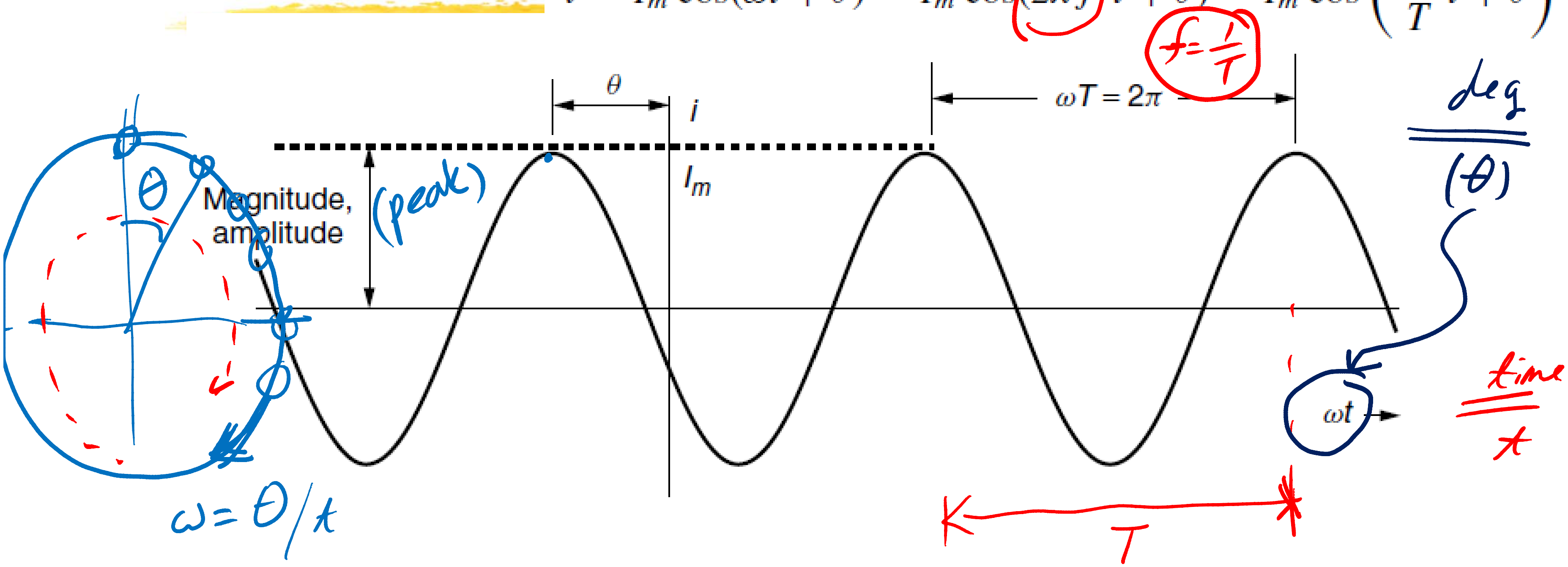
Chapter2: Electric Power

- Sinusoidal Source $i = I_m\cos(\omega t + \theta) = I_m\cos(2\pi f\, t + \theta) = I_m\cos\left(\frac{2\pi}{T}t + \theta\right)$

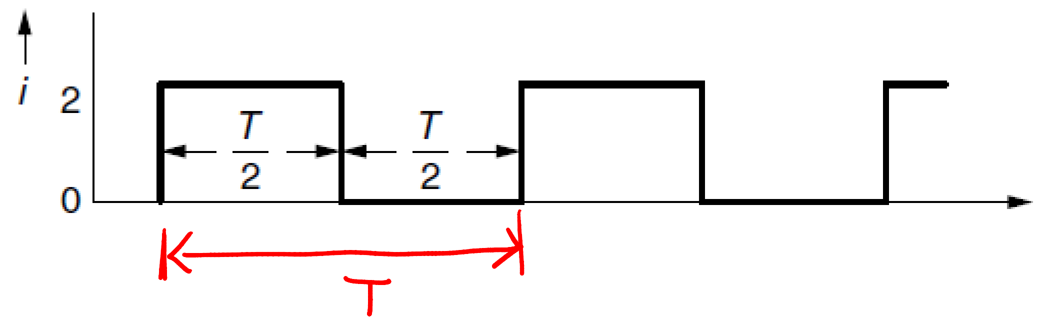


- Instantaneous Power $p = i^2 R$
- Average Power (by effective current) $P_{\text{avg}} = (i^2)_{\text{avg}} R = I_{\text{eff}}{}^2 R$
- RMS current = Effective current $I_{\text{eff}} = \sqrt{(i^2)_{\text{avg}}} = I_{\text{rms}}$

