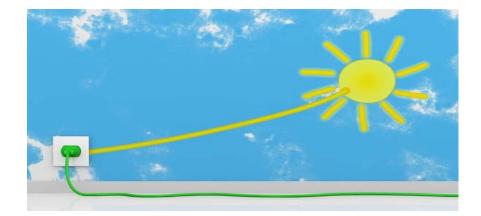
# Chapter 7. Solar Resource



#### Solar Resource

**#** What do we have to know to design a solar system?

- How much ( ) is available?
- ✓Where is the sun in the sky at any ( ) of the day?
- Solar intensity ("( Solar Radiation)

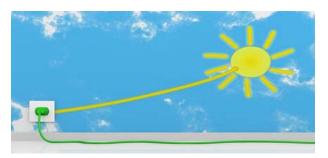
 $\bigtriangleup$  Insolation on (

)" = Incident

► (

) Daily Insolation

) surfaces



# Solar Spectrum # Sun 1.4 million kilometer diameter 4 <sup>1</sup>H + 2 e --> <sup>4</sup>He + 2 neutrinos + 6 photons Thermonuclear furnace fusing hydrogen atoms into helium 3.8 x 10<sup>20</sup> MW electromagnetic energy radiation into space Solar Spectrum (scaled to have the total area under the curve is 1.37 kW/m<sup>2</sup>, which is the solar insolation just outside the earth's

atmosphere: > 8000 km)

⊠Visible Light ( )%

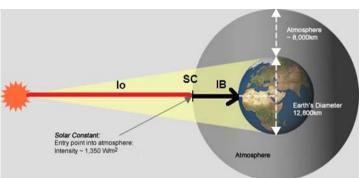
(wavelength: 0.38 um (violet) – 0.78 um (Red))

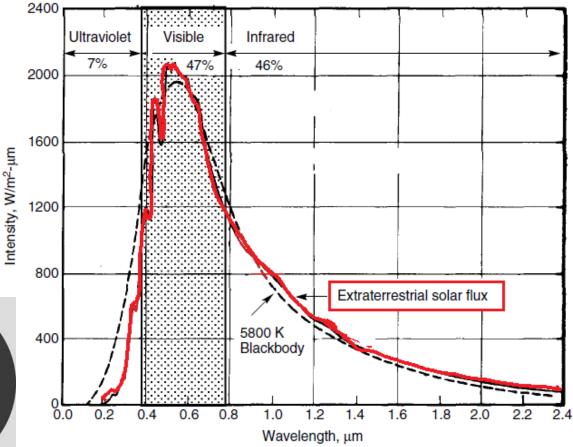
⊠IR ( ) %

#### Solar Spectrum

#### **#** Extraterrestrial Solar

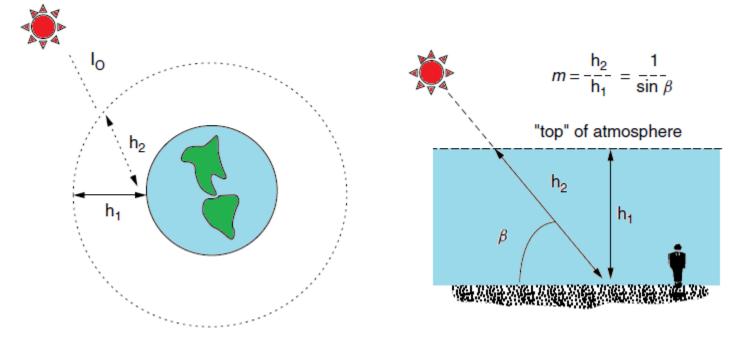
Spectrum (scaled to have the total area under the curve is 1.37 kW/m<sup>2</sup>, which is the solar insolation just outside the earth's atmosphere)





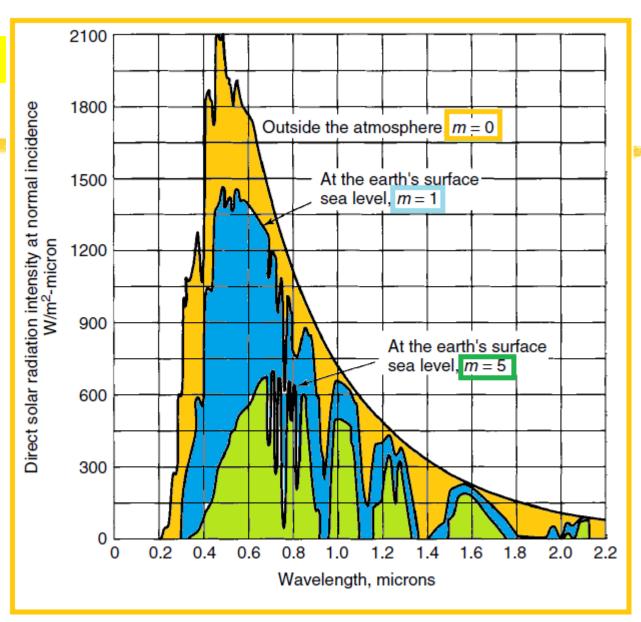
#### Solar Spectrum

- Extraterrestrial Solar radiation is absorbed in the atmosphere
- Hength of path of sun's rays has to be considered
  - △ h2: actual path length
  - h1: minimum possible length (when sun is directly overhead)
  - $\bigtriangleup$  Air Mass Ratio (**m**) & Sun's Altitude Angle ( $\beta$ )



## Solar Spectrum

- AM0: (Outside the atmosphere)
   Extraterrestrial solar spectrum
- AM1: Air mass ratio of 1--- Sun is directlyoverhead
- AM1.5: Average solar spectrum at the earth's surface
  - UV 2%, Visible 54%, IR 44%
  - Spectrum shifts some toward longer wavelength
  - What angle?

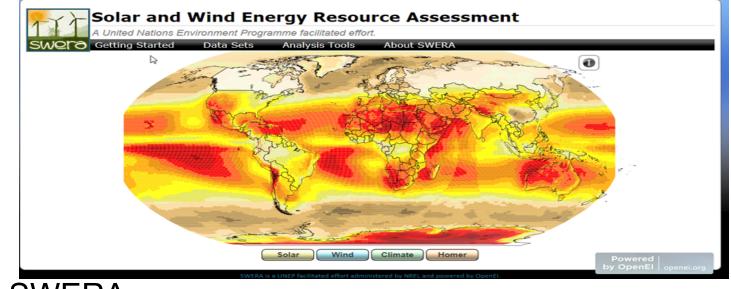


#### Solar Energy Resources

**SWERA**(Solar and Wind Energy Resource Assessment)

% https://openei.org/wiki/Solar\_and\_Wind\_Ene rgy\_Resource\_Assessment\_(SWERA)

http://maps.nrel.gov/re-atlas





#### The Earth's Orbit

- Earth revolves around the sun in an elliptical orbit
- ℜ One revolution per very 365.25 days
- Hearest point to Sun ("Perihelion"): 147 million km on (
- Farthest point from Sun ("Aphelion"): 152 million km on ( )
- Bistance between Sun and Earth on a given day
  - n: day number (1: Jan 1, 365: Dec 31)
  - Angle in **degrees**

$$d = 1.5 \times 10^8 \left\{ 1 + 0.017 \sin \left[ \frac{360(n - 93)}{365} \right] \right\} \text{ km}$$

2 for Apr 15?

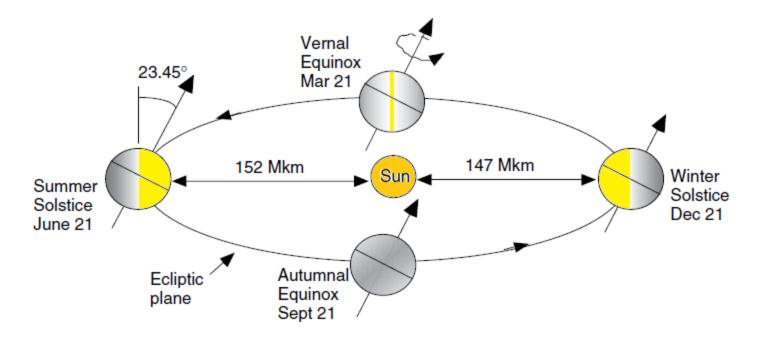
Day Nu	mbers for the	First Day of Eacl	h Month
January	n = 1	July	n = 182
February	n = 32	August	n = 213
March	n = 60	September	n = 244
April	n = 91	October	n = 274
May	n = 121	November	n = 305
June	n = 152	December	<i>n</i> = 335

) 2

3

#### The Earth's Orbit

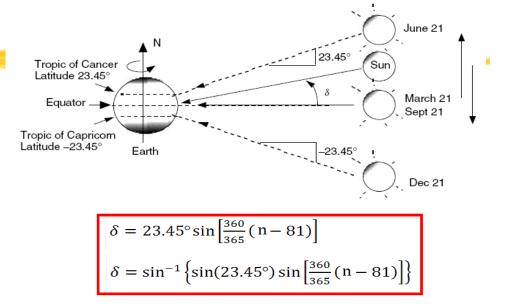
- **Earth's Spin Axis: 23.45 degrees**
- # Equinox (Equal day and night): 12 hours each for March 21 and September 21
- Winter Solstice: December 21 axis angle ("Inclination" of North Pole) is highest away from the sun
- **Summer Solstice: June 21 Inclination is closest to the sun.**



#### **Solar Declination**

Solar angles as seen from the surface of the earth → Ancient perspective: Fixed Earth and Sun Moving Up and Down View

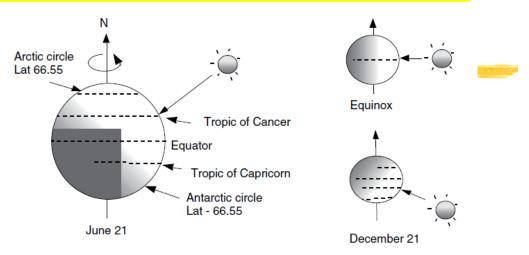
**Solar declination:** "angle between the sun's rays and the earth's <u>equatorial plane</u>, the latitude at which the sun is directly overhead at <u>midday</u>. Declination values are positive when the sun is north of the equator (March 21 to September 23) and negative when the sun is south of the equator. Maximum and minimum values are +0.409 radians (+23.45 degrees) and -0.409 radians (-23.45 degrees)."



