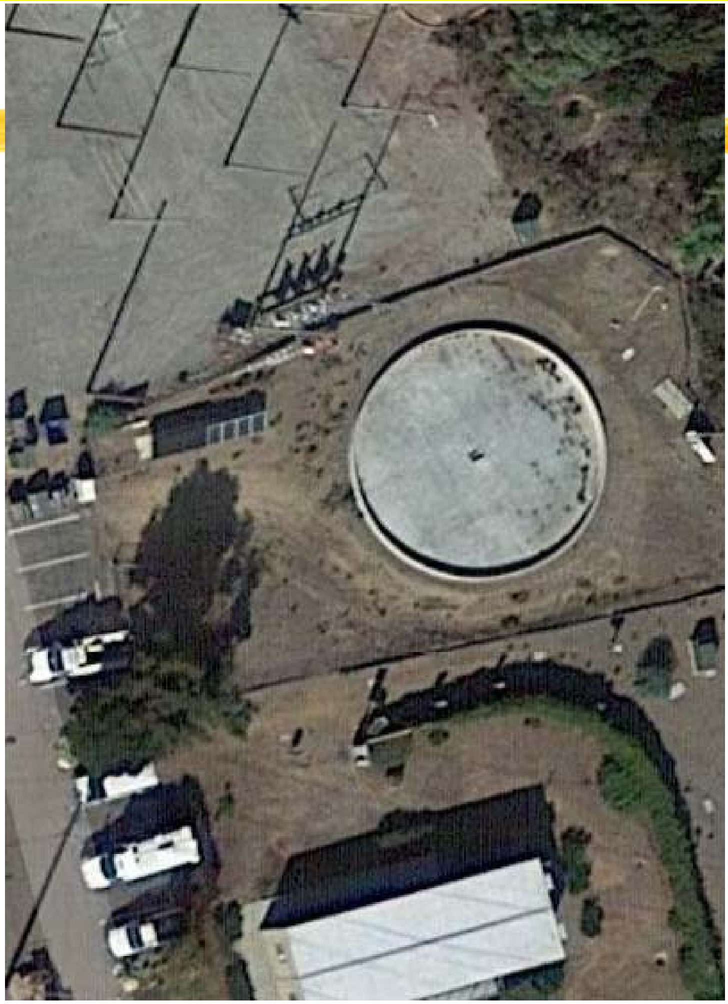






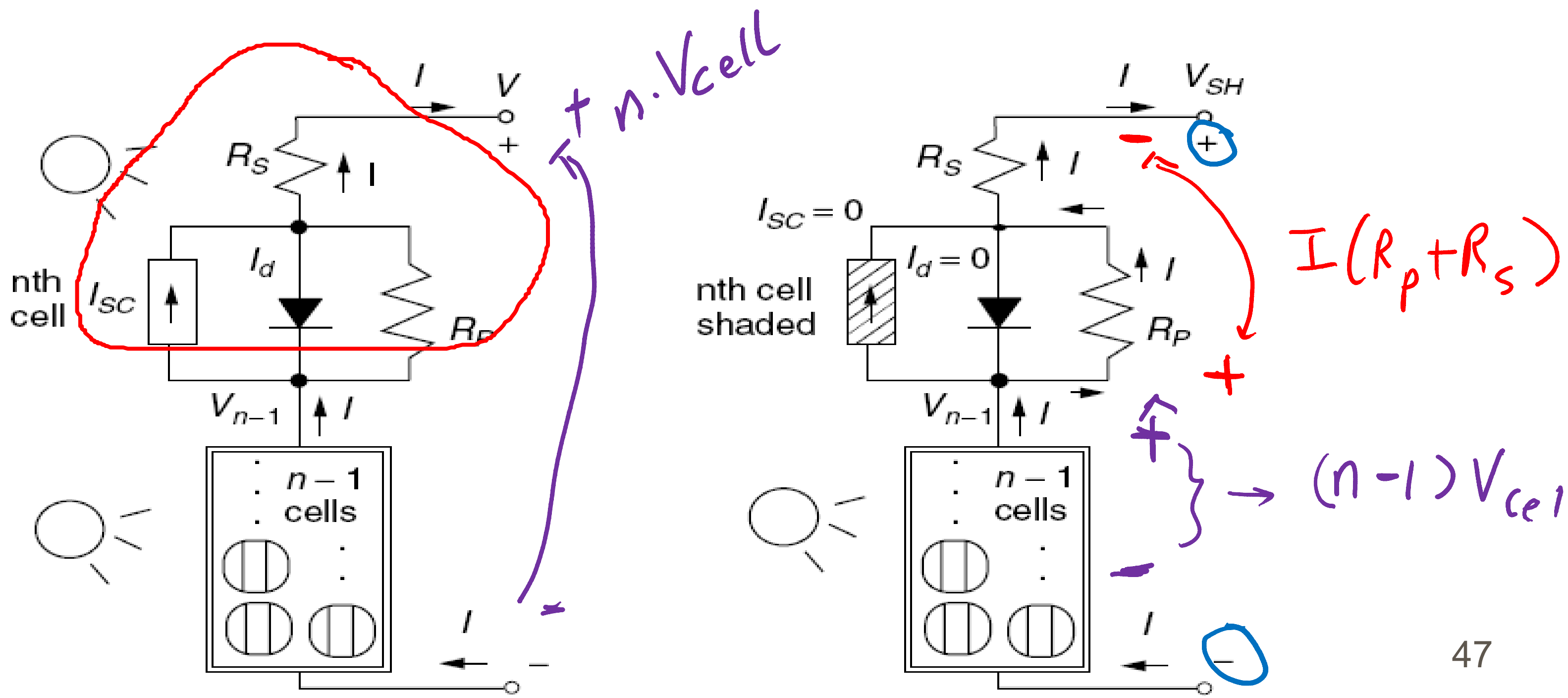
Floating Solar Cells

Floating Solar Cells – Not always easy



Physics of Shading

- All cells under sun
  - The same current flows through each cell
- Top cell under shade
  - The current source is reduced to zero for the cell
  - Now the current from other cells must flow through Rp (and Rs), which drop the voltage, instead of adding voltage.

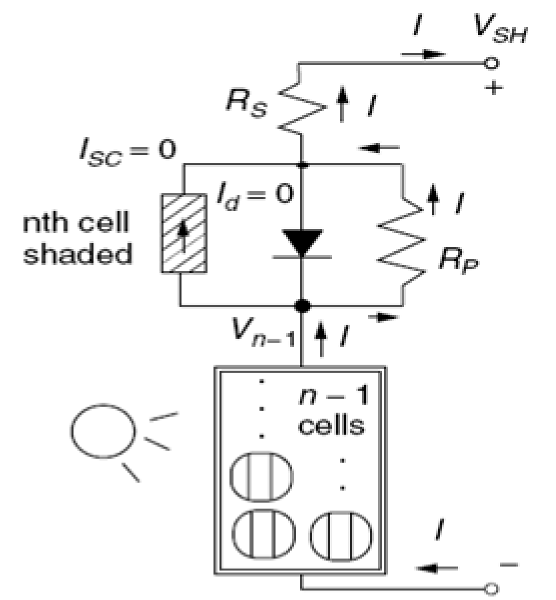


Physics of Shading

$$V_{SH} = V_{n-1} - I(R_P + R_S)$$

$$V_{n-1} = \left(\frac{n-1}{n}\right) V$$

$$V_{SH} = \left(\frac{n-1}{n}\right) V - I(R_P + R_S)$$



The drop in voltage Δ V at any given current I

$$\Delta V = V - V_{SH}$$

$$= V - \left(1 - \frac{1}{n}\right) V + I(R_P + R_S)$$

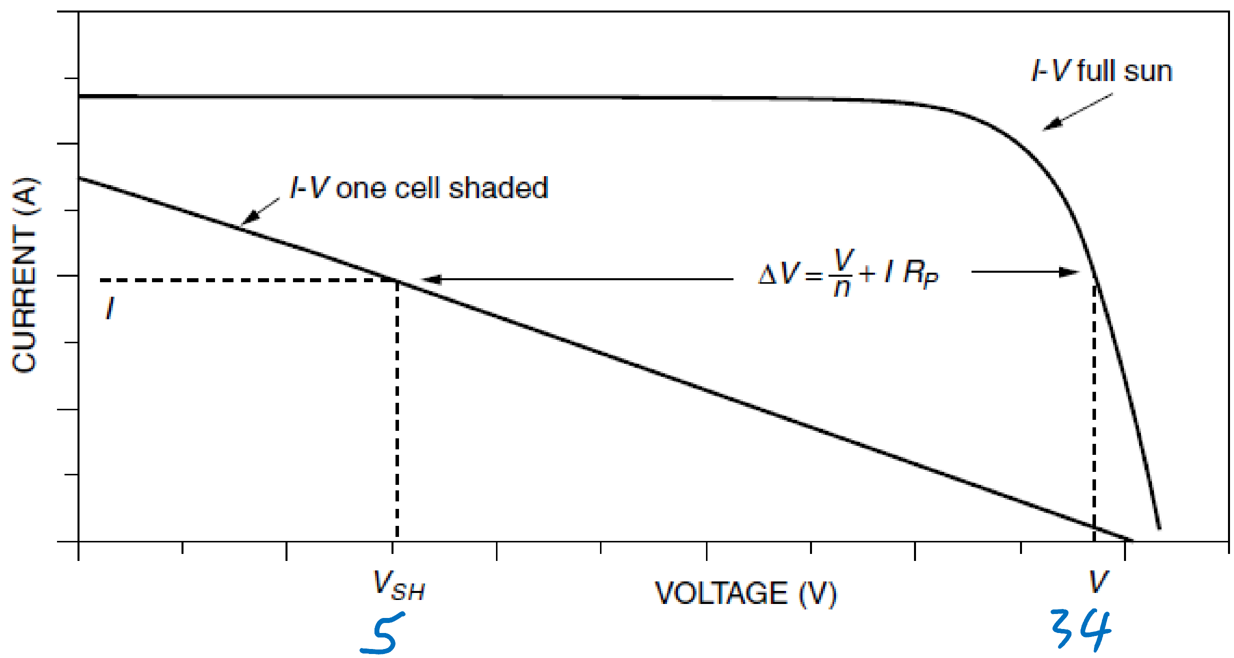
$$= \frac{V}{n} + I(R_P + R_S)$$

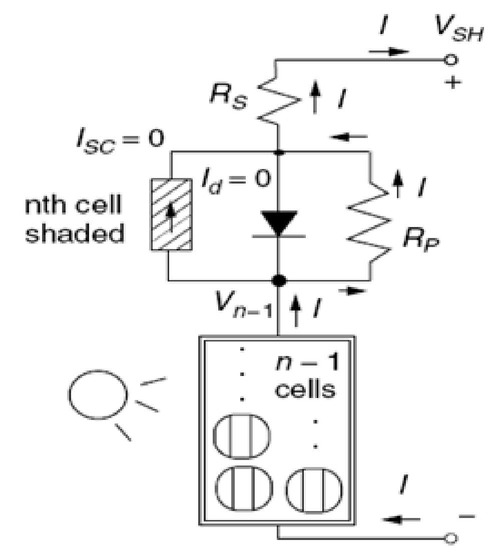
parallel resistance R_P is so much greater than the series resistance R_S,

$$\Delta V \cong \frac{V}{n} + I R_P$$

Physics of Shading

- Huge impact on module voltage of shade

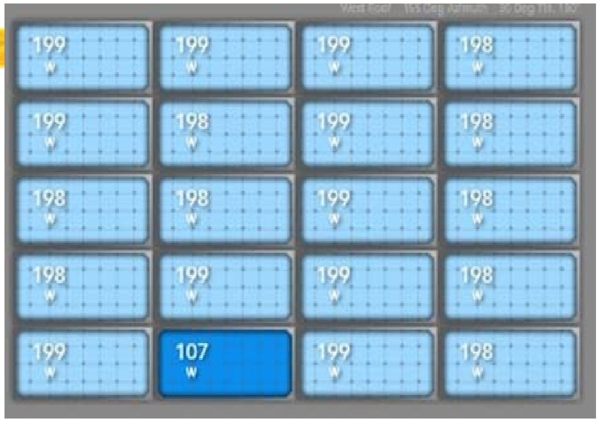




Effect of shading one cell in an n-cell module. At any given current, module voltage drops from V to $V - \Delta V$.

Shading Effect - Summary

- All cells under sun
  - The same current flows through each cell
- Example: **a PV module charging a battery**
  - **Top cell under shade**
  - The current source is reduced to zero for the cell
  - Now the current from other cells must flow through Rp (parallel resistance of the shaded cell), which drops the voltage, instead of adding voltage.
  - Output [Power] of a PV module can be reduced dramatically when even a small portion of it is shaded.
  - Even a single cell under shade in a long string of cells can easily cut **output power** by **more than half.**
  - External **diode**s mitigate the impacts of shading



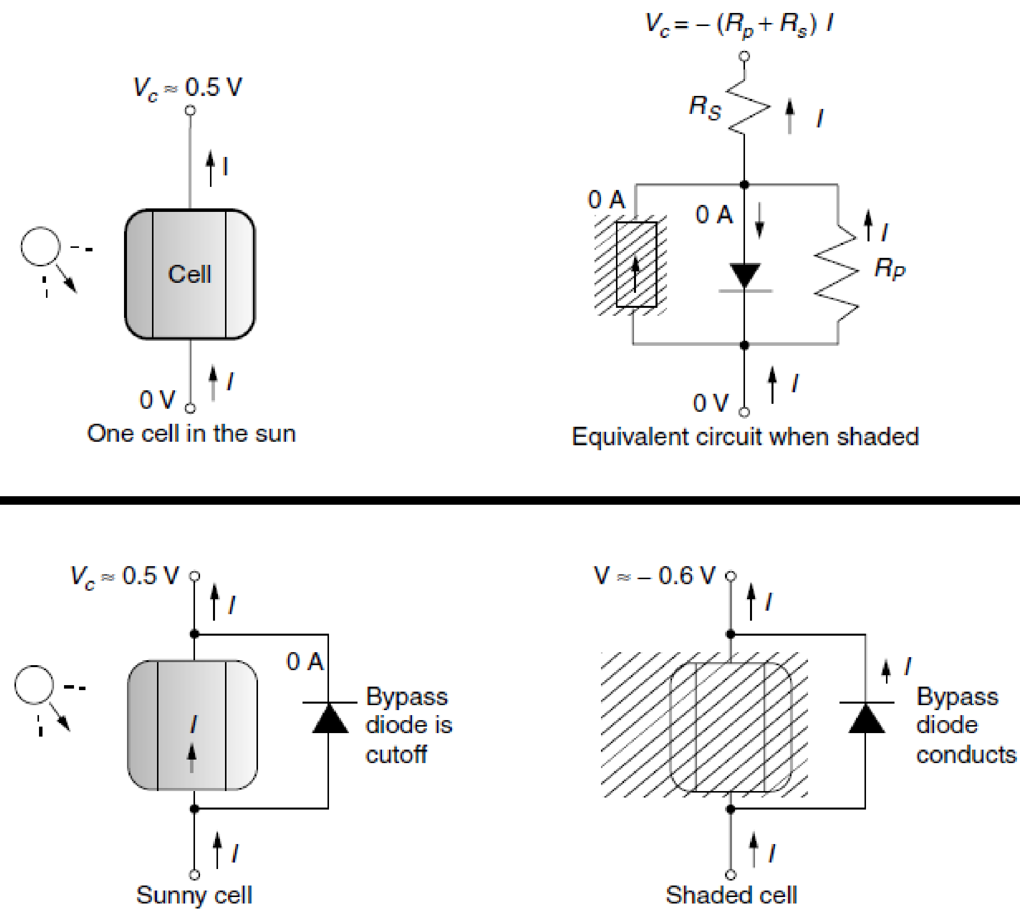
Physics of Shading

- **Example**: **Impacts of Shading on a PV Module.** The 36 series connected cell PV module (with 1-sun insolation [1 kW/m^2], each cell's short-circuit current I_{SC} = 3.4 A , 25◦C reverse saturation current $I_0 = 6 \times 10^{-10}$ A) had a parallel resistance per cell of R_P = 6.6 Ω and series resistance Rs = 0.005 . In full sun and at current I = 2.14 A the output voltage was found there to be V = 19.41 V. If one cell is shaded and this current somehow stays the same, then:
- a. What would be the new module output voltage and power?
- b. What would be the voltage drop across the shaded cell?
- c. How much power would be dissipated in the shaded cell?

$$\Delta V \cong \frac{V}{n} + I R_P$$

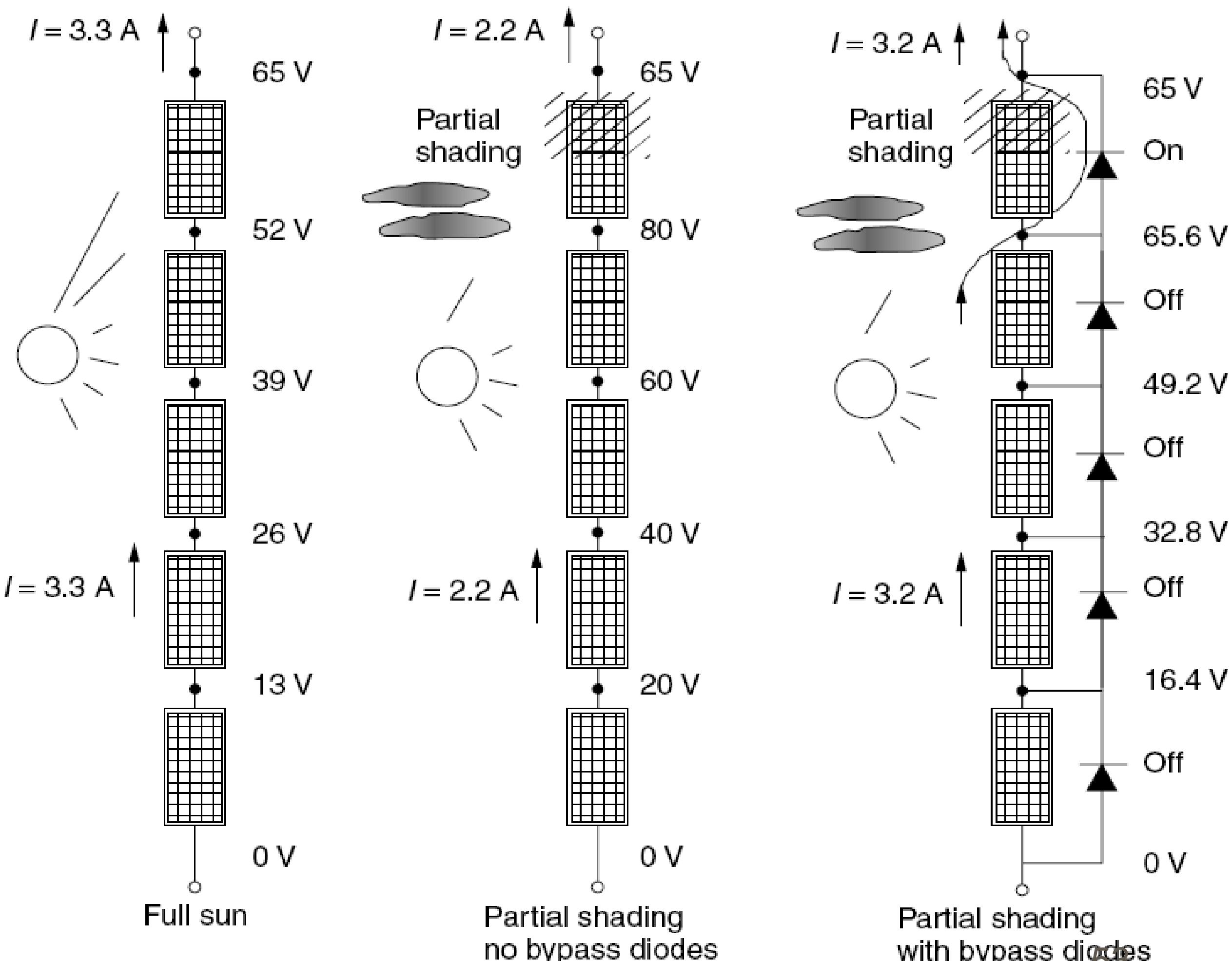
Shade Mitigation by Bypass Diode

- Voltage drop problem can be corrected by adding a bypass diode across each cell.



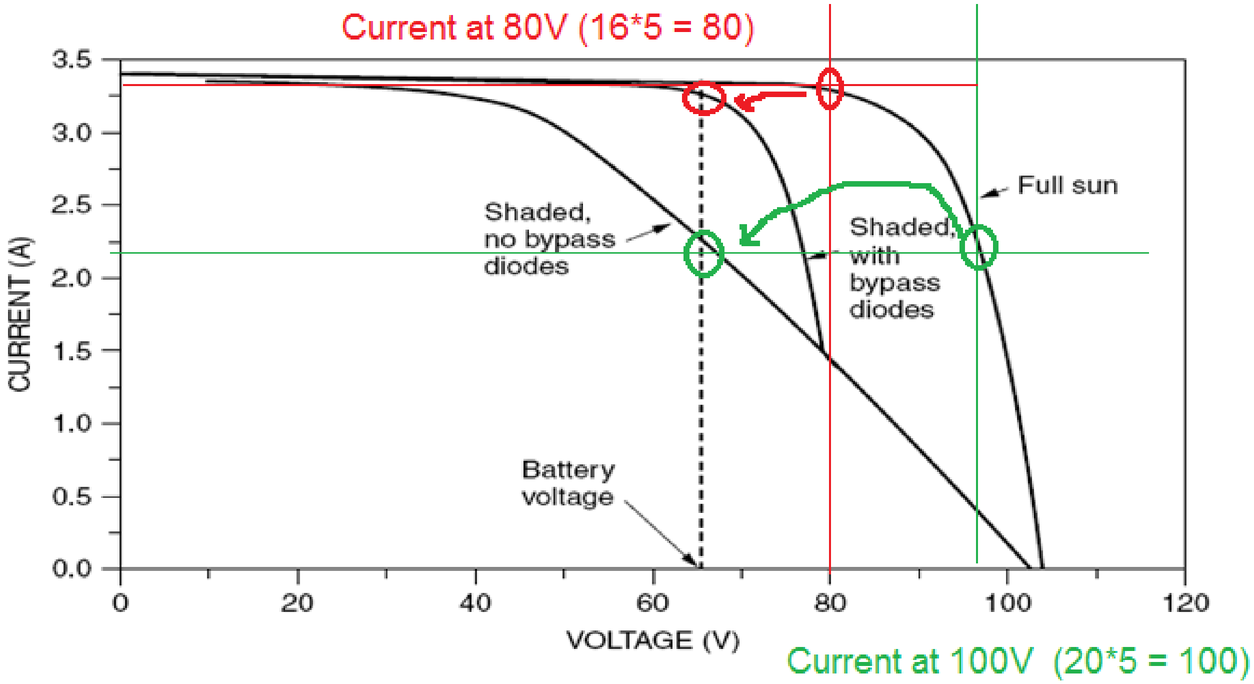
Mitigation by Bypass Diode

- Rp= 6.6 Ω
- **Without Bypass Diode**
  - 1. Voltage Drop across the shaded elements
  - 2. Each cells to compensate the voltage drop by increasing each cell voltage
  - 3. The increased cell voltage decreases the cell current (by the I-V curve)
- **With Bypass Diode**
  - 1. The voltage drop at the shaded cells are minimal (Regular Diode Drop of 0.6 V)
  - 2. The voltage increase in each cell is trivial
  - 3. The decrease in cell current is negligible

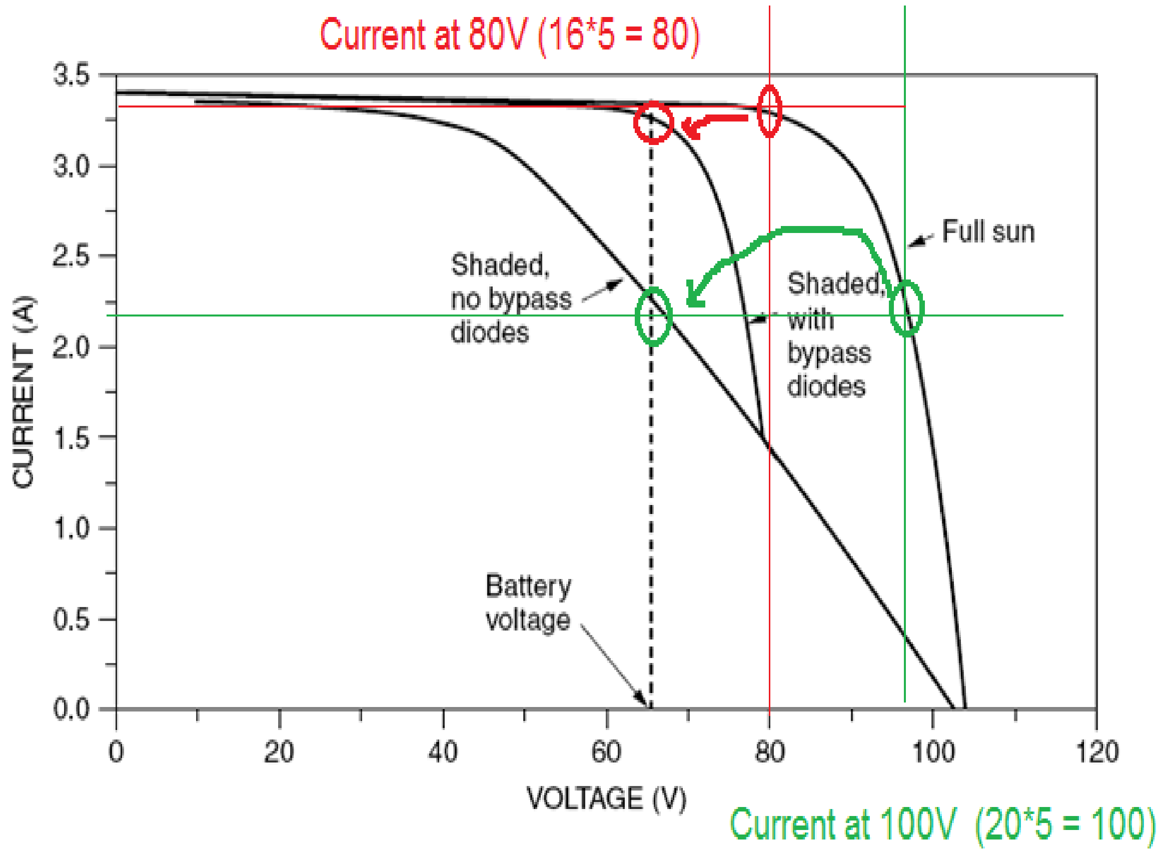


Impact of Bypass Diode

- A case: Five (5) PV modules in series delivering 65V to a battery bank – one module has 2 shaded cells.
- Charging current drops to 2.2A from 3.3A
- With a bypass diode, the current is recovered to 3.2 A



Impact of Bypass Diode - Details

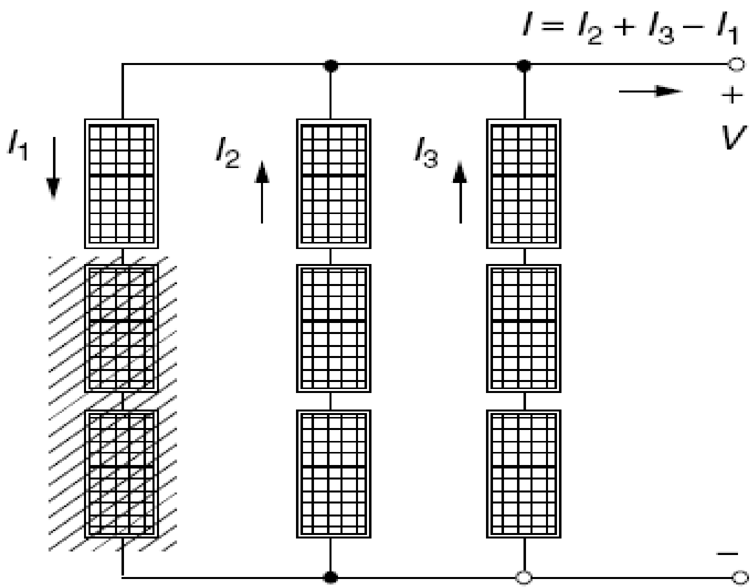


With bypass diodes,
The cell voltage is now 16V each, therefore, the module voltage runs as if 80V, which by the I-V curve,
has the current of 3.2 A.

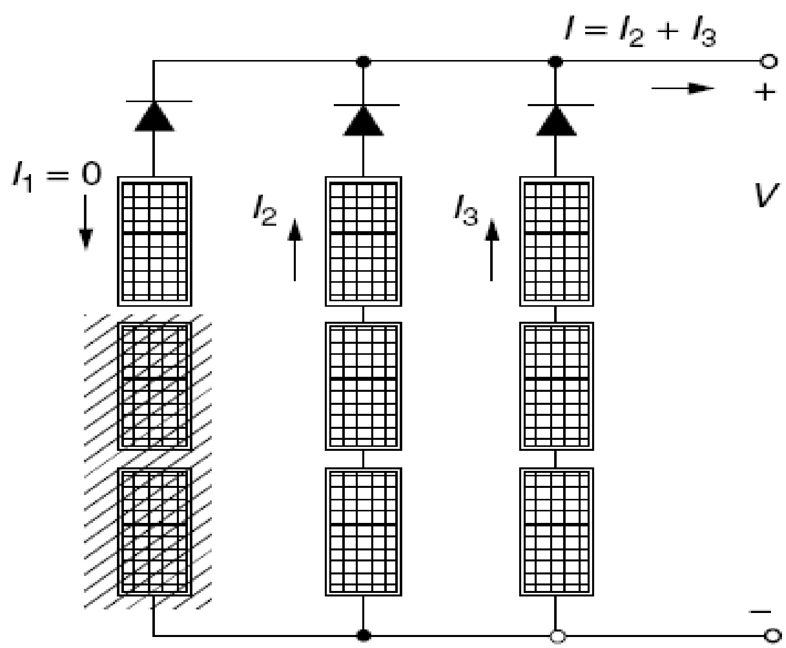
Under shaded cell condition, since unaffected cells have to make out 80V, the Cell voltage is now Considered as 20V -> which would be interpreted as the module voltage runs at 100V ,and corresponding current is 2.2A.

Partial Cell under Performance – Blocking Diode

- In Parallel Combination of strings of cells: Separate the malfunctioning or shaded string of cells by blocking (or "Isolation") diode at the top of each string