Solar Declination $\delta$ for the 21 <sup>st</sup> Day of Each Month (degrees)												
Month: $\delta$ :				-	-		-		-			Dec -23.4

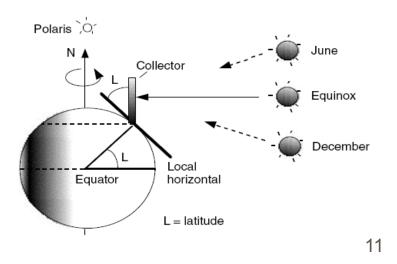
## **Solar Declination**

Solar Noon": Sun is directly over the local meridian (longitude)

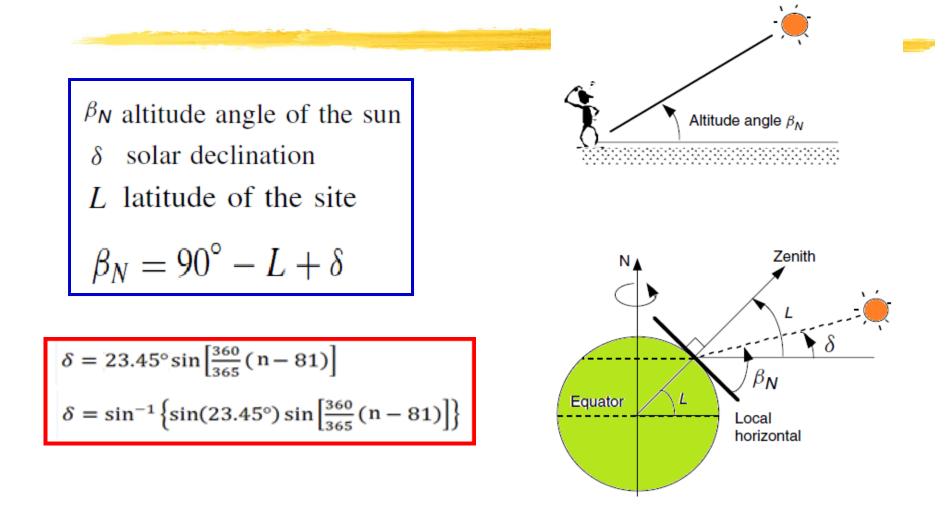


- A good rule of thumb of solar panel for the best annual performance

  - Tilt it up at an angle equal to the local latitude



#### Altitude Angle of the Sun and Solar Declination

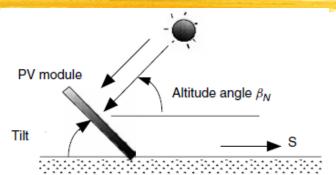


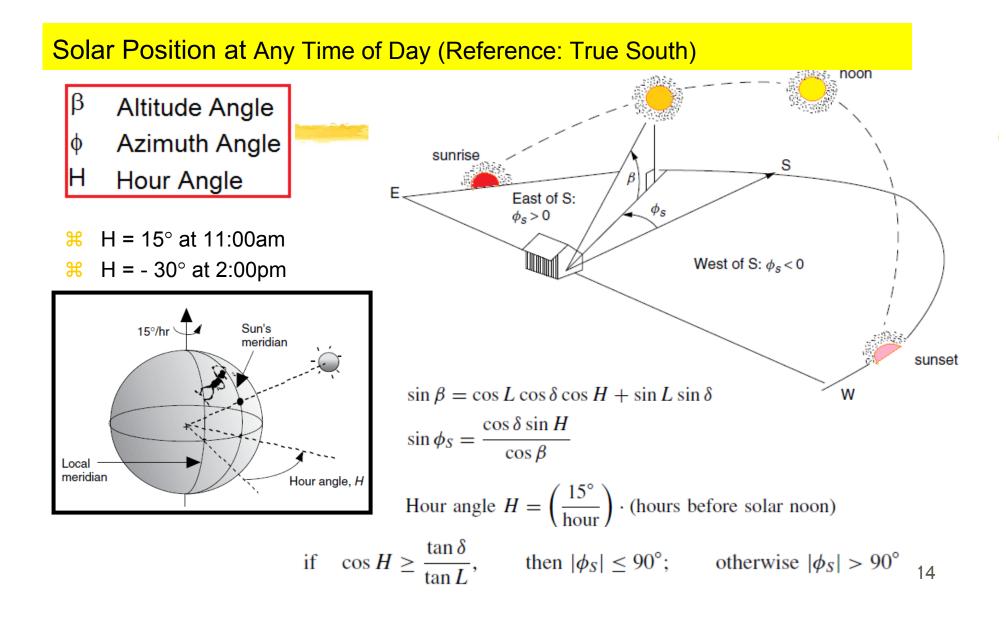
#### Example

Find the optimum tilt angle for a south-facing PV module in Tucson (latitude 32.1°) at solar noon on March 1.

$$\delta = [23.45^{\circ} \sin\left[\frac{360}{365}(n-81)\right]$$
$$\delta = \sin^{-1}\left\{\sin(23.45^{\circ}) \sin\left[\frac{360}{365}(n-81)\right]\right\}$$

$$\beta_N$$
 altitude angle of the sun  
 $\delta$  solar declination  
*L* latitude of the site  
 $\beta_N = 90^\circ - L + \delta$ 





#### Solar Position at Any Time of Day: Example

**Constitution:** Find the altitude angle ( $\beta$ ) and azimuth angle( $\phi$ ) for the sun at 3:00PM in Boulder, CO (Lat = 40°) on the summer solstice.

$$\sin \beta = \cos L \cos \delta \cos H + \sin L \sin \delta$$
  

$$\sin \phi_S = \frac{\cos \delta \sin H}{\cos \beta}$$
  
Hour angle  $H = \left(\frac{15^\circ}{\text{hour}}\right) \cdot \text{(hours before solar noon)}$   
if  $\cos H \ge \frac{\tan \delta}{\tan L}$ , then  $|\phi_S| \le 90^\circ$ ;  
otherwise  $|\phi_S| > 90^\circ$ 

$$\delta = 23.45^{\circ} \sin\left[\frac{360}{365}(n-81)\right]$$
$$\delta = \sin^{-1}\left\{\sin(23.45^{\circ})\sin\left[\frac{360}{365}(n-81)\right]\right\}$$

#### Solar Position at Any Time of Day: SOLUTION

**Constitution:** Find the altitude angle ( $\beta$ ) and azimuth angle( $\phi$ ) for the sun at 3:00PM in Boulder, CO (Lat = 40°) on the summer solstice.

#### **H** Answer:

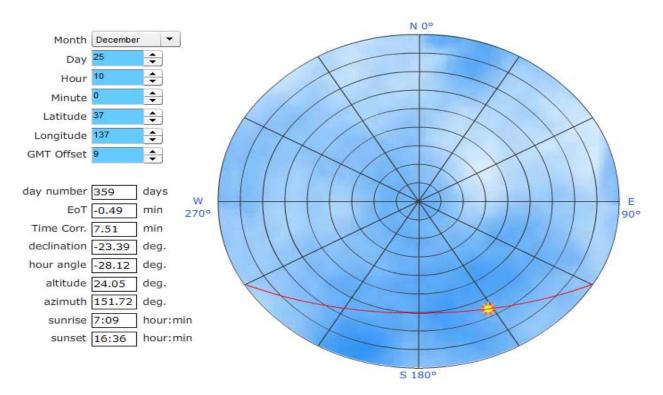
 $\sin \beta = \cos L \cos \delta \cos H + \sin L \sin \delta$   $\sin \phi_S = \frac{\cos \delta \sin H}{\cos \beta}$ Hour angle  $H = \left(\frac{15^\circ}{\text{hour}}\right) \cdot \text{(hours before solar noon)}$ if  $\cos H \ge \frac{\tan \delta}{\tan L}$ , then  $|\phi_S| \le 90^\circ$ ; otherwise  $|\phi_S| > 90^\circ$   $\delta = 23.45^{\circ} \sin\left[\frac{360}{365}(n-81)\right]$  $\delta = \sin^{-1}\left\{\sin(23.45^{\circ})\sin\left[\frac{360}{365}(n-81)\right]\right\}$ 

#### **Sun Position Calculator**

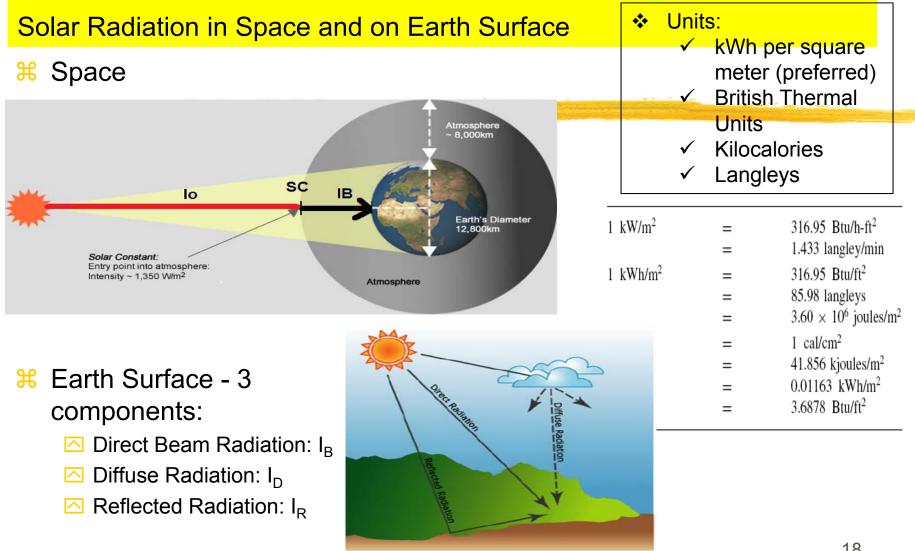
PVEDUCATION.ORG

PV CDROM Christiana Honsberg and Stuart Bowden

- Sample Example
- Weblink: <u>http://pveducation.org/pvcdrom/properties-of-sunlight/sun-position-calculator</u>



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Source: esri.com

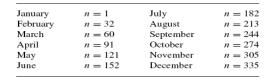
#### **Clear Sky Beam Radiation**

- **#** Extraterrestrial Solar Insolation ( $I_0$ ) $I_0 = SC \cdot \left[ 1 + 0.034 \cos \left( \frac{360n}{365} \right) \right]$  $\bigtriangleup SC$ : Solar constant $\bigtriangleup n$ : day number1.377 kW/m<sup>2</sup>
- **#** Portion of the beam reaching the earth surface ("horizontal surface") ( $I_{\rm B}$ )

$$I_B = A e^{-km}$$

degree<sup>O</sup>

 $(W/m^2)$ 



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△ A: Apparent extraterrestrial flux

$$A = 1160 + 75 \sin \left[ \frac{360}{365} (n - 275) \right] \quad (W/m^2)$$

$$\land k : optical depth \qquad k = 0.174 + 0.035 \sin \left[ \frac{360}{365} (n - 100) \right]$$

$$\land m: \text{ air mass ratio}$$

$$\land \beta: \text{ altitude angle of the sun} \qquad m = \frac{1}{\sin \beta}$$

#### **Clear Sky Beam Radiation**

- H A: Apparent extraterrestrial flux
- Ħ *k* : optical depth
- H *m*: air mass ratio
- $\beta$ : altitude angle of H the sun