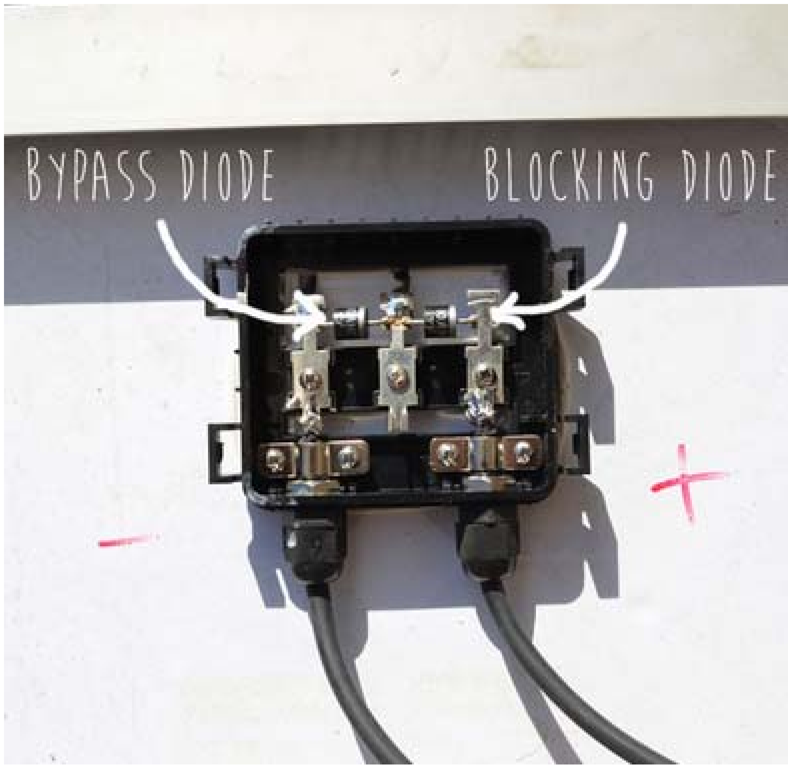


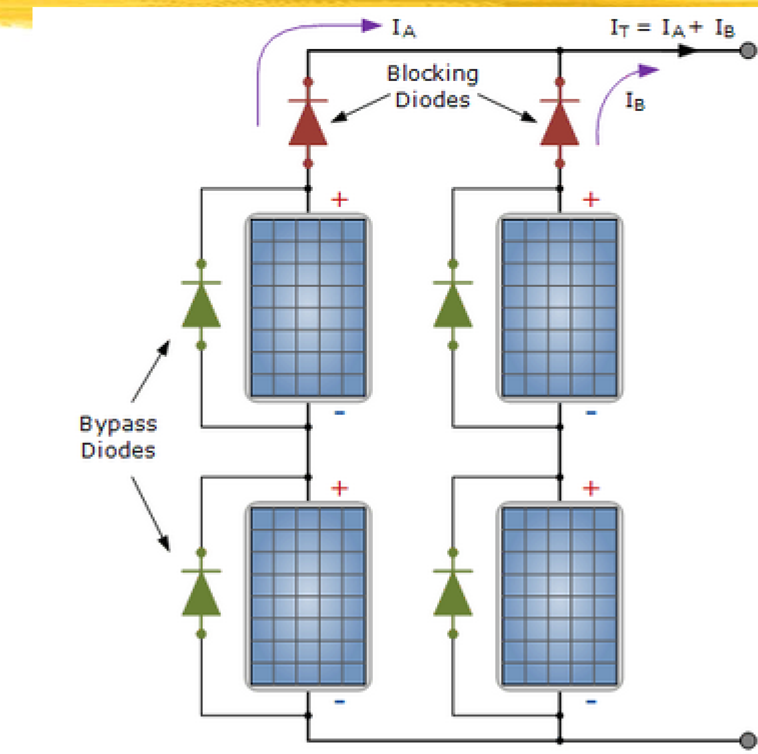
Without blocking diodes



With blocking diodes

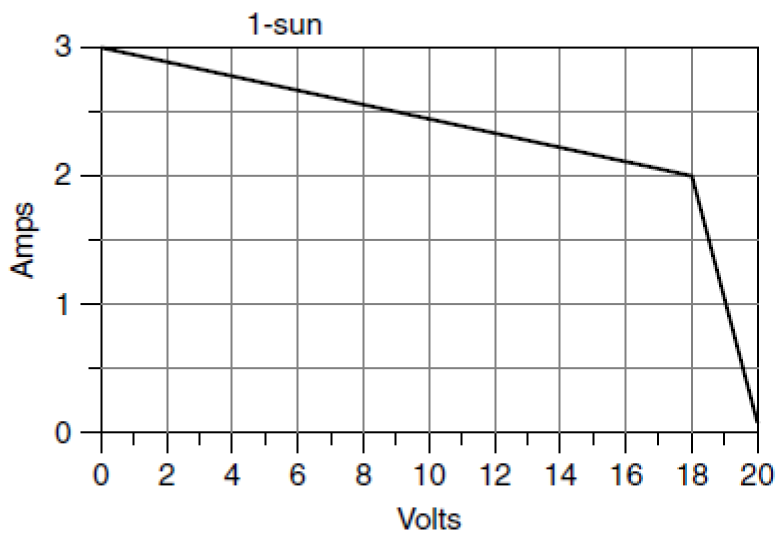
Bypass and Blocking Diode

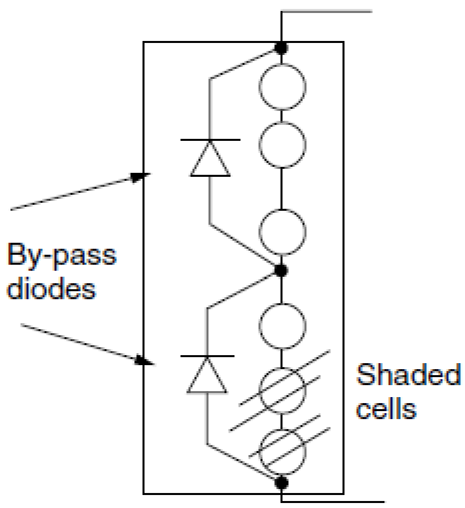




Shaded I-V Curve

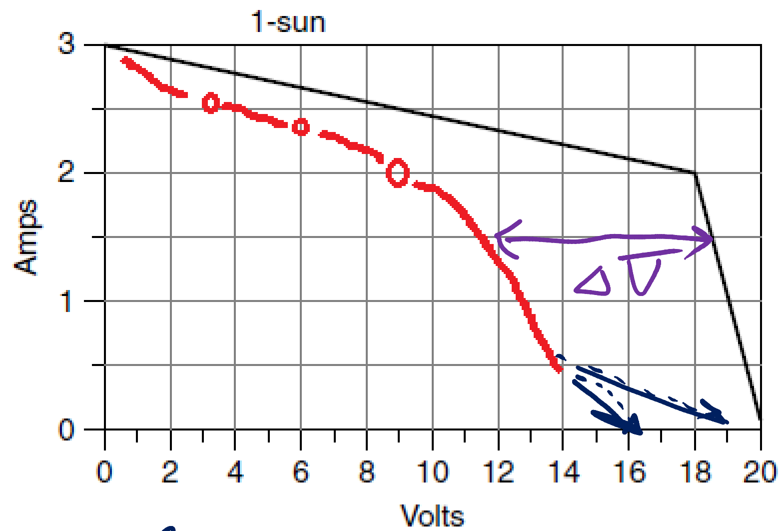
- Suppose a PV module has the 1-sun I-V curve shown below. Within the module itself, the manufacturer has provided a pair of bypass diodes to help the panel deliver some power even when many of the cells are shaded. Each diode bypasses half of the cells, as shown. You may consider the diodes to be "ideal;" that is, they have no voltage drop across them when conducting.
- Suppose there is enough shading on the bottom cells to cause the lower diode to start conducting. **Draw the new "shaded" I-V curve for the module.**

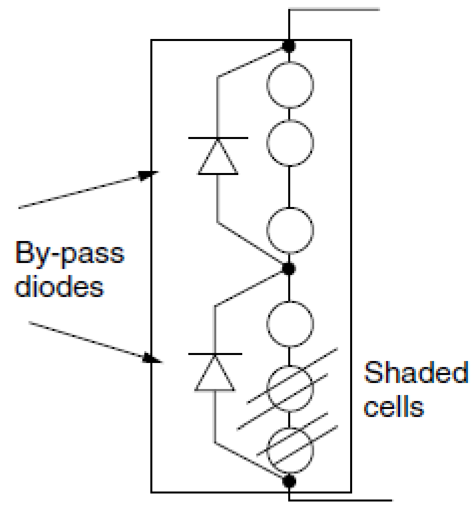




Shaded I-V Curve

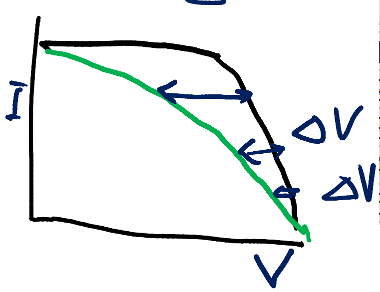
- Suppose there is enough shading on the bottom cells to cause the lower diode to start conducting. Draw the new "shaded" I-V curve for the module.





$$\Delta V = \frac{V}{n} + I \cdot R_p$$

$$\Delta V \propto I \cdot R_{ps}$$

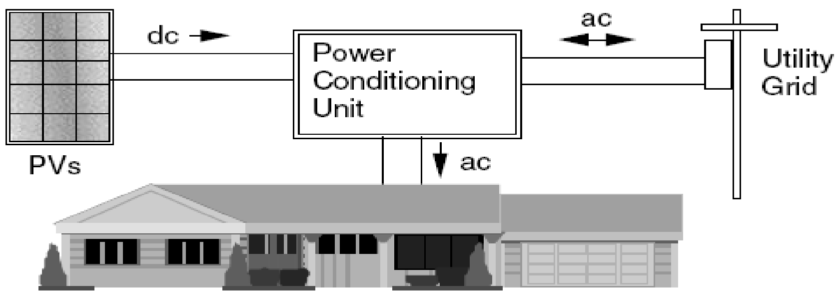


	A	B	C	D	E	F	G
1	Vfull	Afull	Veach	Vshaded	Vshadeeach	Veq	Ashade
2	3	2.75	0.5	3	1	6	2.6
3	6	2.6	1	6	2	12	2.4
4	9	2.5	1.5	9	3	18	2
5							

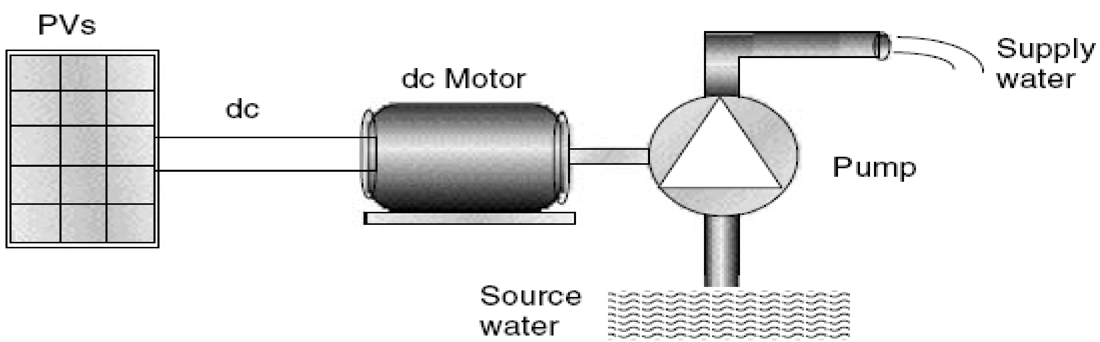
Chapter 9: Photovoltaic (PV) Systems

PV System Configurations

- Utility connected PV System: Feed/get power directly from/to the utility grid and PV

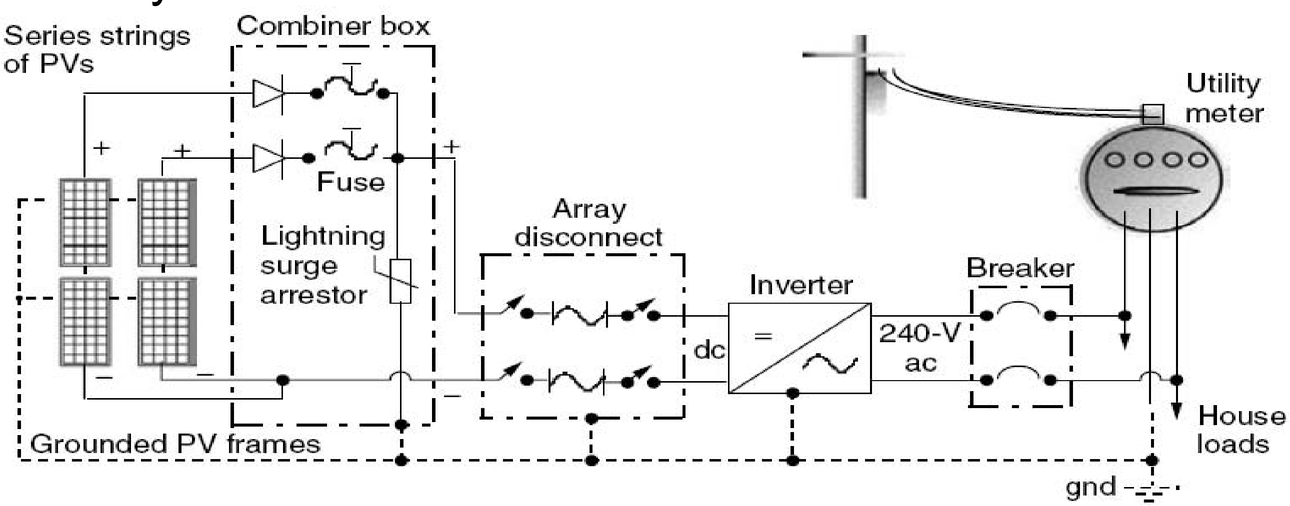


- Stand-alone system: Charge batteries (with or without Generator backup) and serves load

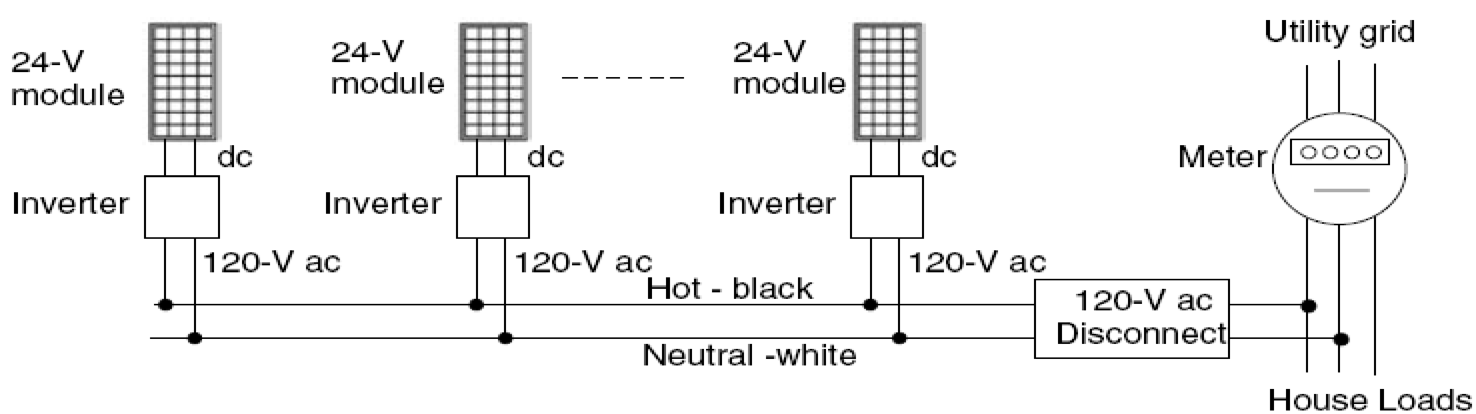


Grid-Connected PV System

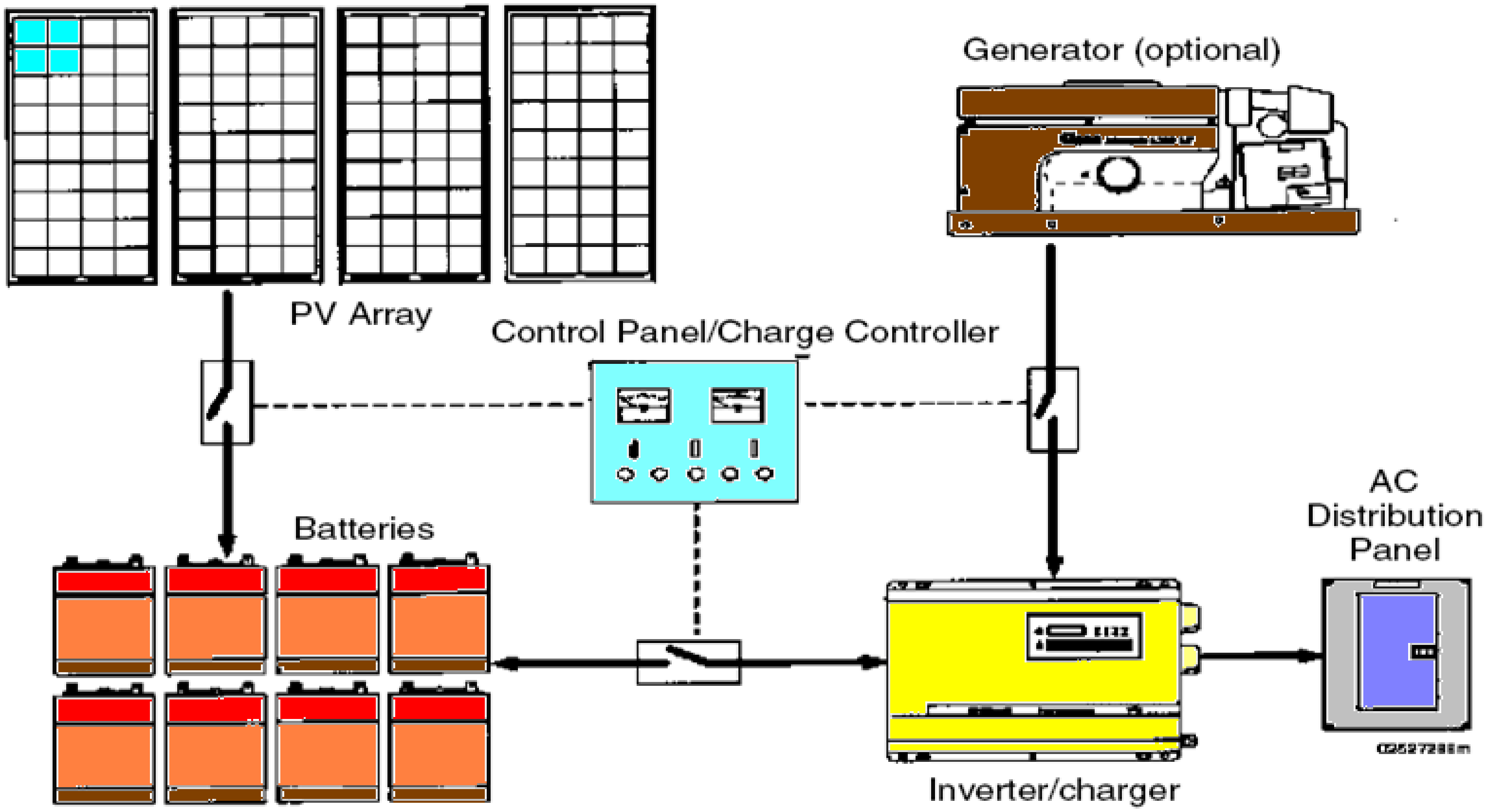
⌘ Combined Inversion system



⌘ Separate Small Inversion System

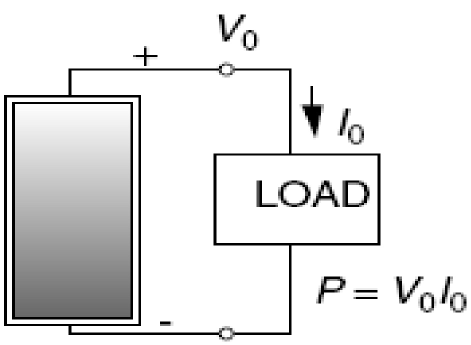


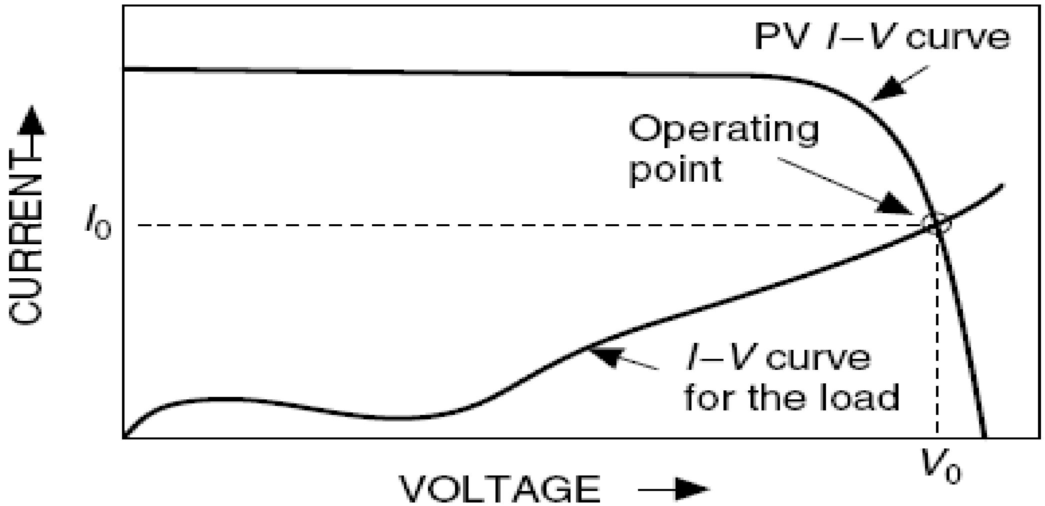
Example Stand-Alone PV System



Operating Point

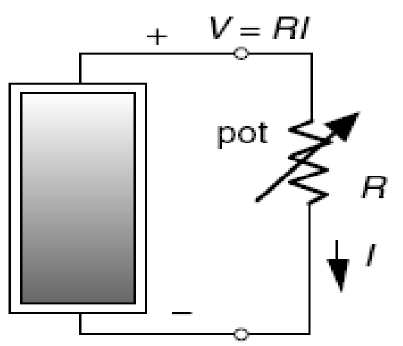
- PV Cell's I-V Curve
- Load's I-V Curve (DC load case)
- The intersection point is the operating point.

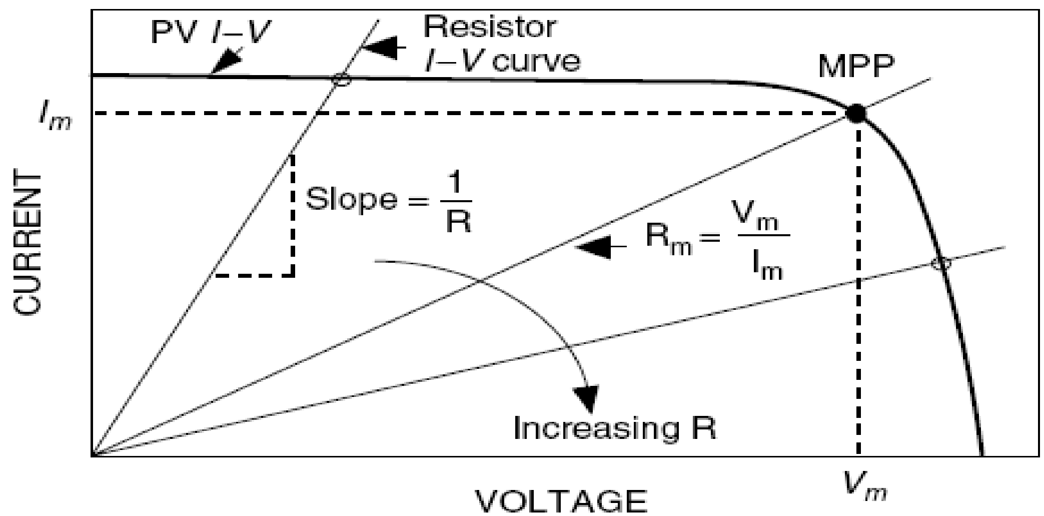




Operating Point - Example

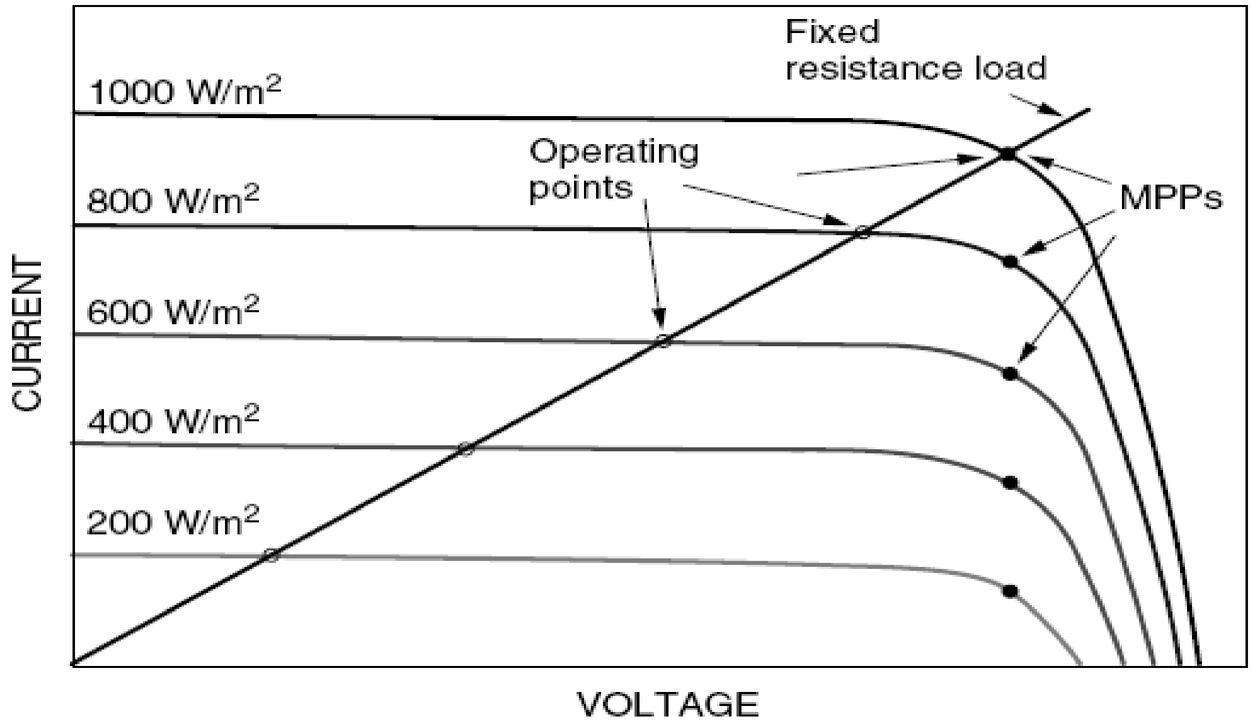
- PV Cell's I-V Curve
- Load's I-V Curve
- Example of Simple Resistive Load (R)
- Changes in Operating Points by the changes in resistance
- Rm: Resistance at Maximum Power Point – good only for test condition





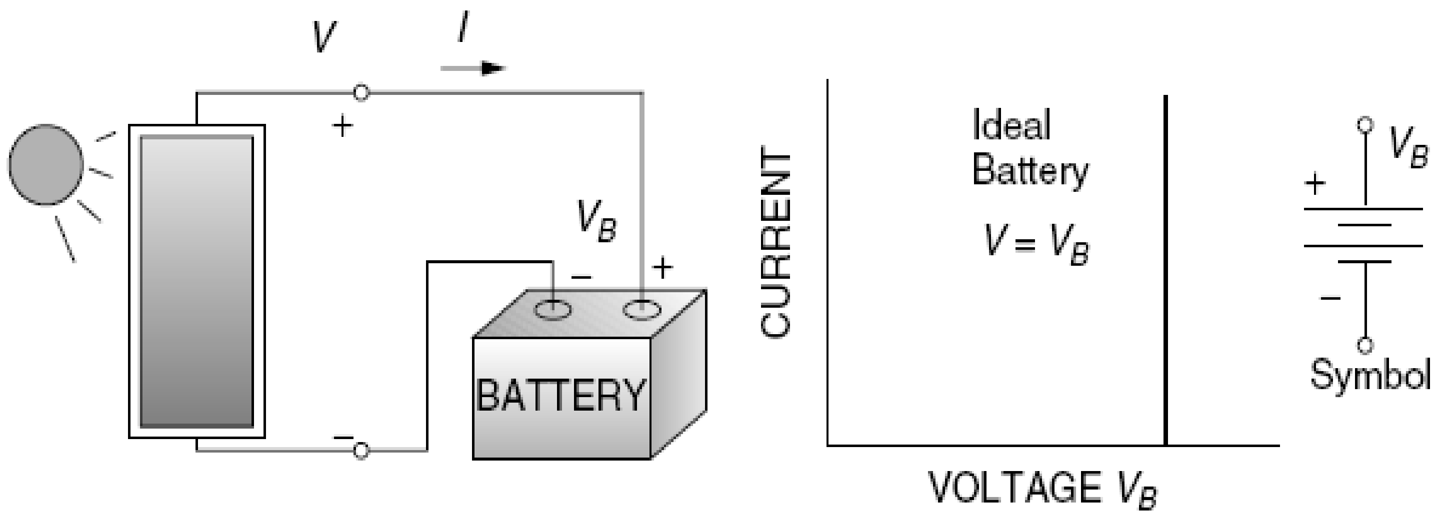
Operating Point Change over Insolation

- With **fixed resistance** (Rm), the operating point moves down off the MPP as the Insolation condition changes and the PV is **less efficient**
- **In need of Maximum Power Tracker**



Battery I-V Curve

- Ideal: Voltage remains constant no matter how much current is drawn
- I-V Curve: Straight up-and-down line

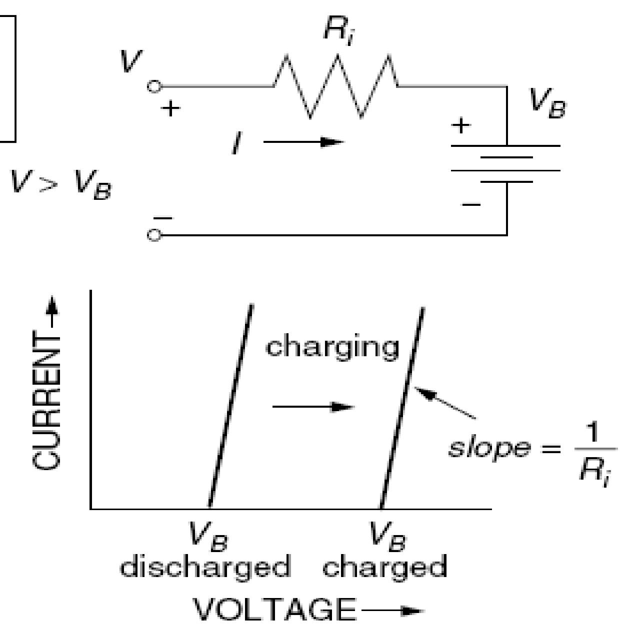


Battery I-V Curve

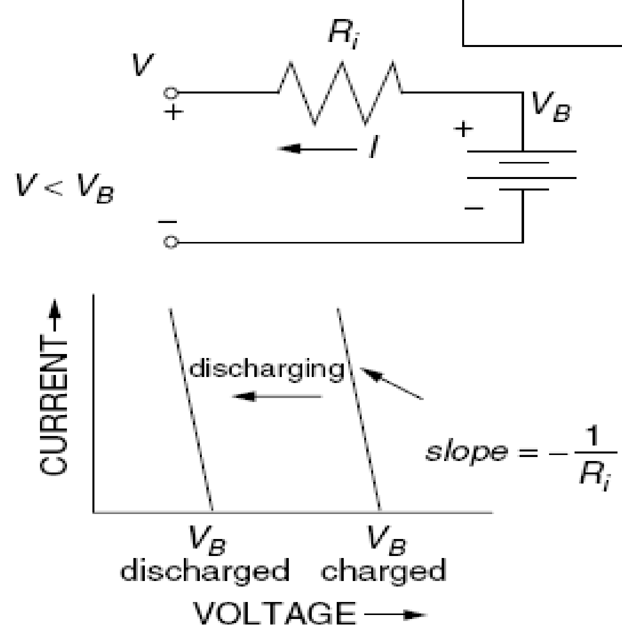
- Real Battery
- Real battery has internal resistance: $V = V_B + R_i * I$
  - Charging: Applied voltage must be bigger than V_B
  - Discharging: Output Voltage is less than V_B.

$$V = V_B + R_i I$$

$$V = V_B - R_i I$$



(a) Charging



(b) Discharging

Battery Charging --- Example

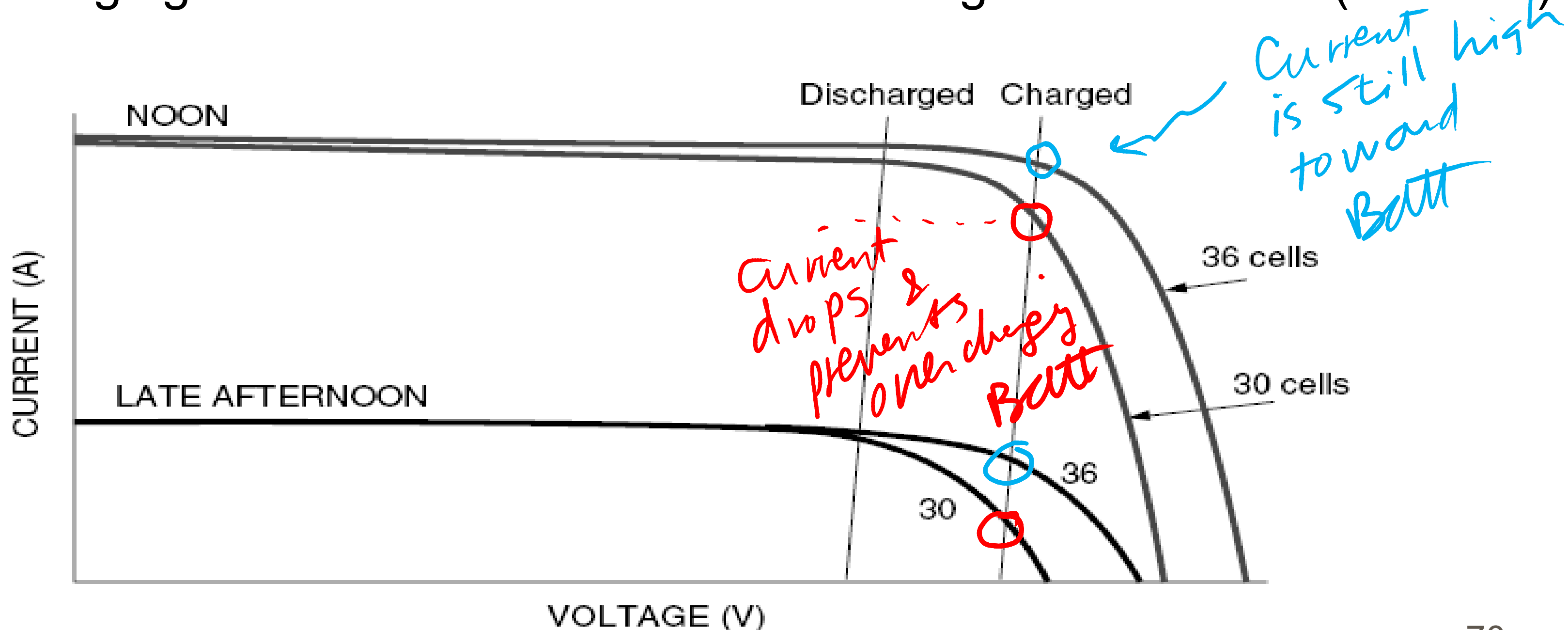
- There is a nearly depleted 12-V lead-acid battery which has an open-circuit voltage of 11.7 V and an internal resistance of 0.03 Ω.
  - (a) What voltage would a PV module operate at if it is delivering 6 A to the battery?
  - (b) If 20 A is drawn form a fully charged battery with open-circuit voltage 12.7 V, what voltage would the PV module operate at? (Remember that the PV operating voltage is determined by the battery voltage)

$$V = V_B + R_i I$$

$$V = V_B - R_i I$$

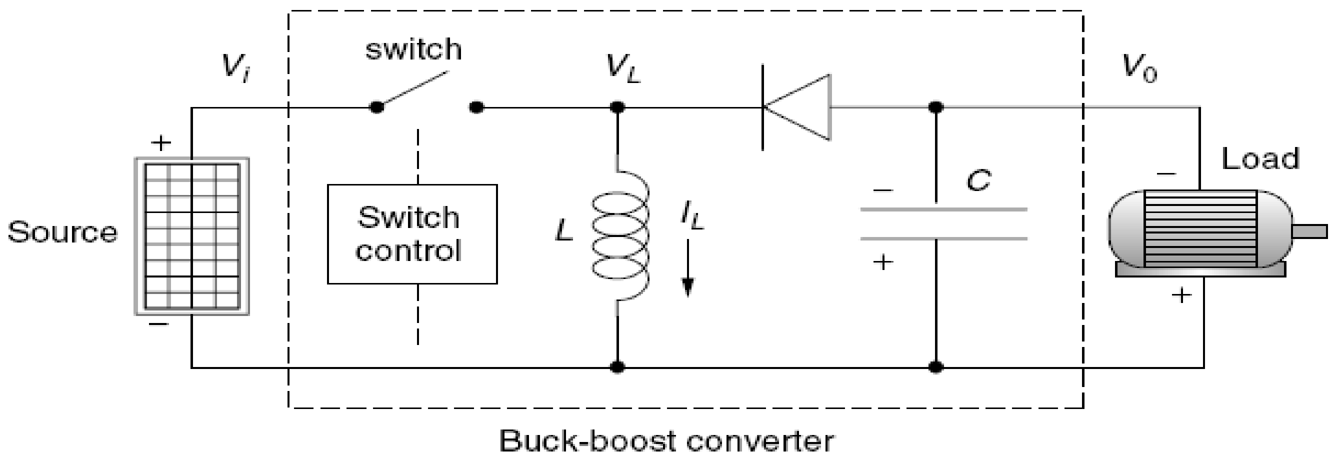
Charging and Discharging

- Charging moves I-V curve toward the right during the day (from PV) → So current lowers and prevents overcharging
- 30 cell module has lower current when charged – preventing over-charge
- Discharging moves I-V curve toward left during late afternoon (from PV)