April

May

June

n = 91

n = 121

n = 152

November

December

Optical Depth k, Apparent Extraterrestrial Flux A, and the Sky

Diffuse Factor C for the 21<sup>st</sup> Day of Each Month

Month:	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
$A (W/m^2)$ :	1230	1215	1186	1136	1104	1088	1085	1107	1151	1192	1221	1233
<i>k</i> :	0.142	0.144	0.156	0.180	0.196	0.205	0.207	0.201	0.177	0.160	0.149	0.142
<i>C</i> :	0.058	0.060	0.071	0.097	0.121	0.134	0.136	0.122	0.092	0.073	0.063	0.057

Source: ASHRAE (1993).

n = 274

n = 305

n = 335

#### **Clear Sky Beam Radiation – Example**

**Question:** Find the direct beam solar radiation normal to the sun's rays ("horizontal surface") at solar noon on a clear day on Howard University campus (latitude ???? degrees) on May 21.

$$I_B = Ae^{-km}$$

Air mass ratio 
$$m = \frac{1}{\sin \beta}$$

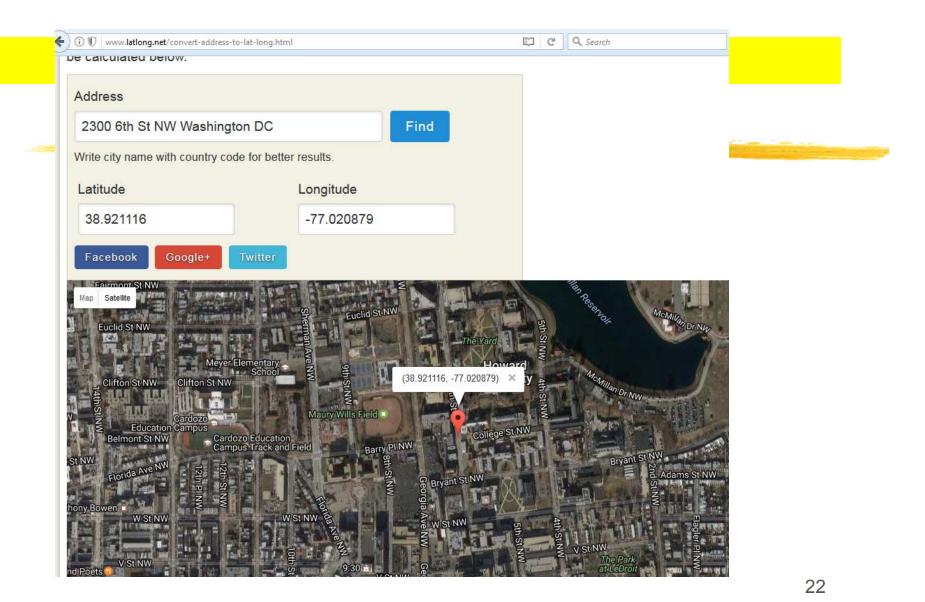
$$A = 1160 + 75 \sin\left[\frac{360}{365}(n - 275)\right] \quad (W/m^2)$$
$$k = 0.174 + 0.035 \sin\left[\frac{360}{365}(n - 100)\right]$$

$\delta = 23.45^{\circ} \sin\left[\frac{360}{365}(n-81)\right]$
$\delta = \sin^{-1} \left\{ \sin(23.45^{\circ}) \sin \left[ \frac{360}{365} (n - 81) \right] \right\}$

January	n = 1	July	n = 182
February	n = 32	August	n = 212
March	n = 60	September	n = 24
April	n = 91	October	n = 27
May	n = 121	November	n = 30
June	n = 152	December	n = 33

altitude angle of the sun at solar noon.

$$\beta_N = 90^\circ - L + \delta$$



#### **Clear Sky Beam Radiation – Example**

 $\Re$  Question: Find the direct beam solar radiation normal to the sun's rays ("horizontal surface") at solar noon on a clear day on Howard University campus (latitude 38.92116 degrees) on May 21.

$$I_{B} = Ae^{-km}$$

$$\delta = 23.45^{\circ} \sin\left[\frac{360}{365}(n-81)\right]$$

$$\delta = \sin^{-1}\left\{\sin(23.45^{\circ})\sin\left[\frac{360}{365}(n-81)\right]\right\}$$
mass ratio
$$m = \frac{1}{\sin\beta}$$

Air m

$$A = 1160 + 75 \sin\left[\frac{360}{365}(n - 275)\right] \quad (W/m^2)$$
$$k = 0.174 + 0.035 \sin\left[\frac{360}{365}(n - 100)\right]$$

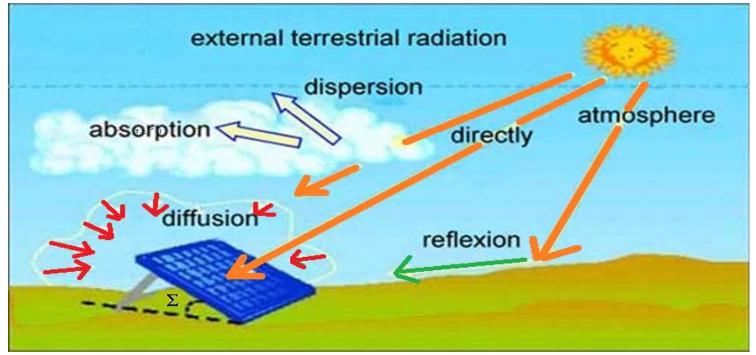
altitude angle of the sun at solar noon.

$$\beta_N = 90^\circ - L + \delta$$

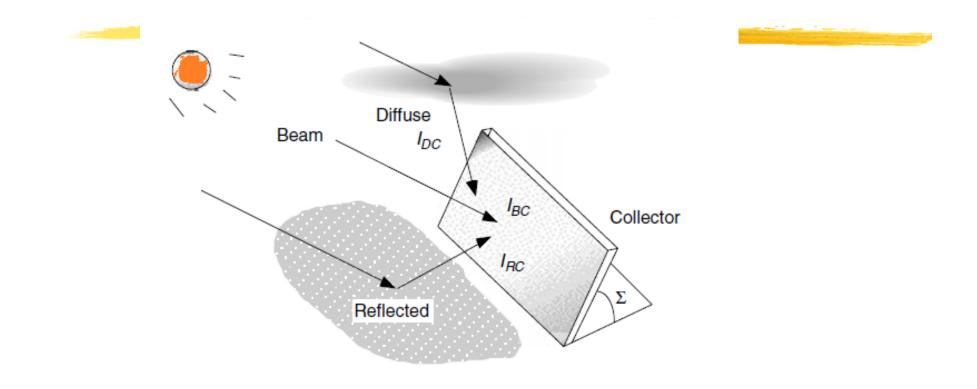
January	n = 1	July	n = 182
February	n = 32	August	n = 213
March	n = 60	September	n = 244
April	n = 91	October	n = 274
May	n = 121	November	n = 305
June	n = 152	December	n = 335

#### **Radiation on collector**

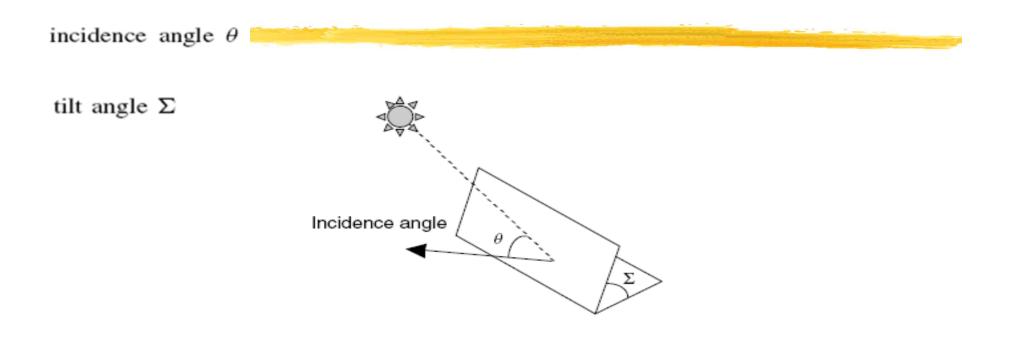
- **Collector Surface:** 
  - Beam radiation: I<sub>BC</sub>
  - Diffuse radiation: I<sub>DC</sub>
  - △ Reflected radiation: I<sub>RC</sub>



# Total Clear Sky Insolation on Collector Surface



#### Beam Radiation on Collector – Simple Case



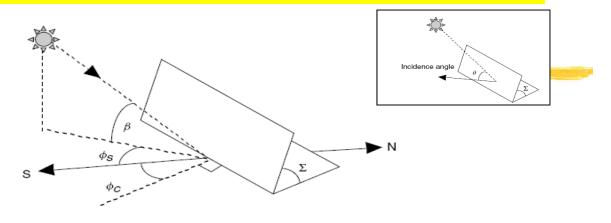
$$I_{BC} = I_B \cos \theta$$

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#### Beam Radiation on Collector - Complicated

incidence angle  $\theta$ collector azimuth angle  $\phi_C$ altitude angle  $\beta$ solar azimuth angle  $\phi_S$ tilt angle  $\Sigma$ 

- **Solar altitude:**  $\beta$
- **H** Normal to vertical surface:  $\phi_c$



$$I_{BC} = I_B \cos \theta$$

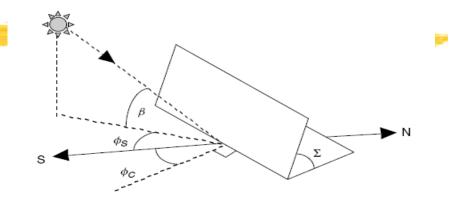
$$\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma$$

beam insolation on a horizontal surface  $I_{BH}$ 

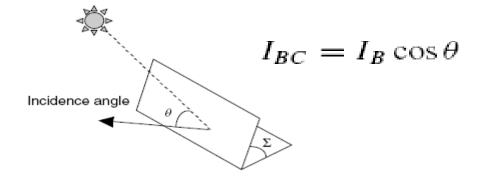
$$I_{BH} = I_B \cos(90^\circ - \beta) = I_B \sin \beta$$

#### Beam Radiation on **Collector - Example**

Question: At solar noon in Atlanta (latitude 33.7) on May 21, the altitude angle of the sun was found to be 76.4 degrees and the clear-sky beam insolation was found to be 902 W/m<sup>2</sup>. Find the beam insolation at that time on a collector that faces 20 degrees toward the southeast with tipped angle at 52 degrees.



$$\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma$$

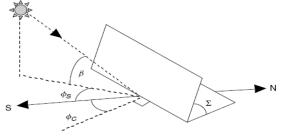


incidence angle  $\theta$ collector azimuth angle  $\phi_C$ altitude angle  $\beta$ solar azimuth angle  $\phi_S$ tilt angle  $\Sigma$ 

#### Beam Radiation on Collector - Example

**Question:** At solar noon in Atlanta (latitude 33.7) on May 21, the altitude angle of the sun was found to be 76.4 degrees and the clear-sky beam insolation was found to be 902 W/m<sup>2</sup>. Find the beam insolation at that time on a collector that faces 20 degrees toward the southeast with tipped angle at 52 degrees.