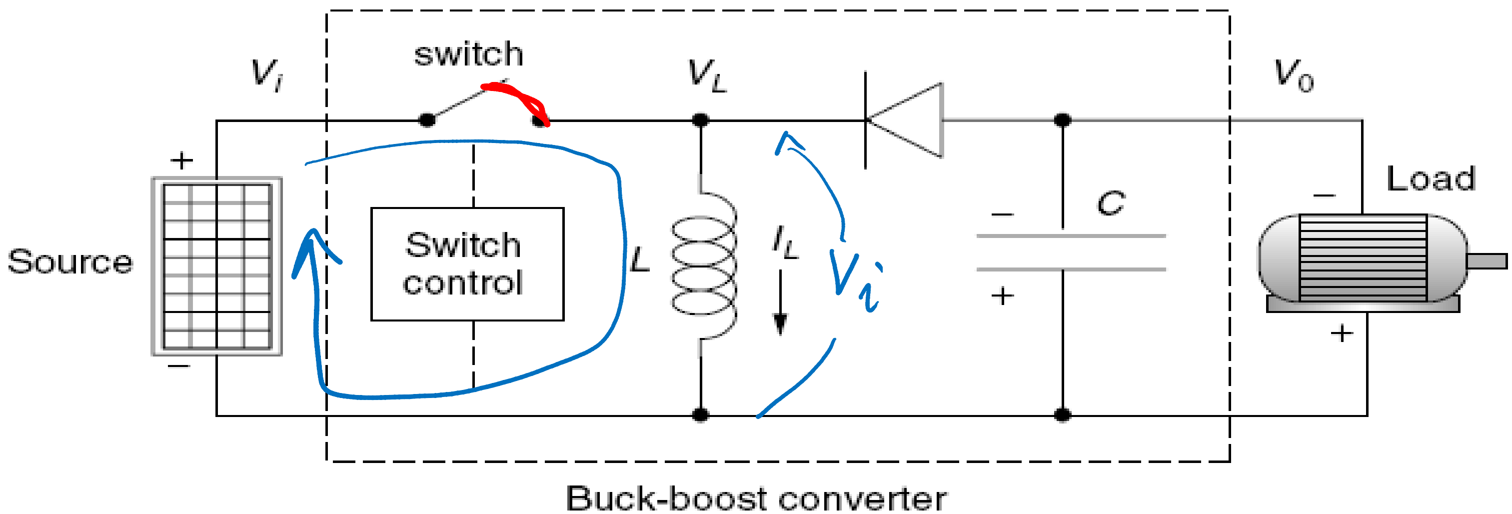
Voltage Control

- Benefit of operating PV near the knee (MPP) of the I-V Curve throughout the ever-changing daily conditions
- Conversion of DC voltages → Switched mode dc-to-dc converter {on-off switch to allow current to pass or block}
- Boost Converter: Step-up
- Buck Converter: Step-Down
- Buck-Booster Converter: Combination



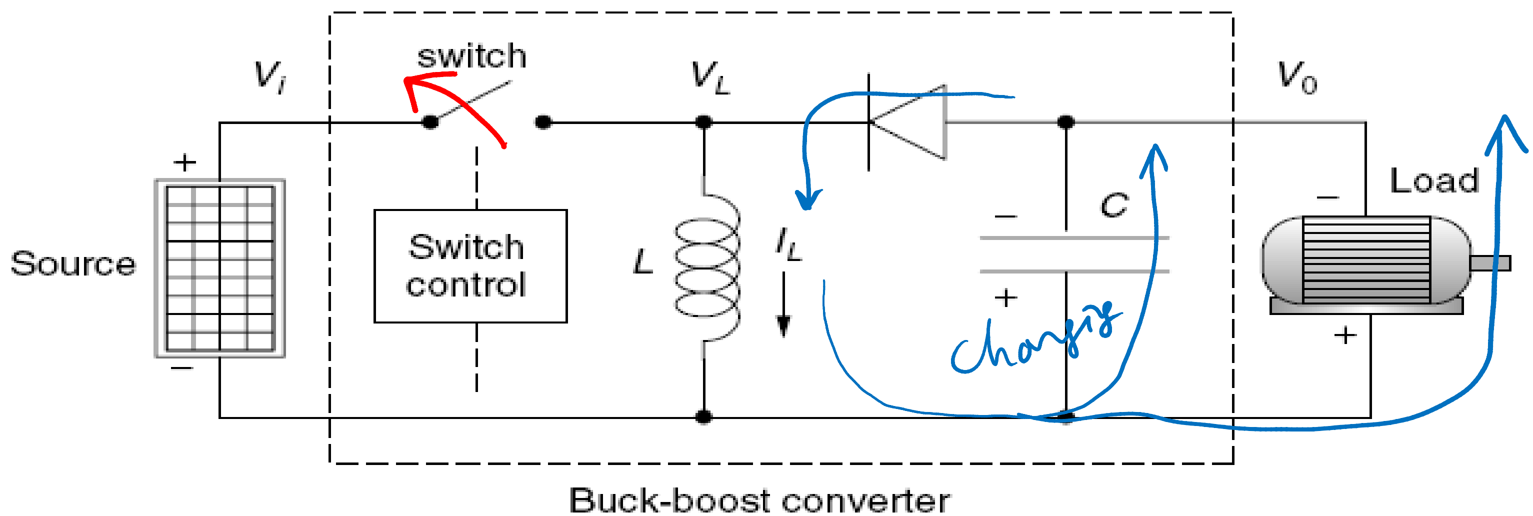
Voltage Control - 1

- When the switch is closed, the input voltage Vi is applied across the inductor, driving current IL through the inductor. All of the source current goes through the inductor since the diode blocks any flow to the rest of the circuit. During this portion of the cycle, energy is being added to the magnetic field in the inductor as current builds up. If the switch stayed closed, the inductor would eventually act like a short-circuit and the PVs would deliver short-circuit current at zero volts.



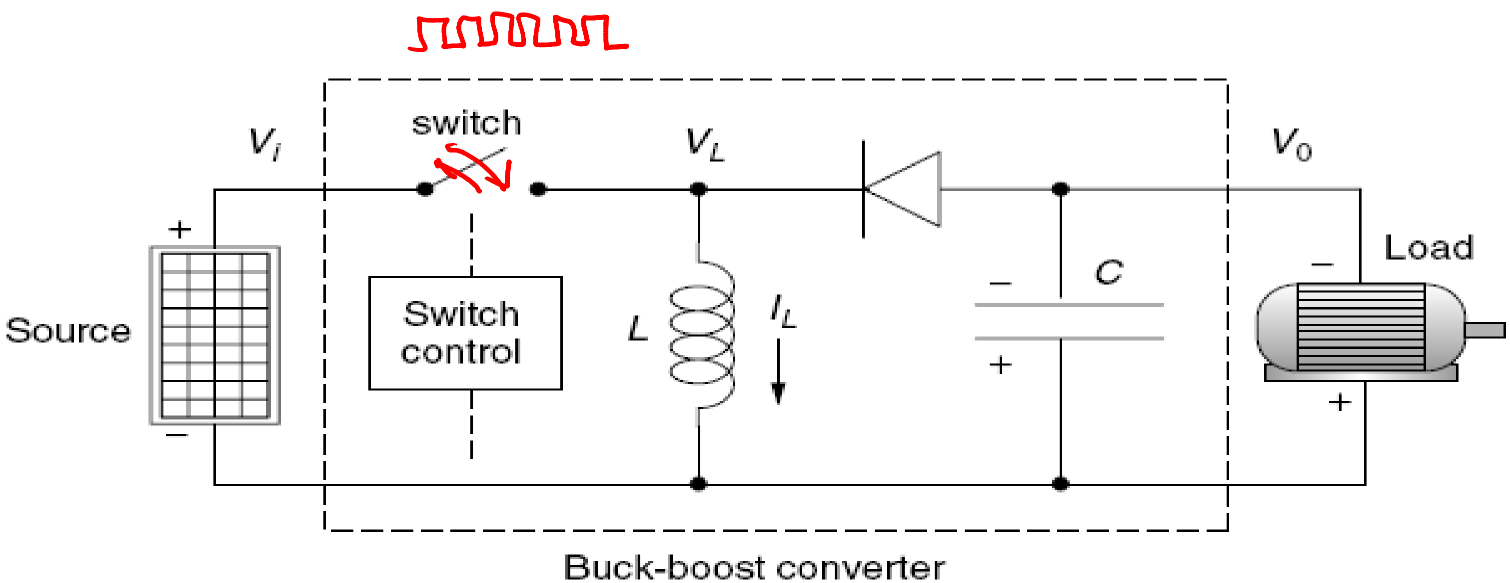
Voltage Control - 2

- When the switch is opened, current in the inductor continues to flow as the magnetic field begins to collapse (remember that current through an inductor cannot be changed instantaneously—to do so would require infinite power). Inductor current now flows through the capacitor, the load, and the diode. Inductor current charging the capacitor provides a voltage (with a polarity reversal) across the load that will help keep the load powered after the switch closes again.



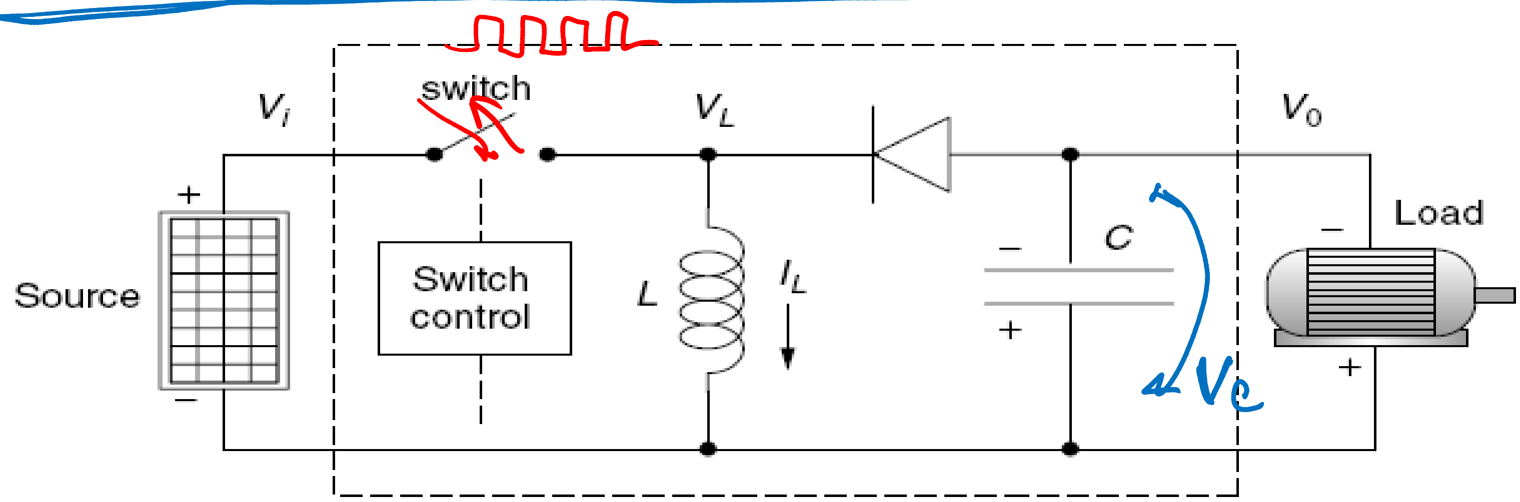
Voltage Control - 3

- If the switch is cycled quickly enough, the current through the inductor doesn't have a chance to drop much while the switch is open before the next jolt of current from the source. With a fast enough switch and a large enough inductor, the circuit can be designed to have nearly constant inductor current. That's our first important insight into how this circuit works: Inductor current is essentially constant.



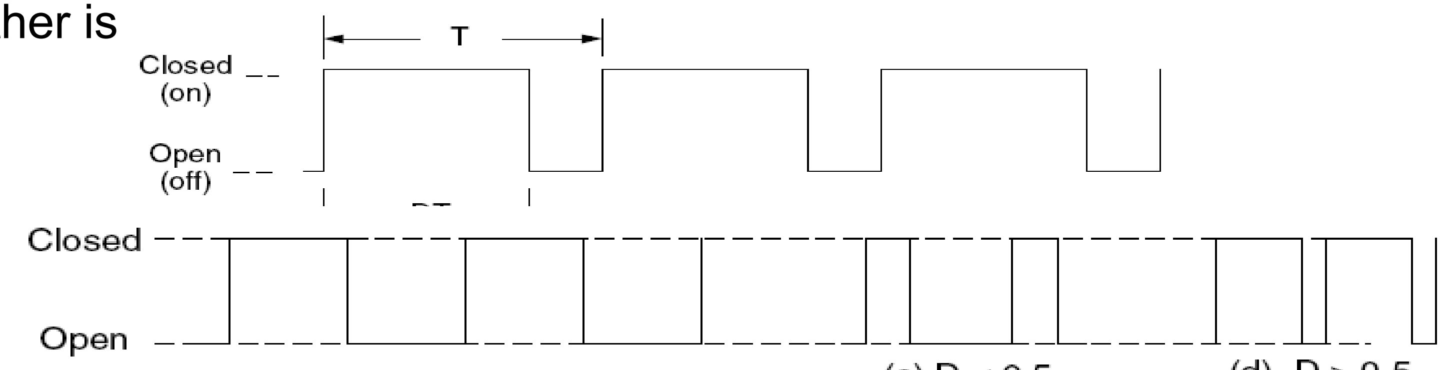
Voltage Control - 4

- If the switch is cycled quickly enough, the voltage across the capacitor doesn't have a chance to drop much while the switch is closed before the next jolt of current from the inductor charges it back up again. Capacitors, recall, can't have their voltage change instantaneously so if the switch is cycling fast enough and the capacitor is sized large enough, the output voltage across the capacitor and load is nearly constant. We now have our second insight into this circuit: Output voltage Vo is essentially constant (and opposite in sign to Vi)



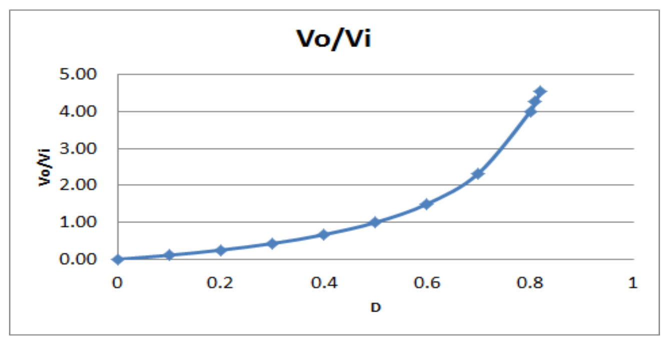
Input – Output Voltage by Duty Cycle

- The duty cycle of the switch itself controls the relationship between the input and output voltages of the converter.
- The duty cycle *D* (0 < *D* < 1) is the fraction of the time that the switch is closed. This variation in the fraction of time the switch is in one state or the other is