 $\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma$ 



 $I_{BC} = I_B \cos \theta$ 

#### **Diffuse Radiation on Collector**

₭ Sky diffuse factor (C)

$$C = 0.095 + 0.04 \sin\left[\frac{360}{365}(n - 100)\right]$$

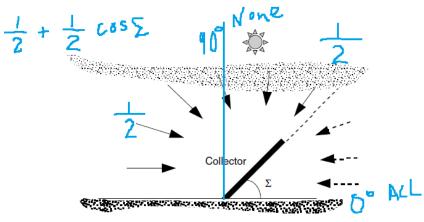
January	n = 1	July	n = 182
February	n = 32	August	n = 213
March	n = 60	September	n = 244
April	n = 91	October	n = 274
May	n = 121	November	n = 305
June	n = 152	December	n = 335

- n: day number
- Biffuse insolation on a Horizontal surface is proportional to the direct radiation

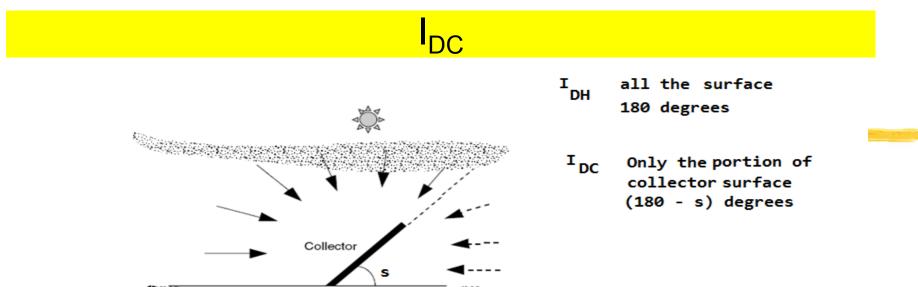
$$I_{DH} = C I_B$$

Biffuse Radiation on collector

$$I_{DC} = I_{DH} \left( \frac{1 + \cos \Sigma}{2} \right) = C I_B \left( \frac{1 + \cos \Sigma}{2} \right)$$



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If S=0; Collector gets full diffuse radiation.

If S=90; collecter gets 1/2 of the diffuse radiation.

mathematical expression  $\frac{1 + \sin(90-S)}{2}$ 

sin (90-S)=sin90\*cosS - cos90\*sinS

=cosS

Finally,  $I_{DC} = \frac{1 + \cos S}{2}$ 

#### **Diffuse Radiation on Collector - Example**

- Question: At solar noon in Atlanta (latitude 33.7) on May 21, the altitude angle of the sun was found to be 76.4 degrees and the clear-sky beam insolation was found to be 902 W/m<sup>2</sup>. Find the diffuse radiation at that time on a collector that faces 20 degrees toward the southeast with tipped angle at 52 degrees.
- **Find the Diffuse Radiation** (I<sub>DC</sub>)

January	n = 1	July	n = 182
February	n = 32	August	n = 213
 March	n = 60	September	n = 244
April	n = 91	October	n = 274
May	n = 121	November	n = 305
June	n = 152	December	n = 335

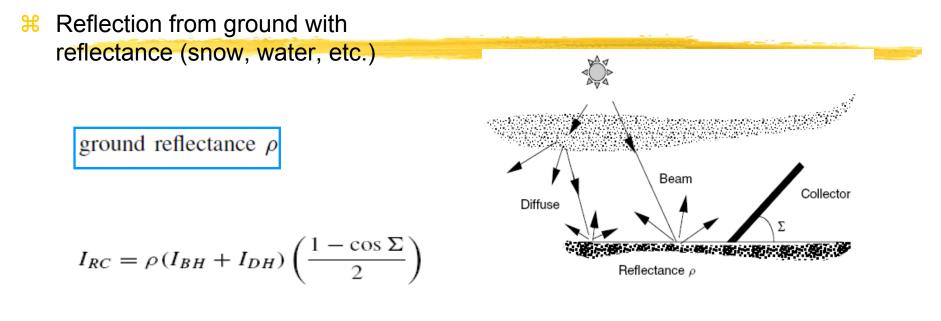
$$C = 0.095 + 0.04 \sin\left[\frac{360}{365}(n - 100)\right]$$

$$I_{DH} = C I_B$$

$$I_{DC} = I_{DH} \left( \frac{1 + \cos \Sigma}{2} \right) = C I_B \left( \frac{1 + \cos \Sigma}{2} \right)$$

 $\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma$ 

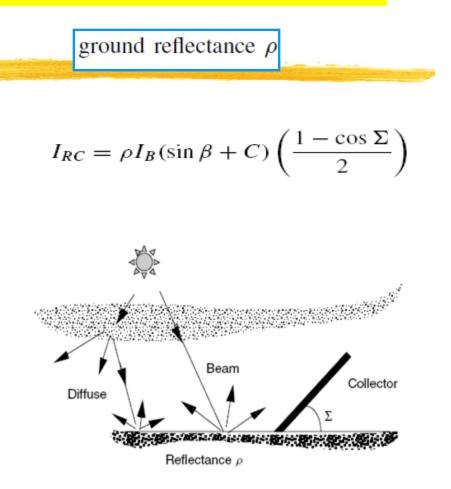
#### **Reflected Radiation on Collector**



$$I_{RC} = \rho I_B (\sin \beta + C) \left( \frac{1 - \cos \Sigma}{2} \right)$$

#### **Reflected Radiation on Collector: Example**

**Question:** At solar noon in Atlanta (latitude 33.7) on May 21, the altitude angle of the sun was found to be 76.4 degrees and the clear-sky beam insolation was found to be 902 W/m<sup>2</sup>. Find the **reflected radiation** at that time on a collector that faces 20 degrees toward the southeast with tipped angle at 52 degrees, if the reflectance of the surfaces in front of the panel is 0.2.



#### **Total Radiation on Collector**

**Combination** of all three: Radiation striking a collector on a clear day

$$I_{C} = I_{BC} + I_{DC} + I_{RC}$$

$$I_{C} = Ae^{-km} \left[ \cos\beta\cos(\phi_{S} - \phi_{C})\sin\Sigma + \sin\beta\cos\Sigma + C\left(\frac{1 + \cos\Sigma}{2}\right) + \rho(\sin\beta + C)\left(\frac{1 - \cos\Sigma}{2}\right) \right]$$

$$I_{B} = Ae^{-km} \qquad I_{BC} = I_{B}\cos\theta \qquad A = 1160 + 75\sin\left[\frac{360}{365}(n - 275)\right] \quad (W/m^{2})$$

$$C = 0.095 + 0.04\sin\left[\frac{360}{365}(n - 100)\right] \qquad k = 0.174 + 0.035\sin\left[\frac{360}{365}(n - 100)\right]$$

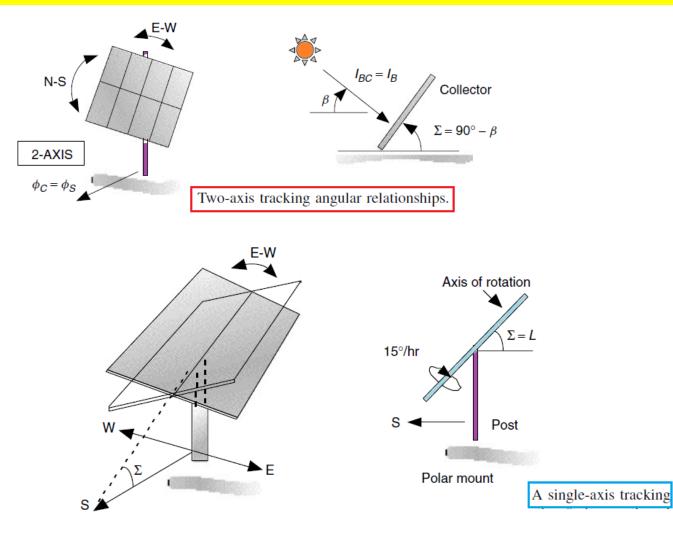
$$\beta_{N} = 90^{\circ} - L + \delta$$

$$\cos\theta = \cos\beta\cos(\phi_{S} - \phi_{C})\sin\Sigma + \sin\beta\cos\Sigma \qquad \delta = 23.45^{\circ}\sin\left[\frac{360}{365}(n - 81)\right]$$

$$\delta = \sin^{-1}\left\{\sin(23.45^{\circ})\sin\left[\frac{360}{365}(n - 81)\right]\right\}$$

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# **Tracking System**





# Class Activity - 7

Compare the 40° latitude, clear sky insolation on a collector at solar noon on the summer solstice for (a) fixed titled angle of 40° facing south, (b) single axis polar mount, and (c) two-axis tracking. Ignore the reflected insolation.

**#** Answer:

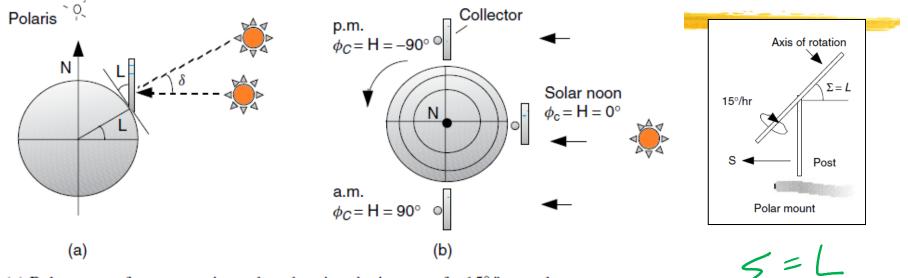
△(a) Ic (fixed) =	[	] W/m²
△(b) Ic (1-axis) =	[	] W/m <sup>2</sup>
○ (c) Ic (2-axis) =	[	] W/m²

# Fixed Collector (No Tracking)

$$I_{BC} = I_B \cos \theta \qquad \cos \theta = \cos \beta \cos(\phi_S - \phi_C) \sin \Sigma + \sin \beta \cos \Sigma$$
$$I_{DC} = CI_B \left(\frac{1 + \cos \Sigma}{2}\right) \qquad \qquad I_{BH} = I_B \cos(90^\circ - \beta) = I_B \sin \beta$$
$$I_{RC} = \rho (I_{BH} + I_{DH}) \left(\frac{1 - \cos \Sigma}{2}\right) \qquad \qquad I_{DH} = CI_B$$

### **One-Axis Polar Mount Tracking**

**One-Axis, Polar Mount:** 



(a) Polar mount for a one-axis tracker showing the impact of a 15°/h angular rotation of the collector array.