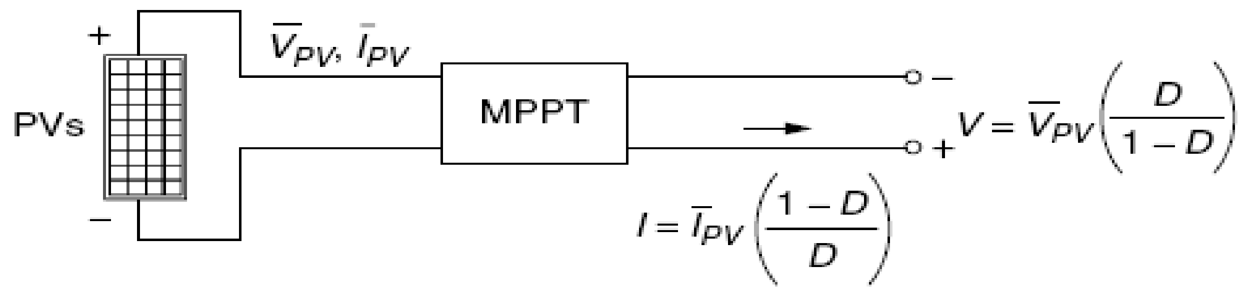


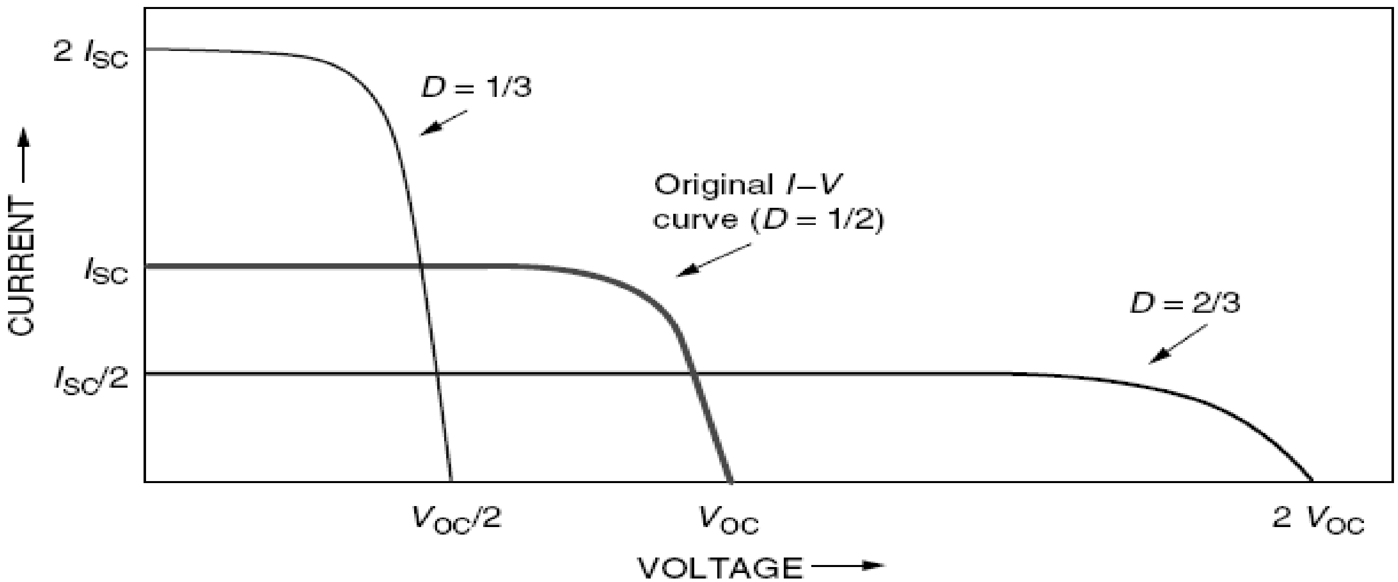
(b) D = 0.5 (c) D < 0.5 (d) D > 0.5

$$\frac{V_0}{V_i} = -\left(\frac{D}{1-D}\right)$$



MPPT and PV I-V with Duty Cycle





Home PV – Experience

Home photovoltaic systems for physicists

Thomas W. Murphy Jr

Installing a modest photovoltaic system and using it to run a suite of appliances can be educational and immensely satisfying. This brief how-to guide will help get you started.

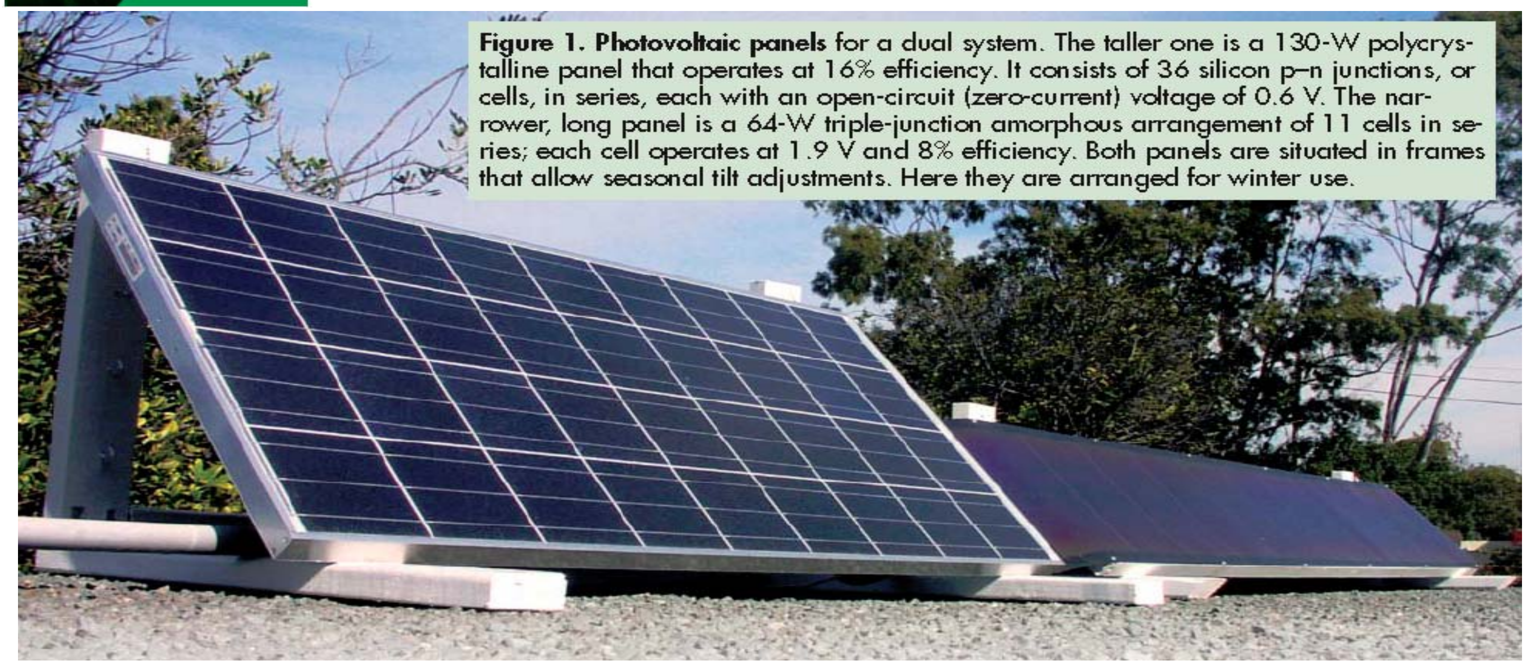


Figure 1. Photovoltaic panels for a dual system. The taller one is a 130-W polycrystalline panel that operates at 16% efficiency. It consists of 36 silicon p–n junctions, or cells, in series, each with an open-circuit (zero-current) voltage of 0.6 V. The narrower, long panel is a 64-W triple-junction amorphous arrangement of 11 cells in series; each cell operates at 1.9 V and 8% efficiency. Both panels are situated in frames that allow seasonal tilt adjustments. Here they are arranged for winter use.

- Tom Murphy, "Home Photovoltaic systems for physicists" Physics Today, July 2008.

HOME PV

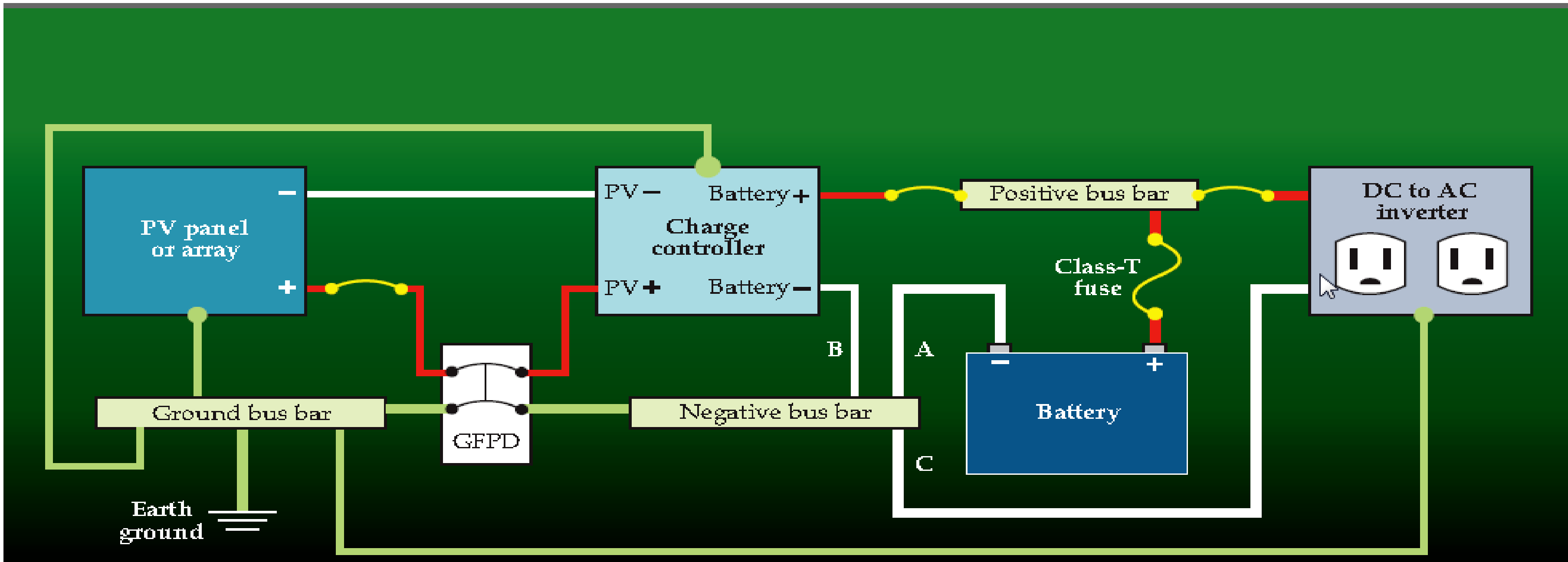


Figure 2. Circuit diagram for a standalone photovoltaic system. The breakers (yellow arcs) serve as overcurrent protection and disconnect simultaneously, but they can be implemented as fuses and disconnect switches. If the PV panels are not attached to a dwelling, the ground-fault protection device (GFPD) can be eliminated and the same bus bar, a metal strip that allows branch connections, can be used for negative and ground. All modules have grounded frames. In-line shunts can be placed at locations A, B, and C for measuring net battery current, solar input current, and load current, respectively—although, since those currents obey a sum rule, any two will suffice. Ground wires are green, negative white, and positive red.

A HOME PV

Figure 3. Power center for my dual PV system. Only half the parts are necessary for a more typical single system. Cut off at the bottom edge of the photo are two 12-V, 150-Ah batteries (one for each system), and immediately above them are 110-A class-T fuses. At left are two charge controllers. The black one is a maximum power-point tracking charge controller, as described in the text; it is certainly overkill for this small system. In the upper right are two 400-W inverters. Below them is an MPPT charge controller not currently used. To the left of the inverters is the monitoring system, capable of measuring two voltages and three currents. In the center is the exposed breaker box, showing three breakers per system, four shunts for current measurement (I use only three at a time), and for each panel, connected grounded bus bars, metal strips that allow branch connections. Ground wires are green, negative white, and positive red. The green extension cords on the right deliver AC power to appliances in the house. This particular system implementation does not require a ground-fault protection device.

A Home PV

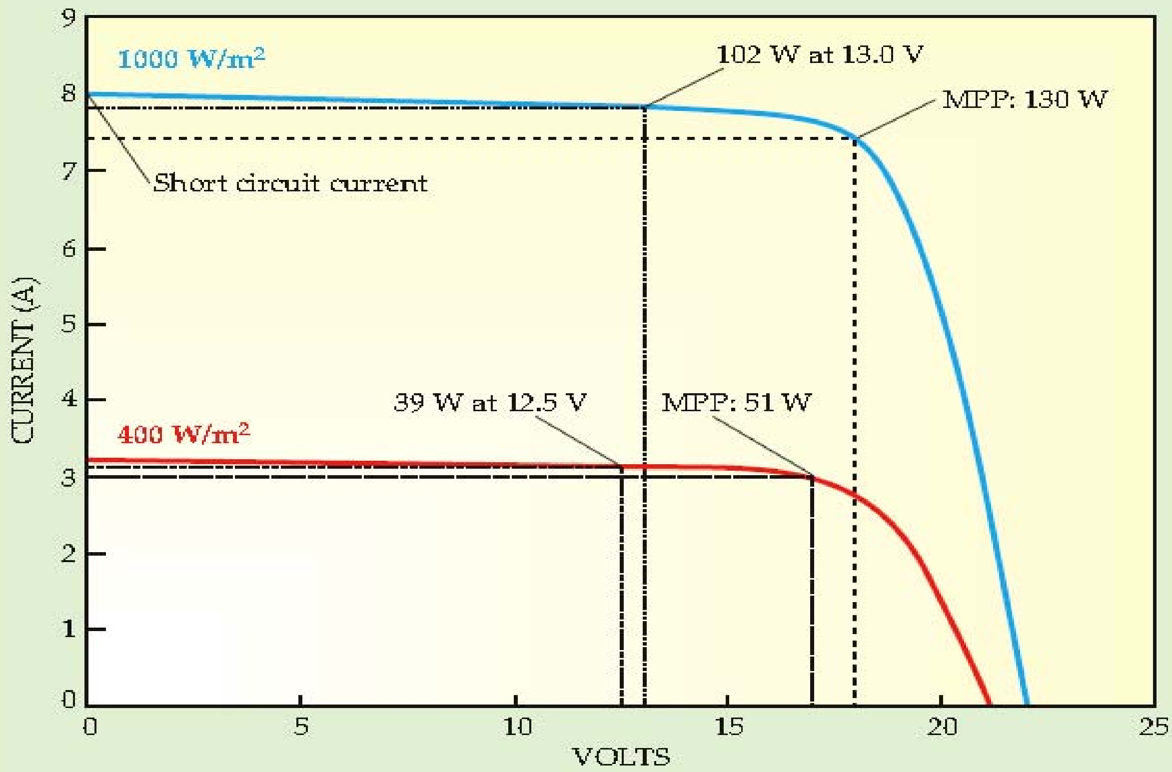


Figure 4. Current–voltage curves for a photovoltaic panel rated at 130 W at 25 °C. The blue curve shows full illumination; the red curve 40% illumination. Typical PV panels, with 36 cells in series, have an open-circuit voltage of around 22 V. Maximum power is typically delivered at around 15–18 V. At lower voltages, the PV current saturates; in full sun the saturation current is called the short-circuit current. The maximum power point (MPP) is indicated for both curves, as is the reduced power achieved by extracting current at typical battery voltages.

A Home PV

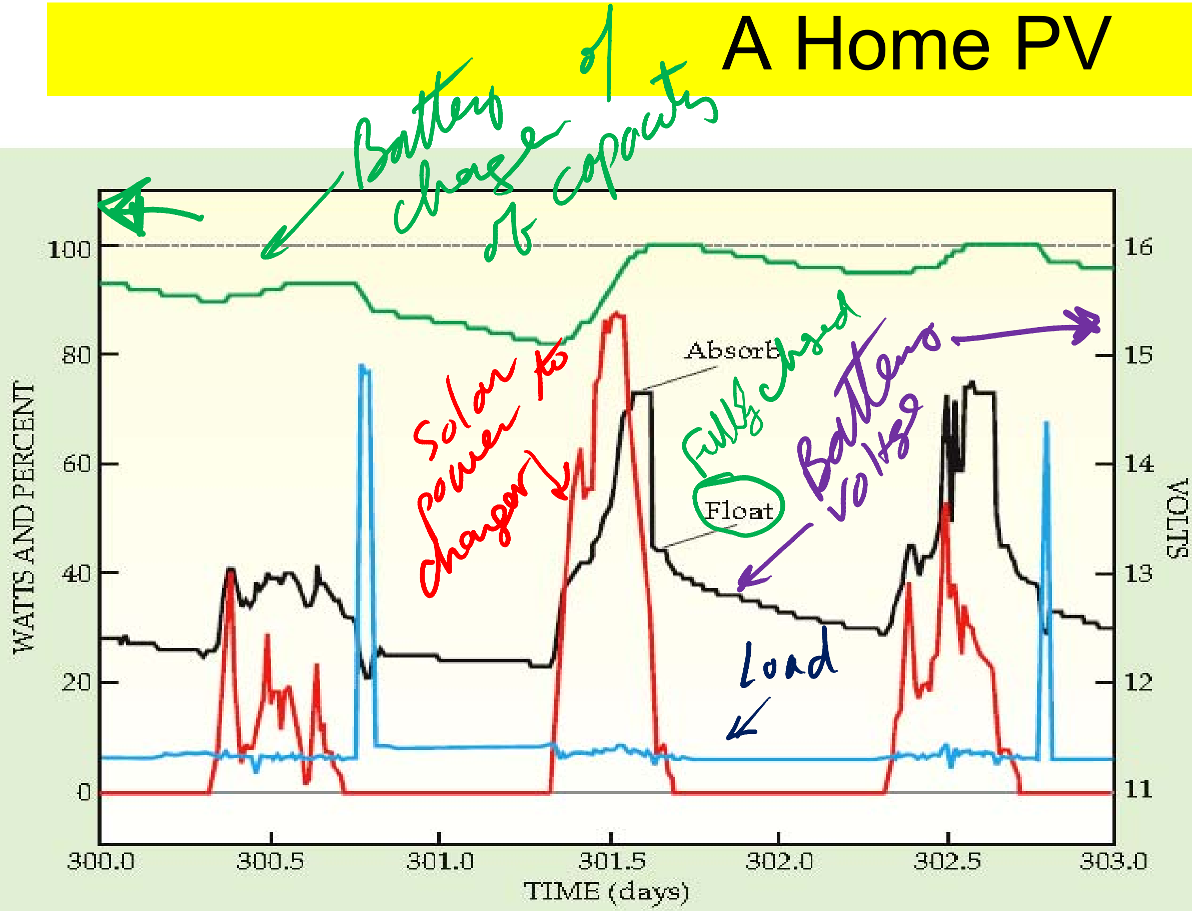


Figure 5. Sample data for a 130-W solar panel powering an entertainment system. Day 300 had a heavy but variable overcast; day 301 was mostly sunny; day 302 was cloudy. The red curve traces the solar power delivered by the panel to the charge controller, and the blue curve traces the load. Note the constant "off" load plus brief intervals of television usage. The black curve is battery voltage, as indicated on the right-hand scale. The battery reached absorption stage on days 301 and 302 and also showed intervals of float stage—indicating a fully charged battery—at the end of both days. The green curve indicates battery charge as a percent of capacity. The first day made only a small positive contribution to the battery charge, but the sunny day that followed made up for the deficit. Note that the falling edge of the solar power on the sunny day is not due to a decline in illumination, but reflects the diminishing current demand in maintaining absorption-stage voltage. Additional data samples are available at http://physics.ucsd.edu/~tmurphy/pv_for_pt.html.