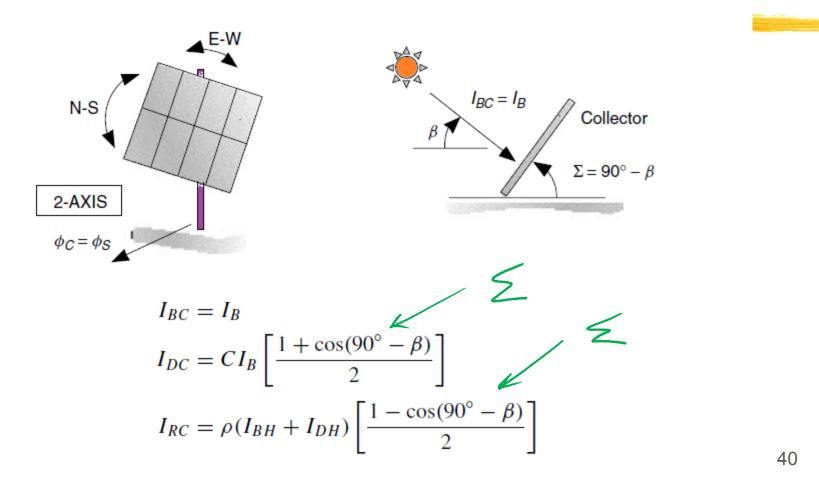
(b) Looking down on North Pole.

B=90-L+8

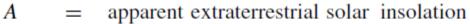
 $I_{BC} = I_B \cos \delta$  $I_{DC} = CI_B \left[ \frac{1 + \cos(90^\circ - \beta + \delta)}{2} \right]$  $1 - \cos(90^\circ - \beta +$  $I_{RC} = \rho (I_{BH} + I_{DH})$ 

### **Two-Axis Tracking**

**Two-Axis Tracking:** 



- $I_0$  = extraterrestrial solar insolation
- m = air mass ratio
- $I_B$  = beam insolation at earth's surface



- k = atmospheric optical depth
- C = sky diffuse factor
- $I_{BC}$  = beam insolation on collector
- $\theta$  = incidence angle
- $\Sigma$  = collector tilt angle
- $I_H$  = insolution on a horizontal surface
- $I_{DH}$  = diffuse insolation on a horizontal surface
- $I_{DC}$  = diffuse insolation on collector
- $I_{RC}$  = reflected insolation on collector
- $\rho$  = ground reflectance
- $I_C$  = insolation on collector
- n = day number
- $\beta$  = solar altitude angle
- $\delta$  = solar declination
- $\phi_S$  = solar azimuth angle (+ = AM)
- $\phi_C$  = collector azimuth angle (+ = SE) 41

 $I_0 = 1370 \left[ 1 + 0.034 \cos \left( \frac{360n}{365} \right) \right]$ (W/m<sup>2</sup>)  $m = \frac{1}{\sin\beta}$  $I_B = Ae^{-km}$  $A = 1160 + 75 \sin \left[ \frac{360}{365} (n - 275) \right] (W/m^2)$  $k = 0.174 + 0.035 \sin\left[\frac{360}{365}(n - 100)\right]$  $I_{BC} = I_B \cos \theta$  $\cos\theta = \cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma$  $I_{BH} = I_B \cos(90^\circ - \beta) = I_B \sin \beta$  $I_{DH} = CI_R$ 

$$C = 0.095 + 0.04 \sin\left[\frac{360}{365}(n - 100)\right]$$
$$I_{DC} = I_{DH}\left(\frac{1 + \cos \Sigma}{2}\right) = I_B C\left(\frac{1 + \cos \Sigma}{2}\right)$$
$$I_{RC} = \rho I_B(\sin \beta + C)\left(\frac{1 - \cos \Sigma}{2}\right)$$
$$I_C = I_{BC} + I_{DC} + I_{RC}$$
$$I_C = Ae^{-km}\left[\cos\beta\cos(\phi_S - \phi_C)\sin\Sigma + \sin\beta\cos\Sigma + C\left(\frac{1 + \cos\Sigma}{2}\right) + \rho(\sin\beta + C)\left(\frac{1 - \cos\Sigma}{2}\right)\right]$$

Two-Axis Tracking:

$$I_{BC} = I_B$$

$$I_{DC} = CI_B \left[ \frac{1 + \cos(90^\circ - \beta)}{2} \right]$$

$$I_{RC} = \rho (I_{BH} + I_{DH}) \left[ \frac{1 - \cos(90^\circ - \beta)}{2} \right]$$

**One-Axis, Polar Mount:** 

$$I_{BC} = I_B \cos \delta$$
$$I_{DC} = CI_B \left[ \frac{1 + \cos(90^\circ - \beta + \delta)}{2} \right]$$
$$I_{RC} = \rho (I_{BH} + I_{DH}) \left[ \frac{1 - \cos(90^\circ - \beta + \delta)}{2} \right]$$

# Class Activity - 7

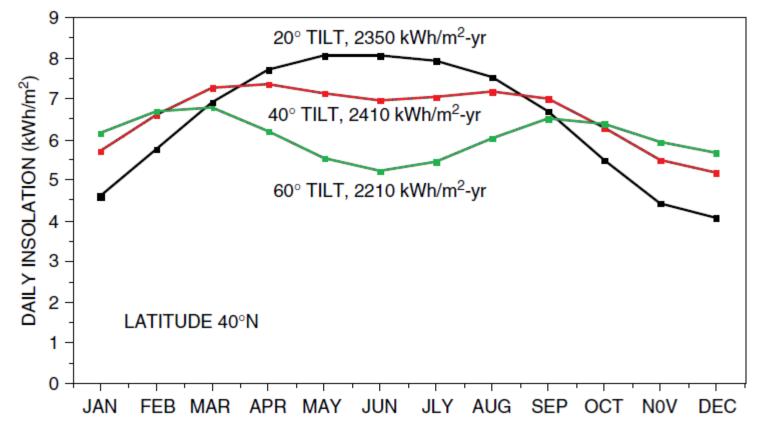
Compare the 40° latitude, clear sky insolation on a collector at solar noon on the summer solstice for (a) fixed titled angle of 40° facing south, (b) single axis polar mount, and (c) two-axis tracking. Ignore the reflected insolation.

**#** Answer:

△(a) Ic (fixed) =	[	] W/m²
△(b) Ic (1-axis) =	[	] W/m <sup>2</sup>
○ (c) Ic (2-axis) =	[	] W/m²

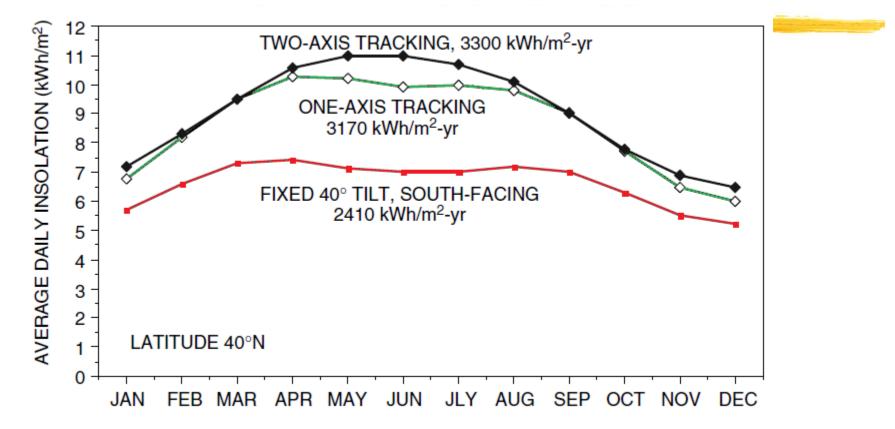
# **Monthly Clear-Sky Insolation**

- **What we have calculated so far?** Instantaneous insolation
- **Baily (and Yearly) Insolation on South-Facing Collectors**



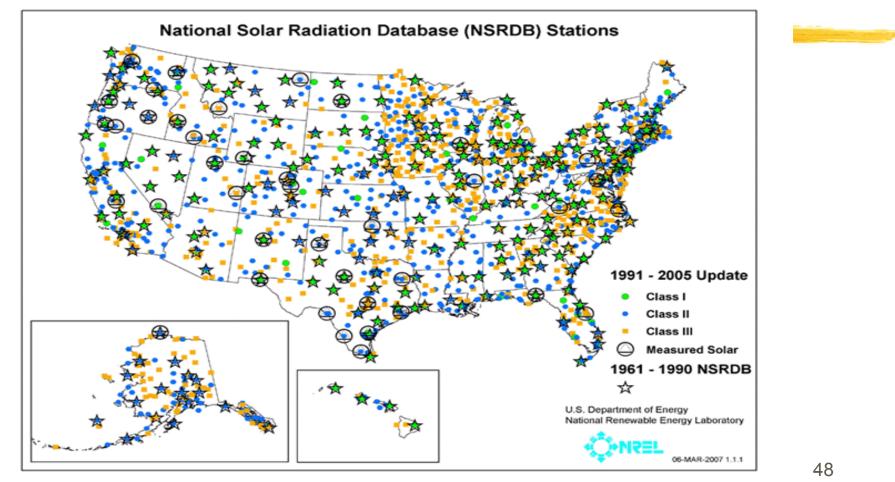
# **Monthly Clear-Sky Insolation**

**Baily (and Yearly) Insolation on Tracking & South-Faced Collectors** 



# **Solar Radiation Measurement Stations**

**239** National Solar Radiation Database Stations (NSRDB)



### **Pyranometer and Pyrheliometer**

 Pyranometer: measures total radiation arriving from all directions, direct and diffuse component





**Pyrheliometer**: measures only direct radiation





### **Average Daily Insolation**

- ₭ Sky is not always clear
- Estimate of average insolation that strikes a tilted collector under real conditions at a particular site
  - Atmospheric clearness
  - Solar hour angle from sunrise to sunset
  - 🔼 Average Tilt Factor
  - $\square$   $I_{C} = I_{BC} + I_{DC} + I_{RC}$  (direct + Diffuse + reflection) on collector surface
- Starting Point: Working on horizontal insolation (on horizontal surface) (since primary measurement data is on horizontal insolation I<sub>H</sub>)
- $\Re$   $H_{H} = I_{DH} + I_{BH}$  (Horizontal Insolation = Horizontal Diffuse + Horizontal Beam)
- $\exists I_{DC} \leftarrow I_{DH} \& I_{RC} \leftarrow I_{H}$  (already discussed)
- $\Re$  Question is how to get I<sub>BC</sub> from I<sub>H</sub>
  - We decompose the total horizontal insolation in to its diffuse component and beam components
  - Clearness index K<sub>T</sub>

### Decomposition of Total Horizontal Insolation (I<sub>H</sub>)

- Clearness index (K<sub>T</sub>): Ratio of average horizontal insolation at a site (I<sub>H</sub>) to the extraterrestrial insolation on a horizontal surface above the site and just outside the atmosphere (I<sub>o</sub>)
- **Here are a set of**  $I_0$ : averaging the product of normal radiation and the SIN of the solar hour angle from sunrise and sunset:
- Correlation between Clearness Index and Diffuse Radiation SC: Solar constant 1.377 kW/m<sup>2</sup>

$$\overline{I}_0 = \left(\frac{24}{\pi}\right) \operatorname{SC}\left[1 + 0.034 \cos\left(\frac{360n}{365}\right)\right] (\cos L \cos \delta \sin H_{SR} + H_{SR} \sin L \sin \delta)$$

Biffuse and Reflected Radiation on a tilted collector surface

$$\frac{\overline{I}_{DH}}{\overline{I}_{H}} = 1.390 - 4.027K_{T} + 5.531K_{T}^{2} - 3.108K_{T}^{3}$$
$$\overline{I}_{DC} = \overline{I}_{DH} \left(\frac{1 + \cos\Sigma}{2}\right) \qquad \overline{I}_{RC} = \rho \overline{I}_{H} \left(\frac{1 - \cos\Sigma}{2}\right)$$

 $K_T = \frac{I_H}{\overline{I}_0}$ 

 $H_{SR} = \cos^{-1}(-\tan L \tan \delta)$ 

SUNRISE HOUR ANGLE - The sunrise hour angle is the hour angle, expressed in degrees, when the sun's center reaches the horizon.

### **Conversion to Beam Radiation on Collector**

He average beam radiation on a horizontal surface (I<sub>BH</sub>) can be found by subtracting the diffuse portion  $(I_{DH})$  from the total  $(I_{H})$ :

$$\overline{I}_{H} = \overline{I}_{DH} + \overline{I}_{BH} \longrightarrow \overline{I}_{BH} = \overline{I}_{H} - \overline{I}_{DH}$$

H Conversion of horizontal beam radiation (I<sub>BH</sub>) to the beam radiation on collector  $(I_{BC})$ :

$$I_{BH} = I_B \sin \beta$$

$$I_{BC} = I_B \cos \theta \longrightarrow I_{BC} = I_{BH} \left(\frac{\cos \theta}{\sin \beta}\right) = I_{BH} R_B$$

$$\theta \text{ is the incidence angle between the collector and beam}$$

$$\beta \text{ is the sun's altitude angle}$$

$$R_B \text{ is beam tilt factor}$$
Average value of Beam Tilt Factor (R<sub>B</sub>):
Instantaneous beam tilt factor
$$R_B = \left(\frac{\cos \theta}{\sin \beta}\right)^{1/2}$$
Incidence angle i

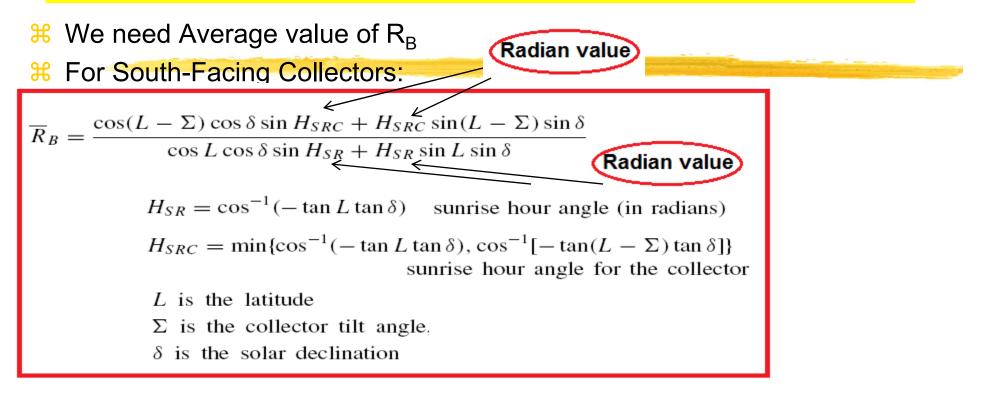
Instantaneous beam tilt factor

Σ

52

H

### Average value of Beam Tilt Factor ( $R_B$ )

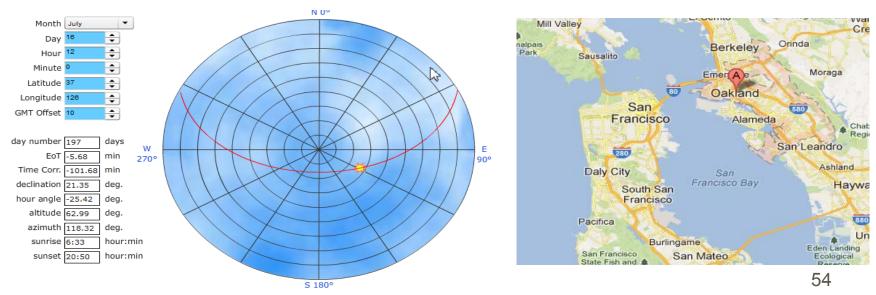


### **Final Equation for Insolution Striking a collector** $\overline{I}_C = \overline{I}_{BC} + \overline{I}_{DC} + \overline{I}_{RC}$

$$\overline{I}_{C} = \overline{I}_{H} \left( 1 - \frac{\overline{I}_{DH}}{\overline{I}_{H}} \right) \cdot \overline{R}_{B} + \overline{I}_{DH} \left( \frac{1 + \cos \Sigma}{2} \right) + \rho \overline{I}_{H} \left( \frac{1 - \cos \Sigma}{2} \right)$$

### **Calculation of Average Daily Insolation**

- Herein Average Monthly Insolation on a Tilted Collector
- # Average horizontal insolation (I<sub>H</sub>) in Oakland, California (latitude 37.73°N) in July 16 is 7.32 kWh/m<sup>2</sup>-day. Assume ground reflectivity of 0.2.
- **# Question**: Estimate the average insolation on a south-facing collector at a tilt angle of 30° with respect to the horizontal.



### **Calculation of Average Daily Insolation**

- 0. Target
- $\Re$  1. Sun declination ( $\delta$ ) for July 16 (n=197)
- $\approx$  2. Sunrise Hour Angle (H<sub>SR</sub>) using L=37.73°
- ∺ 3. Extraterrestrial Insolation (I₀) (with SC=1.37 kW/m²)
- ∺ 4. Clearness Index (K<sup>T</sup>)
- ∺ 5. Horizontal Diffuse Radiation (I<sub>DH</sub>)
- $\Re$  6. Diffuse Radiation on the Collector ( $I_{DC}$ )
- $\Re$  7. Reflected Radiation on the Collector (I<sub>RC</sub>)
- $\aleph$  8. Horizontal Beam Radiation (I<sub>BH</sub>)
- $\Re$  9. Sunrise Hour Angle on the Collector (H<sub>SRC</sub>)
- $\approx$  10. Beam Tilt Factor (R<sub>B</sub>)
- 11. Beam Radiation on the Collector ( $I_{BC}$ )
- $\approx$  12. Total Insolation on the Collector (I<sub>C</sub>)

# **Solution - Details**

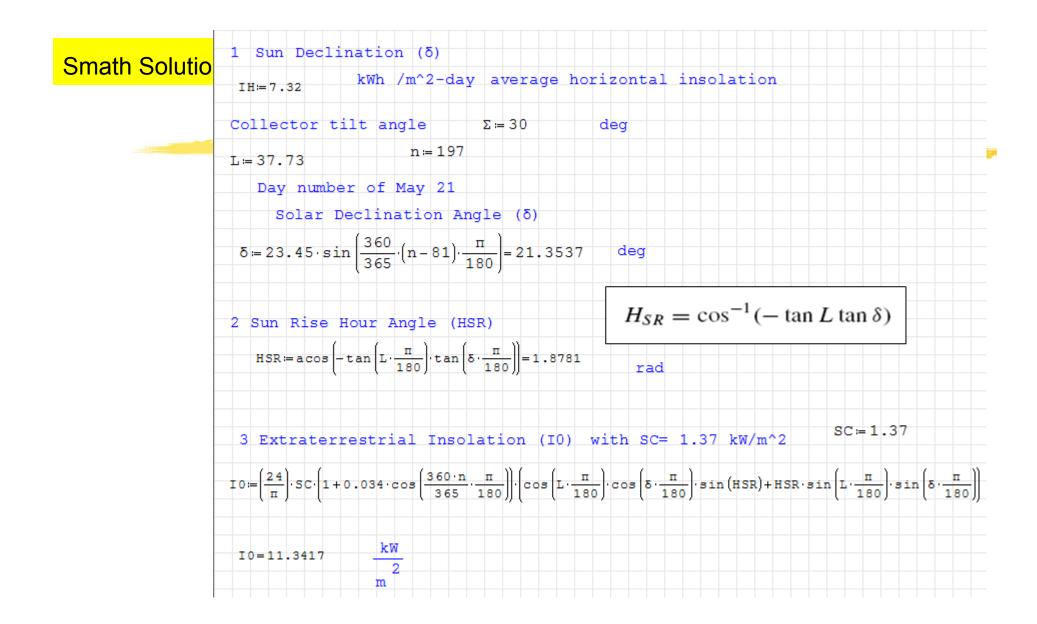
July 16 (n = 197):