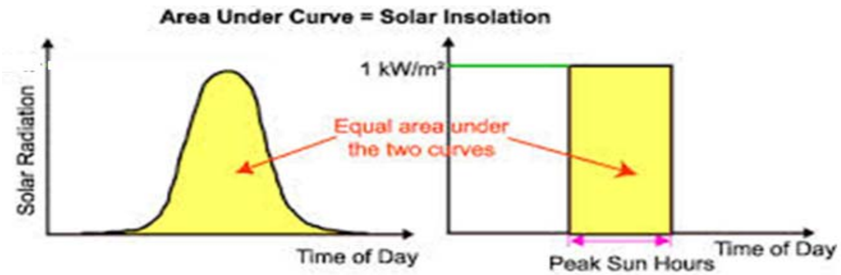
Estimation of PV Performance

- "1-sun" ("peak sun hour")of insolation is defined as 1 kW/m^2
- (EX)5.6 kWh/m^2-day = 5.6 h/day of 1-sun = 5.6 h of "peak sun"
- P_{ac} =AC power delivered by an array under 1-sun insolation.
- Daily kWh delivered = [**rated AC power**]*[number of hours of peak sun]
- **Rated AC Power = Rated DC Power * Conversion_Efficiency**

$$\text{Energy (kWh/day)} = \text{Insolation}\left(\frac{\text{kWh/m}^2}{\text{day}}\right) \cdot A\ (\text{m}^2) \cdot \overline{\eta}$$

A is the area of the PV array

$\overline{\eta}$ is the average system efficiency over the day.



DC power from the system 1-sun of insolation,

$$P_{dc}(\text{kW}) = \left(\frac{1\ \text{kW}}{\text{m}^2}\right) \cdot A\ (\text{m}^2) \cdot \eta_{1-\text{sun}}$$

η_{1-sun} is the system efficiency at 1-sun.

$$P_{ac}(\text{kW}) = P_{dc}(\text{kW}) \cdot \text{[Dc-AC Conversion Efficiency]}$$

$$\text{Energy (kWh/day)} = P_{ac}(\text{kW}) \cdot \text{(h/day of "peak sun")}$$

PV Test Condition: STC and PTC

- STC (Standard Test Condition)
  - 1-sun iiradiance - AM1.5 air mass ratio
  - 25°C **cell** temperature
  - DC output of an array: $P_{DC,STC}$.
  - $P_{ac} = P_{DC,STC}$ * (Conversion Efficiency)
- PTC (PVUSA Test Condition)
  - 1-sun irradiance in the plane of the array
  - 20°C **ambient** temperature
  - Wind speed of 1 m/s
  - AC output of an array: $P_{ac(PTC)}$
  - Better indicator of the actual power delivered in full sun

Energy Calculation with **Rated Power** and **Peak Sun Hour**

- http://rredc.nrel.gov/solar/pubs/redbook/
- The amount of solar energy in hours ("**peak sun**" hours) received each day on an optimally tilted surface during the **worst month** ("design month") of the year.
- **Rated AC Power = Rated DC Power * Conversion_Efficiency**

```
ENERGY = Rated_Power * Conversion_Efficiency * Peak_Sun_Hour/Day * 365 Day/Year
```

PV Energy Delivery Calculation

- Estimate the annual energy delivered by the 1-kW (dc, STC) array in Madison, WI, which south-facing, and has a tilt angle equal to its latitude minus 15°. Assume the dc-to-ac conversion efficiency at 72%.
- Insolation Table for Madison, WI

	Madison, WI					Latitude 43.13°N							
Tilt	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sept	Oct	Nov	Dec	Year
Lat − 15	3.0	3.9	4.5	5.1	5.8	6.2	6.2	5.7	4.8	3.8	2.5	2.3	4.5
Lat	3.4	4.3	4.7	5.0	5.5	5.7	5.8	5.5	4.8	4.0	2.8	2.6	4.5
Lat + 15	3.6	4.4	4.6	4.6	4.8	4.9	5.0	5.0	4.6	4.0	2.9	2.8	4.3
90	3.5	4.0	3.7	3.2	2.9	2.8	2.9	3.2	3.4	3.3	2.6	2.7	3.2
1-Axis (Lat)	3.9	5.0	5.8	6.4	7.3	7.8	7.7	7.1	6.0	4.8	3.2	3.0	5.7
Temp. (°C)	−4.0	−1.1	5.3	13.7	20.5	25.7	28.0	26.4	21.9	15.5	6.7	−1.2	13.1

Solution

- From 72% Conversion efficiency
  - P_{ac}=1.kW*0.72 = 0.72kW
- From the Insolation Table, the annual average insolation is 4.5 kWh/m^2-day
  - Same as 4.5 h "peak sun"/day
- Energy Calculation

$$\text{Energy} = 0.72 \text{ kW} \times 4.5 \text{ h/day} \times 365 \text{ day/yr} = 1183 \text{ kWh/yr}$$

ENERGY = Rated_Power * Conversion_Efficiency * Peak_Sun_Hour/Day * 365 Day/Year

Capacity Factor = [“Peak Sun Hour”/24]

- Capacitor Factor (CF): Ratio with Rated Power
- CF of 0.4 means:
  - the system delivers full-rated power 40% of the time and no power at all the rest of the time, or
  - the system deliver 40% of rated power all of the time.

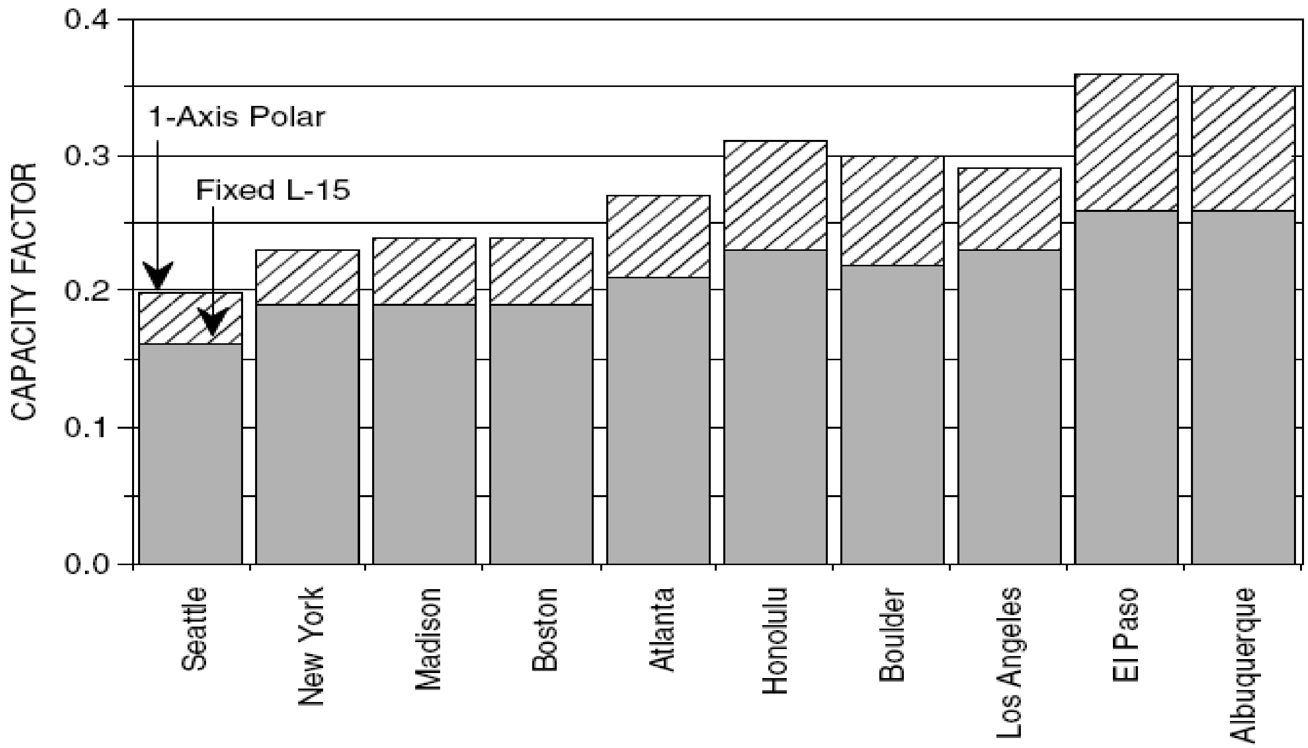
$$\text{Energy (kWh/yr)} = P_{ac}(\text{kW}) \cdot \text{CF} \cdot 8760(\text{h/yr})$$

$$\text{Energy (kWh/day)} = P_{ac}(\text{kW}) \cdot (\text{h/day of “peak sun”})$$

$$\text{Capacity factor (CF)} = \frac{(\text{h/day of “peak sun”})}{24\ \text{h/day}}$$

CFs for a number of U.S. cities

- CF: 0.16 – 0.26 for fixed south-facing panel at tilt L-15
- CF: 0.20 – 0.36 for single axis polar mount tracker



Grid-Connected PV System Sizing

- Assumption
  - Grid power is always available when needed
  - No need of energy storage – No Battery needed
- Questions
  - How many kWh/yr are required?
  - How many peak watts of dc PV power are needed to provide that amount?
  - How much area will that system require?
  - What real components are available ?

PV System Sizing

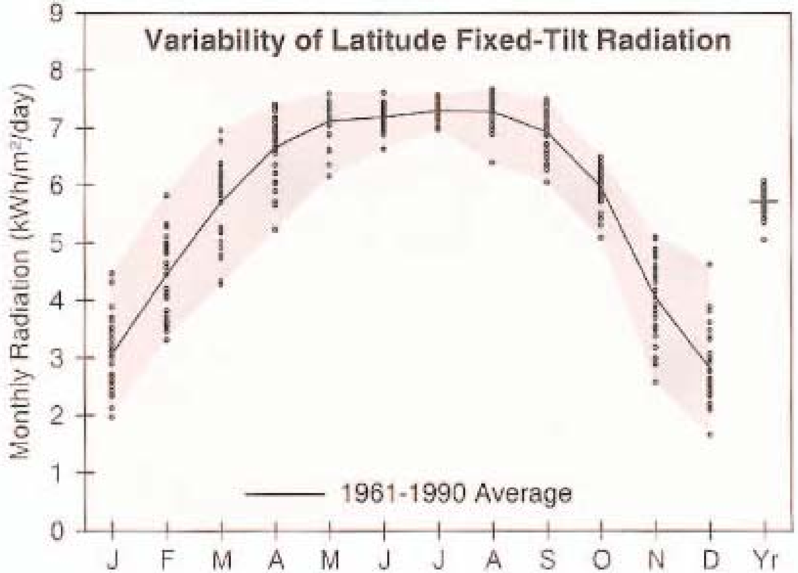
- Example
  - An energy efficient house in Fresno (Latitude at 22^o) is to be fitted with a rooftop PV array that will annually displace all of the 3600 kWh/yr of electricity that the home uses.
  - **Question: How many kW (dc, STC) of panels will be required and what area will be needed?**
  - Assumptions:
    - Roof is south-facing with a moderate tilt angle
    - Annual insolation for L-15 is 5.7kWh/m^2-day
    - Dc-to-ac conversion efficiency at 75%
    - Solar system average 1-sun efficiency at 12.5%

Fresno, CA

WBAN NO. 93193

LATITUDE: 36.77° N
LONGITUDE: 119.72° W
ELEVATION: 100 meters
MEAN PRESSURE: 1004 millibars

STATION TYPE: Primary



Solar Radiation for Flat-Plate Collectors Facing South at a Fixed Tilt (kWh/m²/day), Uncertainty ±9%

Tilt (°)		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Year
0	Average	2.1	3.2	4.7	6.3	7.5	8.1	8.0	7.2	5.9	4.3	2.7	1.9	5.2
	Min/Max	1.7/2.7	2.5/3.9	3.7/5.5	5.1/7.0	6.6/8.0	7.4/8.7	7.6/8.4	6.3/7.6	5.2/6.3	3.9/4.6	2.0/3.2	1.4/2.6	4.7/5.4
Latitude -15	Average	2.8	4.1	5.5	6.8	7.6	7.8	7.9	7.5	6.8	5.5	3.6	2.5	5.7
	Min/Max	1.9/3.9	3.1/5.3	4.2/6.6	5.4/7.5	6.6/8.1	7.2/8.3	7.5/8.2	6.6/7.9	5.9/7.3	4.8/6.0	2.4/4.5	1.6/3.9	5.1/6.0
Latitude	Average	3.1	4.4	5.7	6.7	7.1	7.2	7.3	7.3	6.9	6.0	4.1	2.8	5.7
	Min/Max	2.0/4.5	3.3/5.8	4.3/6.9	5.2/7.4	6.2/7.6	6.6/7.6	7.0/7.6	6.4/7.7	6.1/7.5	5.1/6.5	2.6/5.1	1.7/4.6	5.1/6.1
Latitude +15	Average	3.2	4.5	5.6	6.2	6.3	6.1	6.3	6.6	6.7	6.1	4.2	3.0	5.4
	Min/Max	2.0/4.8	3.3/6.1	4.1/6.9	4.8/6.9	5.4/6.7	5.7/6.4	6.0/6.5	5.8/6.9	5.8/7.2	5.1/6.6	2.6/5.4	1.7/5.0	4.7/5.8

PV Sizing Solution

- Roof is south-facing with a moderate tilt angle
- Annual insolation for L-15 is 5.7kWh/m^2-day
- DC-to-AC conversion efficiency at 75%
- Solar Cell efficiency at 12.5%
- 1. Annual Energy Equation

$$\text{Energy (kWh/yr)} = P_{ac}(\text{kW}) \cdot (\text{h/day @1-sun}) \cdot 365 \text{ days/yr}$$

- 2. AC Power
- 3. DC Power
- 4. Area Calculation

Excel Solution

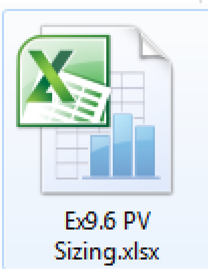
PV Sizing.xlsx - Microsoft Excel

F27

	A	B	C	D	E	F	G
1	peak sun [h/day]	Solar 1-sun effic	DC2AC effi	kWh/yr required to produce	Pac [kW]	Pdc [kW]	Area (m2)
2	**5.7**	**0.125**	**0.75**	**3600**	**1.73035328**	**2.307138**	**18.4571**
3	4.7	0.125	0.75	3600	2.09851355	2.798018	22.38414
4	3.7	0.125	0.75	3600	2.66567938	3.554239	28.43391
5							
6							
7							
8							

=D2/(A2*365)

C	D	E	F	G
DC2AC effi	kWh/yr required to produce	Pac [kW]	Pdc [kW]	Area (m2)
0.75	**3600**	**1.73035328**	**2.307138**	**18.4571**
0.75	3600	2.09851355	2.798018	22.38414
0.75	3600	2.66567938	3.554239	28.43391



Practically, PV and Inverter Modules

Module:	Sharp NE-K125U2	Kyocera KC158G	Shell SP150	Uni-Solar SSR256
Material:	Poly Crystal	Multicrystal	Monocrystal	Triple junction a-Si
Rated power $P_{dc,STC}$:	125 W	158 W	150 W	256 W
Voltage at max power:	26.0 V	23.2 V	34 V	66.0 V
Current at max power:	4.80 A	6.82 A	4.40 A	3.9
Open-circuit voltage V_{OC}:	32.3 V	28.9 V	43.4 V	95.2
Short-circuit current I_{SC}:	5.46 A	7.58 A	4.8 A	4.8
Length:	1.190 m	1.290 m	1.619 m	11.124 m
Width:	0.792 m	0.990 m	0.814 m	0.420 m
Efficiency:	13.3%	12.4%	11.4%	5.5%