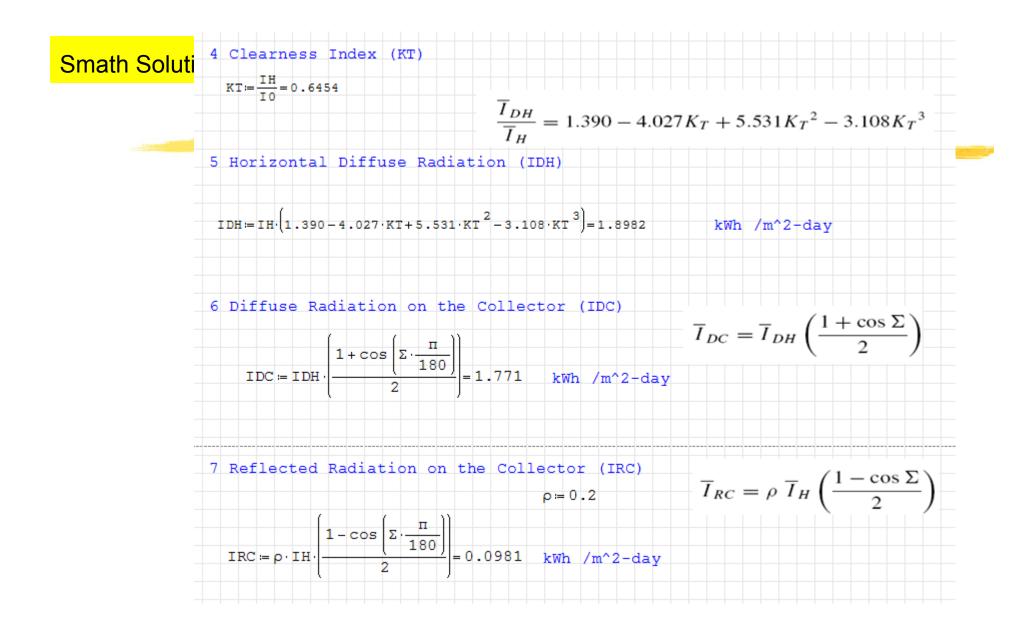
$$\begin{split} \delta &= 23.45 \sin \left[ \frac{360}{365} (n-81) \right] = 23.45 \sin \left[ \frac{360}{365} (197-81) \right] \\ &= 21.35^{\circ} \\ H_{SR} &= \cos^{-1} (-\tan L \tan \delta) \\ &= \cos^{-1} (-\tan 37.73^{\circ} \tan 21.35^{\circ}) = 107.6^{\circ} = 1.878 \text{ radians} \\ \overline{I}_{0} &= \left( \frac{24}{\pi} \right) \text{SC} \left[ 1 + 0.034 \cos \left( \frac{360n}{365} \right) \right] (\cos L \cos \delta \sin H_{SR} + H_{SR} \sin L \sin \delta) \\ &= \left( \frac{24}{\pi} \right) 1.37 \left[ 1 + 0.034 \cos \left( \frac{360 \cdot 197}{365} \right)^{\circ} \right] (\cos 37.73 \cos 21.35^{\circ} \sin 107.6^{\circ} \\ &+ 1.878 \sin 37.73^{\circ} \sin 21.35^{\circ}) \\ &= 11.34 \text{ kWh/m}^{2} \text{-day} \\ K_{T} &= \frac{\overline{I}_{H}}{\overline{I}_{0}} = \frac{7.32 \text{ kWh/m}^{2} \cdot \text{day}}{11.34 \text{ kWh/m}^{2} \cdot \text{day}} = 0.645 \\ \frac{\overline{I}_{DH}}{\overline{I}_{H}} &= 1.390 - 4.027 K_{T} + 5.531 K_{T}^{2} - 3.108 K_{T}^{3} \\ &= 1.390 - 4.027 (0.645) + 5.531 (0.645)^{2} - 3.108 (0.645)^{3} = 0.258 \end{split}$$

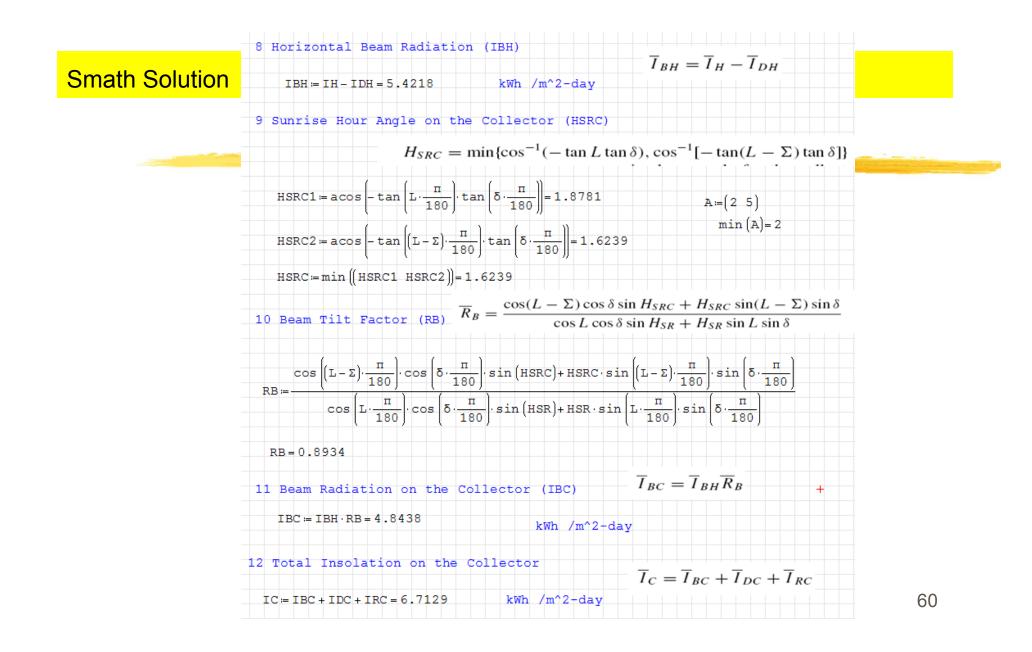
# Solution- Details (Continued)

$$\begin{split} \overline{I}_{DH} &= 0.258 \cdot 7.32 = 1.89 \text{ kWh/m}^2 \text{-day} \\ \overline{I}_{DC} &= \overline{I}_{DH} \left( \frac{1 + \cos \Sigma}{2} \right) = 1.89 \left( \frac{1 + \cos 30^\circ}{2} \right) = 1.76 \text{ kWh/m}^2 \text{-day} \\ \overline{I}_{RC} &= \rho \ \overline{I}_H \left( \frac{1 - \cos \Sigma}{2} \right) = 0.2 \cdot 7.32 \left( \frac{1 - \cos 30^\circ}{2} \right) = 0.10 \text{ kWh/m}^2 \text{-day} \\ \overline{I}_{BH} &= \overline{I}_H - \overline{I}_{DH} = 7.32 - 1.89 = 5.43 \text{ kWh/m}^2 \text{-day} \\ H_{SRC} &= \min\{\cos^{-1}(-\tan L \tan \delta), \ \cos^{-1}[-\tan(L - \Sigma) \tan \delta]\} \\ &= \min\{\cos^{-1}(-\tan 37.73^\circ \tan 21.35^\circ), \ \cos^{-1}[-\tan(37.73 - 30)^\circ \tan 21.35^\circ]\} \\ &= \min\{107.6^\circ, 93.0^\circ\} = 93.0^\circ = 1.624 \text{ radians} \\ \overline{R}_B &= \frac{\cos(L - \Sigma) \cos \delta \sin H_{SRC} + H_{SRC} \sin(L - \Sigma) \sin \delta}{\cos L \cos \delta \sin H_{SR} + H_{SR} \sin L \sin \delta} \\ &= \frac{\cos(37.73 - 30)^\circ \cos 21.35^\circ \sin 93^\circ + 1.624 \sin(37.73 - 30)^\circ \sin 21.35^\circ}{\cos 37.73^\circ \cos 21.35^\circ \sin 107.6^\circ + 1.878 \sin 37.73^\circ \sin 21.35^\circ} \\ &= 0.893 \end{split}$$

$$\overline{I}_{BC} = \overline{I}_{BH} \overline{R}_B = 5.43 \cdot 0.893 = 4.85 \text{ kWh/m}^2 \text{-day}$$
$$\overline{I}_C = \overline{I}_{BC} + \overline{I}_{DC} + \overline{I}_{RC} = 4.85 + 1.76 + 0.10 = 6.7 \text{ kWh/m}^2 \text{-day}$$
57

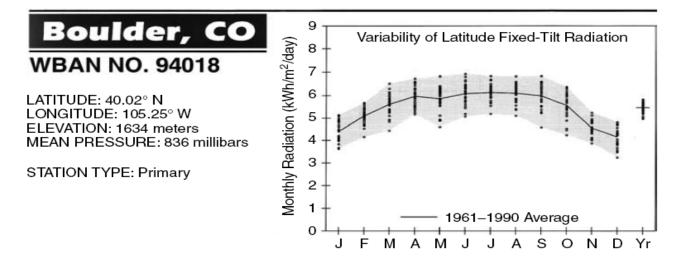






### Calculation is complex, so we need

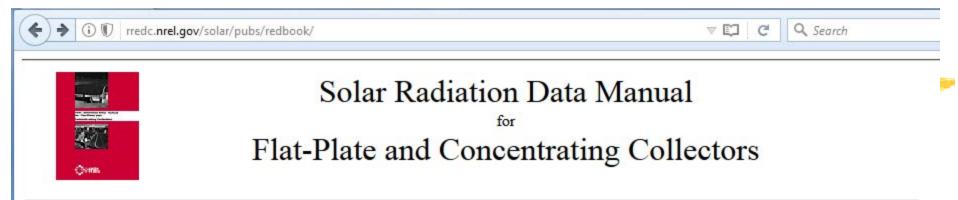
- $\Re$  Spreadsheet or Computer Analysis  $\rightarrow$  Complex
- Pre-computed Data such as Solar Radiation Data Manual for Flat-Place and Concentrating Collectors (NREL, 1994): AVERAGE DAILY INSOLATION Per MONTH



Solar Radiation for Flat-Plate Collectors Facing South at a Fixed Tilt (kWh/m<sup>2</sup>/day), Uncertainty ±9%

Tilt (°)		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Year
0	Average	2.4	3.3	4.4	5.6	6.2	6.9	6.7	6.0	5.0	3.8	2.6	2.1	4.6
0	Min/Max	2.1/2.7	2.8/3.5	3.7/5.0	4.8/6.1	5.1/7.2	5.7/7.8	5.6/7.4	5.2/6.6	4.0/5.5	3.1/4.2	2.3/2.8	1.9/2.3	4.3/4.8
Latitude =15	Average	3.8	4.6	5.4	6.1	6.2	6.6	6.6	6.3	5.9	5.1	4.0	3.5	5.4
	Min/Max	3.2/4.4	3.8/5.1	4.3/6.2	5.3/6.8	4.9/7.3	5.5/7.6	5.6/7.4	5.3/7.1	4.6/6.7	4.0/5.8	3.4/4.6	2.8/4.1	4.9/5.7
Latitude	Average	4.4	5.1	5.6	6.0	5.9	6.1	6.1	6.1	6.0	5.6	4.6	4.2	5.5
	Min/Max	3.6/5.1	4.2/5.7	4.4/6.5	5.2/6.7	4.6/6.8	5.1/6.9	5.2/6.8	5.1/6.8	4.6/6.8	4.2/6.4	3.9/5.2	3.2/4.8	5.0/5.8
Latitude +15	Average	4.8	5.3	5.6	5.6	5.2	5.2	5.3	5.5	5.8	5.7	4.8	4.5	5.3
Latitude +15	Min/Max	3.9/5.6	4.3/5.9	4.4/6.5	4.8/6.2	4.1/6.0	4.4/5.9	4.5/5.9	4.6/6.2	4.4/6.6	4.2/6.5	4.1/5.6	3.5/5.3	4.8/5.6
90	Average	4.5	4.6	4.3	3.6	2.8	2.6	2.7	3.2	4.0	4.6	4.4	4.3	3.8
90	Min/Max	3.6/5.4	3.7/5.2	3.5/5.0	3.0/4.0	2.3/3.1	2.2/2.8	2.3/2.9	2.7/3.6	3.1/4.6	3.4/5.3	3.7/5.1	3.4/5.2	3.4/4.1

### Solar Radiation Data Manual for Flat-Place and Concentrating Collectors (NREL)



The Solar Radiation Data Manual for Flat-Plate and Concentrating Collectors is available on the RReDC in HTML and PDF format. Individual PDF files are available for the main body of the manual and for each of the 50 states, the Pacific Islands (Guam) and Puerto Rico. Compressed files containing the individual PDFS for the manual and the site data tables can be downloaded in three compression formats: PC, Macintosh and Unix. Maps derived of the data represented in the tables are also available for viewing.

### Individual PDFs

- Manual (5.5MB)
- State/Territoy Data Tables

Tables include the "Averages of solar radiation for each of the 360 months during the period of 1961-1990" and "30-year (1961-1990) average of monthly solar radiation".

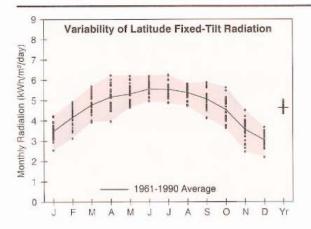
- Alabama (3.5MB)
- Alaska (14MB)
- Arizona (3.5MB)
- Arkansas (zMB)
- California (8.5MB)
- Colorado (5.5MB)
- Connecticut (5MB)
- Delaware (1MB)
- Florida (6MB)
- Georgia (5.5MB)

- Maine (zMB)
- Maryland (1MB)
- Massachusetts (zMB)
- Michigan (11MB)
- Minnesota (4.5MB)
- Mississippi (2MB)
- Missouri (3.5MB)
- Montana (7.5MB)
- Nebraska (4.5MB)

- Oklahoma (zMB)
- Oregon (7.5MB)
- Pacific Islands (1MB)
- Pennsylvania (6.5MB)
- Puerto Rico (1MB)
- Rhode Island (1MB)
- South Carolina (2.5MB)
- South Dakota (3.5MB)
- Tennessee (4.5MB)
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### Solar Radiation Data Manual for Flat-Place and Concentrating Collectors (NREL)

### **K** Average Daily Insolation per month



### Baltimore, MD WBAN NO. 93721



LATITUDE: 39.18° N LONGITUDE: 76.67° W ELEVATION: 47 meters MEAN PRESSURE: 1012 millibars

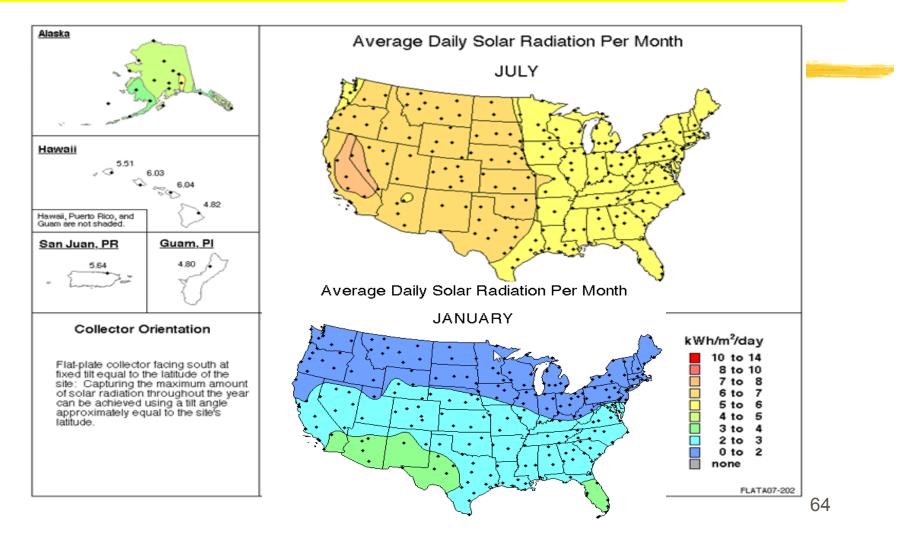
STATION TYPE: Secondary

### Solar Radiation for Flat-Plate Collectors Facing South at a Fixed Tilt (kWh/m²/day), Uncertainty ±9%

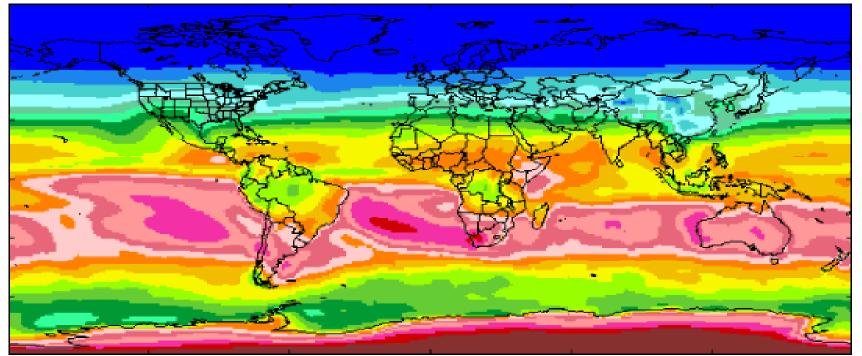
Tilt (°)		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Year
0	Average	2.1	2.9	3.9	4.9	5.6	6.2	6.0	5.3	4.4	3.3	2.2	1.8	4.0
	Min/Max	1.7/2.4	2.4/3.2	3.4/4.5	3.9/5.8	5.0/6.5	5.5/6.9	5.3/6.8	4.7/5.7	3.7/5.0	2.9/3.9	1.8/2.6	1.5/2.0	3.8/4.3
Latitude -15	Average	3.1	3.8	4.6	5.3	5.7	6.0	6.0	5.6	5.0	4.3	3.2	2.7	4.6
	Min/Max	2.3/3.7	3.0/4.4	3.8/5.4	4.1/6.3	5.0/6.6	5.4/6.7	5.2/6.7	4.9/6.0	4.1/5.8	3.5/5.2	2.3/4.0	2.0/3.2	4.3/4.9
Latitude	Average	3.5	4.2	4.8	5.2	5.3	5.6	5.5	5.4	5.1	4.6	3.6	3.1	4.6
	Min/Max	2.5/4.2	3.1/4.9	3.9/5.7	4.0/6.2	4.7/6.2	5.0/6.2	4.9/6.3	4.7/5.8	4.1/5.9	3.6/5.6	2.5/4.5	2.2/3.7	4.4/5.0
Latitude +15	Average	3.7	4.3	4.7	4.8	4.7	4.8	4.9	4.9	4.8	4.6	3.7	3.3	4,4
	Min/Max	2.6/4.6	3.2/5.1	3.8/5.6	3.7/5.8	4.1/5.5	4.3/5.3	4.3/5.5	4.3/5.3	3.9/5.7	3.6/5.7	2.5/4.8	2.3/4.0	4.1/4.8
90	Average	3.4	3.7	3.5	3.0	2.6	2.4	2.5	2.9	3.3	3.7	3.3	3.0	3.1
	Min/Max	2.3/4.4	2.7/4.5	2.8/4.2	2.4/3.6	2.3/2.9	2.2/2.6	2.3/2.8	2.6/3.1	2.7/3.9	2.8/4.6	2.2/4.3	2.0/3.7	2.9/3.4

Axis Tilt (°)		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Year
0	Average	3.0	4.0	5.3	6.4	7.1	7.8	7.7	6.9	5.8	4.6	3.2	2.5	5.4
	Min/Max	2.3/3.7	3.0/4.8	4.2/6.4	4.8/8.1	6.1/8.5	6.5/9.0	6.3/8.9	5.7/7.5	4.7/6.9	3.6/5.7	2.2/4.0	1.9/3.0	5.0/5.9
Latitude -15	Average	3.7	4.7	5.9	6.8	7.2	7.8	7.7	7.1	6.3	5.3	3.9	3.2	5.8
	Min/Max	2.7/4.6	3.4/5.7	4.6/7.2	4.9/8.6	6.1/8.6	6.5/9.0	6.3/9.0	5.9/7.8	5.1/7.5	4.2/6.7	2.6/5.0	2.3/3.9	5.4/6.4
	Averane	4.1	5.0	6.0	67	7.0	75	74	7.0	64	55	4.2	35	5.9

### Average Solar Radiation, Jan/July, Flat, South Facing, Tilted Latitude



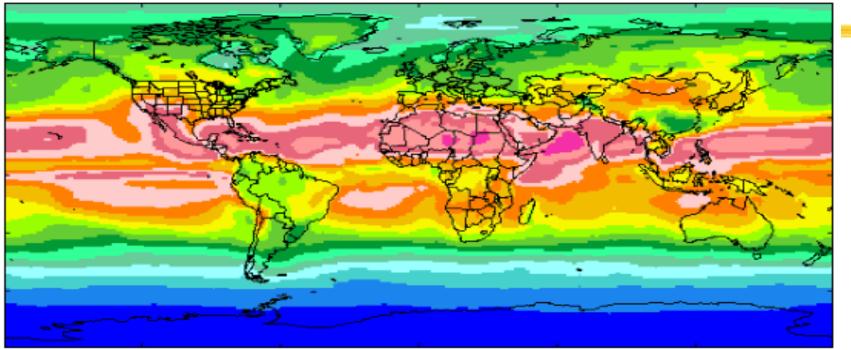
### Solar Insolation Map - January



January 1984–1993



# **Solar Insolation Map - April**

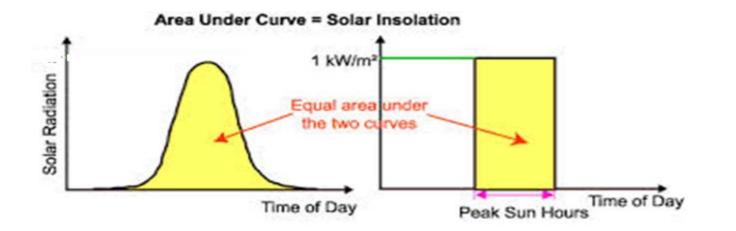


April 1984-1993



### Peak Sun Hours

- $\Re$  Unit in the solar maps is *average kWh/m<sup>2</sup>-day* of insolation.
- Huch simpler approach for PV
- "Peak Sun Hours": Total number of hours per day in which solar irradiance averages 1 kW/m<sup>2</sup> in worst month.



ENERGY = Rated\_Power \* Conversion\_Efficiency \* Peak\_Sun\_Hour/Day \* 365 Day/Year

# **Peak Sun Hour Map**

- # <u>http://www.oynot.com</u>
  /solar-insolationmap.html
- The amount of solar energy in hours ("peak sun" hours) received each day on an optimally tilted surface during the worst month ("design month") of the year at 1 kW/m<sup>2</sup>.



ENERGY = Rated\_Power \* Conversion\_Efficiency \* Peak\_Sun\_Hour/Day \* 365 Day/Year