Manufacturer:	Xantrex	Xantrex	Xantrex	Sunny Boy	Sunny Boy
Model:	STXR1500	STXR2500	PV 10	SB2000	SB2500
AC power:	1500 W	2500 W	10,000 W	2000 W	2500 W
AC voltage:	211–264 V	211–264 V	208 V, 3Φ	198–251 V	198–251 V
PV voltage range MPPT:	44–85 V	44–85 V	330–600 V	125–500 V	250–550 V
Max input voltage:	120 V	120 V	600 V	500 V	600 V
Max input current:	—	—	31.9 A	10 A	11 A
Maximum efficiency:	92%	94%	95%	96%	94%

Sizing Solution -- Continued

- PV Module selection
  - Kyocera KC158G 158-W module: 23.2V
  - Number of modules?
  - From DC Power = 2300W→ 2300/158=14.6
    - 2-string: 23.2x2=46.4V
    - 3-string: 23.2x3=69.6V --- Pick this. Open Circuit voltage (28.9x3=86.7V) is still below 120V max of the STXR2500 inverter
  - 3x5 (15 modules)
- Inverter Module
  - Xantrex STXR2500 Inverter:
    - MPPT Input voltage 44-85V

Module:	Kyocera KC158G
Material:	Multicrystal
Rated power $P_{dc,STC}$:	158 W
Voltage at max power:	23.2 V
Current at max power:	6.82 A
Open-circuit voltage V_{OC}:	28.9 V
Short-circuit current I_{SC}:	7.58 A
Length:	1.290 m
Width:	0.990 m
Efficiency:	12.4%

Manufacturer:	Xantrex
Model:	STXR2500
AC power:	2500 W
AC voltage:	211–264 V
PV voltage range MPPT:	44–85 V
Max input voltage:	120 V
Max input current:	—
Maximum efficiency:	94%

$$\text{Area} = 15 \text{ modules} \times 1.29 \text{ m} \times 0.99 \text{ m} = 19.1 \text{ m}^2 \ (206 \text{ ft}^2)$$

$$P_{dc,STC} = 158 \text{ W/module} \times 15 \text{ modules} = 2370 \text{ W}$$

$$\text{Energy} = 2.37 \text{ kW} \times 0.75 \times 5.7 \text{ h/day} \times 365 \text{ day/yr} = 3698 \text{ kWh/yr}$$

Cost - Example

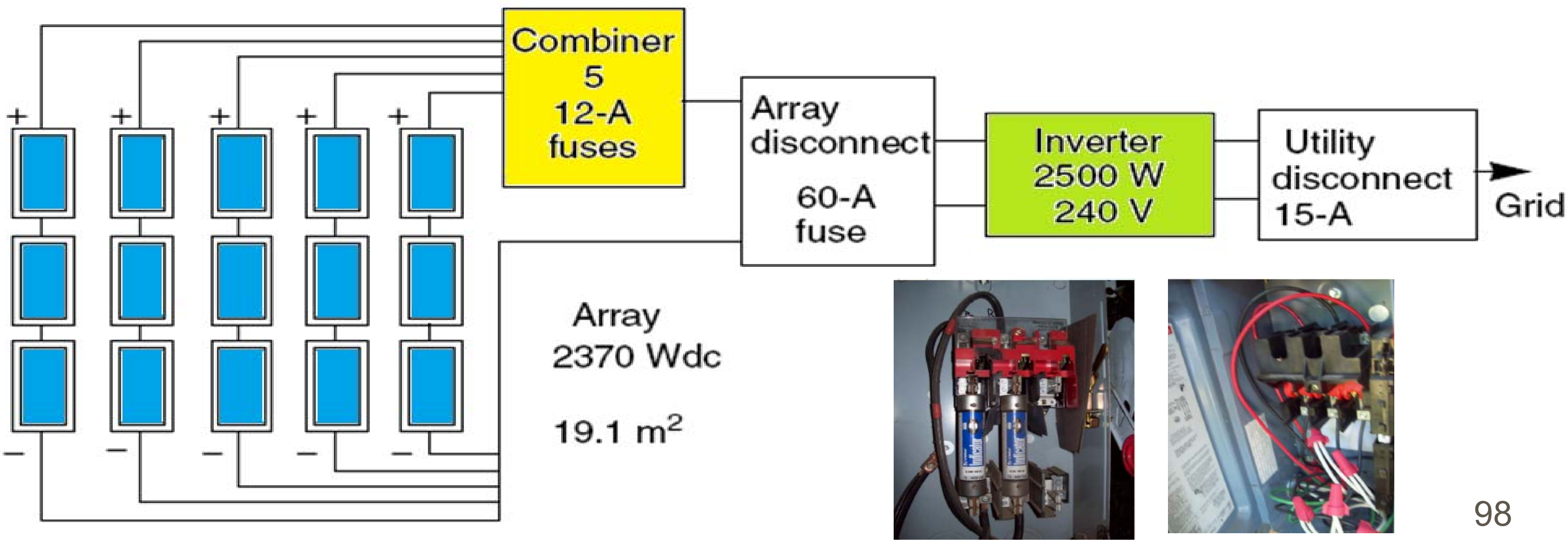
DIY PV System Costs

Item	Cost
28 Evergreen PV modules (blems)	$18,514
PVPowered inverter	2,895
Misc. electrical	2,772
PV support, homemade	2,437
PV rack, homemade	1,554
Travel expense (to pick up parts, equipment)	861
Books	132
Total	$29,165
Less 30% Tax Credit	$8,750
Grand Total	$20,415

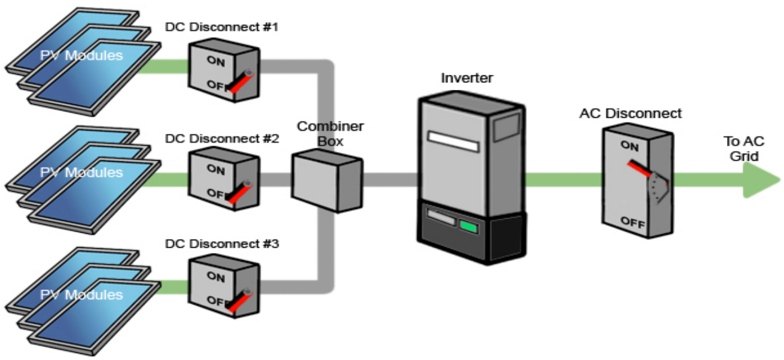
Final Design

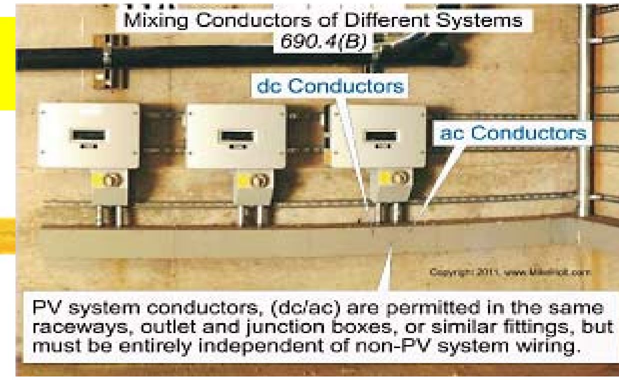
NEC Article 690

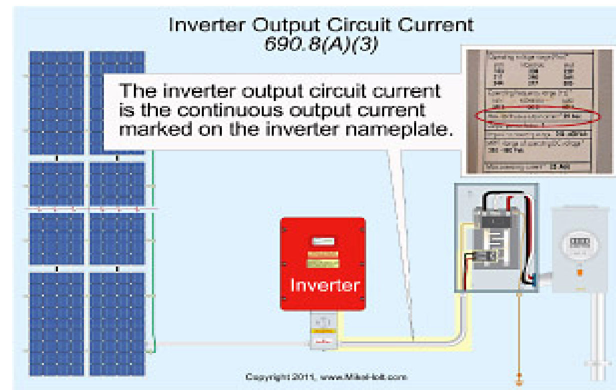
- Other requirements
  - NEC – 600V max voltage limit
  - Fuse and disconnect switch: withstand 125% of expected dc voltage
  - Consider potential exceeded solar insolation: give 125%
  - Combiner fuse rating: (7.58 PV short circuit current)x(1.25)x(1.25) =11.8A
  - Array disconnect switch rating: 11.8Ax 5 = 59.2A
  - Inverter fuse rating (125%): 1.25x[2500W/240V]=13A

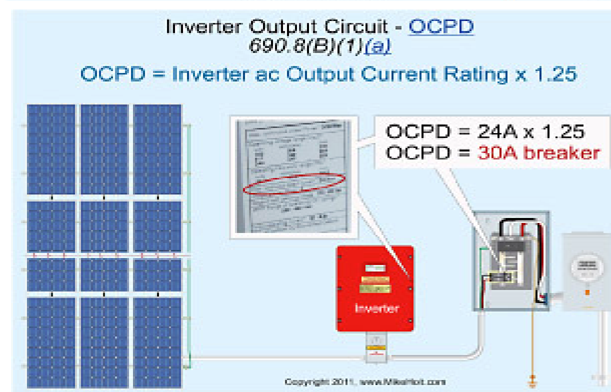


NEC Article 690



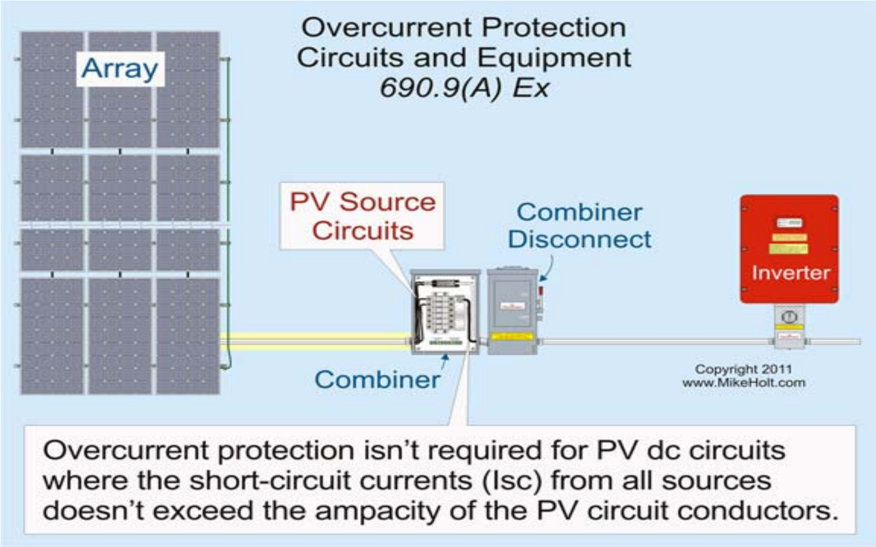


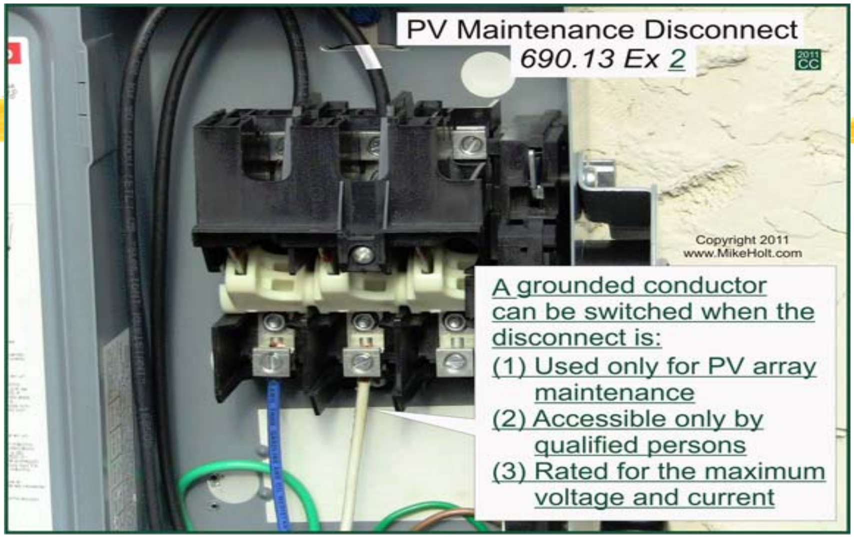


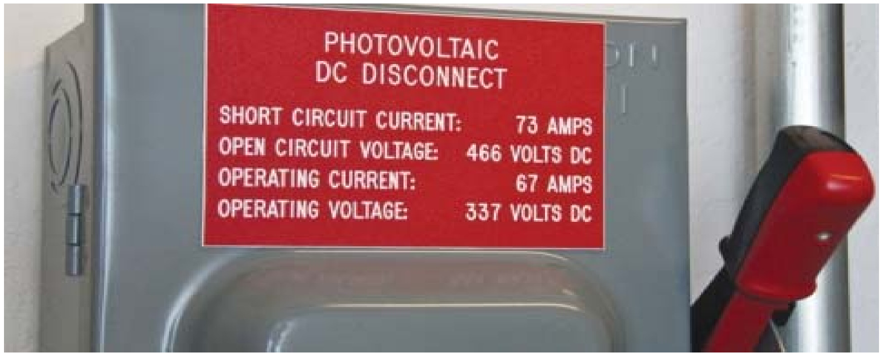


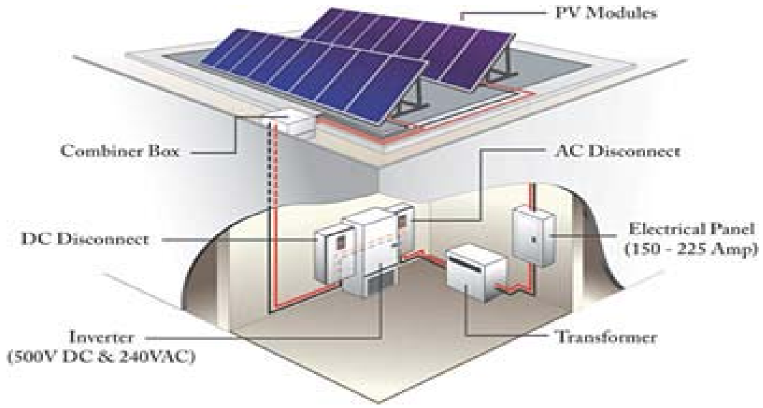
- Article 690, which consists of nine parts, provides electrical requirements for photovoltaic (PV) systems.
- PV source circuits [690.4(B)(1)].
- PV output and inverter circuits [690.4(B)(2)].
- Multiple systems. Conductor of each system where multiple systems are present [690.4(B)(3)].
- Module connection. Arrange module connections so the removal of a module doesn't interrupt the grounded conductor to other PV source circuits [690.4(C)].

NEC Article 690









Grid-Connected PV System Economics

- Estimation of the cost of electricity generated by PV
  - Amortizing cost of **Principal** (**P** \$) over a **period** (***n*** year) with **interest rate of *i*** for Loan payment.
  - **Annual Payment** (**A** \$/yr) divided by **Annual kWh** → \$/kWh
- **CRF** (Capital Recovery Factor):

$$\mathrm{CRF}(i,n) = \frac{i(1+i)^n}{(1+i)^n - 1}$$

- Annual Loan Payment (A):

$$A = P \cdot \mathrm{CRF}(i,n)$$

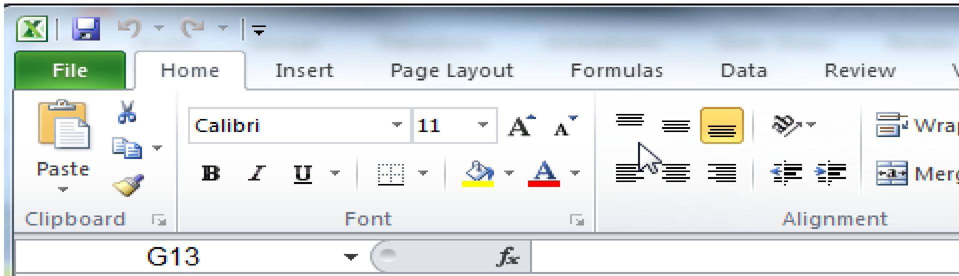
- **Example**: A PV system costs \$16,850 to deliver 4000 kWh/yr. If the system is paid for with a 6% 30-year loan, what would be the cost of electricity, ignoring income tax benefit, loan tax deduction, etc?

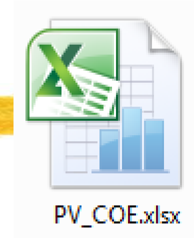
$$\mathrm{CRF}(i,n) = \frac{i(1+i)^n}{(1+i)^n - 1} = \frac{0.06(1.06)^{30}}{(1.06)^{30} - 1} = 0.07265/\mathrm{yr}$$

$$\mathrm{A} = \mathrm{P}\ \mathrm{CRF}(i,n) = \$16{,}850 \times 0.07265/\mathrm{yr} = \$1224/\mathrm{yr}$$

$$\text{Cost of electricity} = \frac{\$1224/\mathrm{yr}}{4000\ \mathrm{kWh/yr}} = \$0.306/\mathrm{kWh}$$

Excel Solution





D2 =(B2*(1+B2)^C2)/((B2+1)^C2-1)

P [$]	i	n	CRF	A [$/yr]	kWh	COE [$/kWh]
16850	0.06	30	0.072649	1224.13	4000	0.31
16850	0.05	30	0.065051	1096.12	4000	0.27
16850	0.04	30	0.05783	974.44	4000	0.24
16850	0.03	30	0.051019	859.67	4000	0.21
16850	0.02	30	0.04465	752.35	4000	0.19
16850	0.01	30	0.038748	652.91	4000	0.16