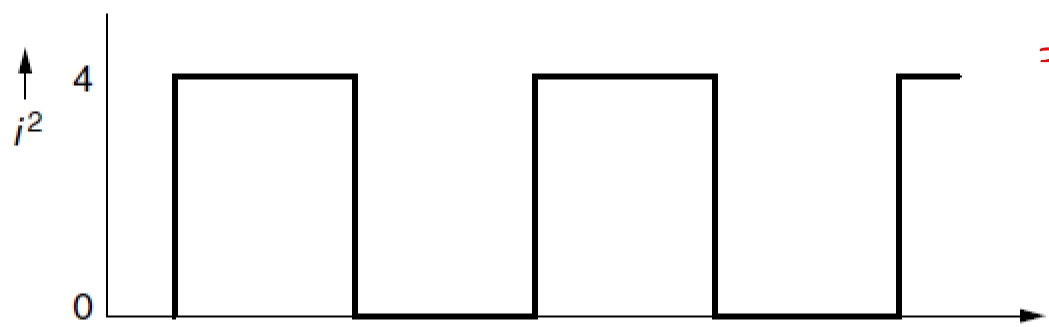
$$I_{\text{rms}} = \sqrt{(i^2)_{\text{avg}}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt}$$

RMS value

- RMS value of a square wave



- ANSWER



$$\sqrt{\frac{1}{T}\int_0^{T/2} i^2\,dt}$$

$$= \sqrt{\frac{1}{T}\left[4t\right]_0^{T/2}}$$

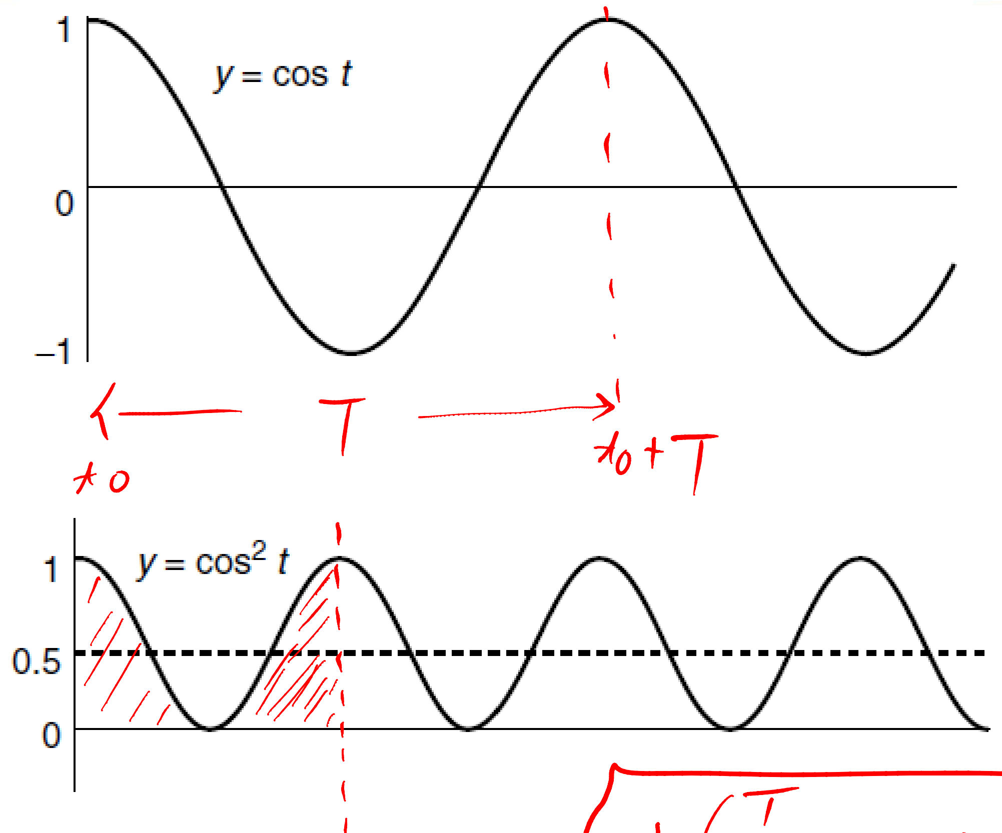
$$= \sqrt{\frac{2T}{T}}$$

$$= \sqrt{2}$$

$$I_{\text{rms}} = \sqrt{(i^2)_{\text{avg}}} = \sqrt{2}A$$

RMS value

- RMS value of a sinusoidal voltage



$$v = V_m \cos \omega t$$

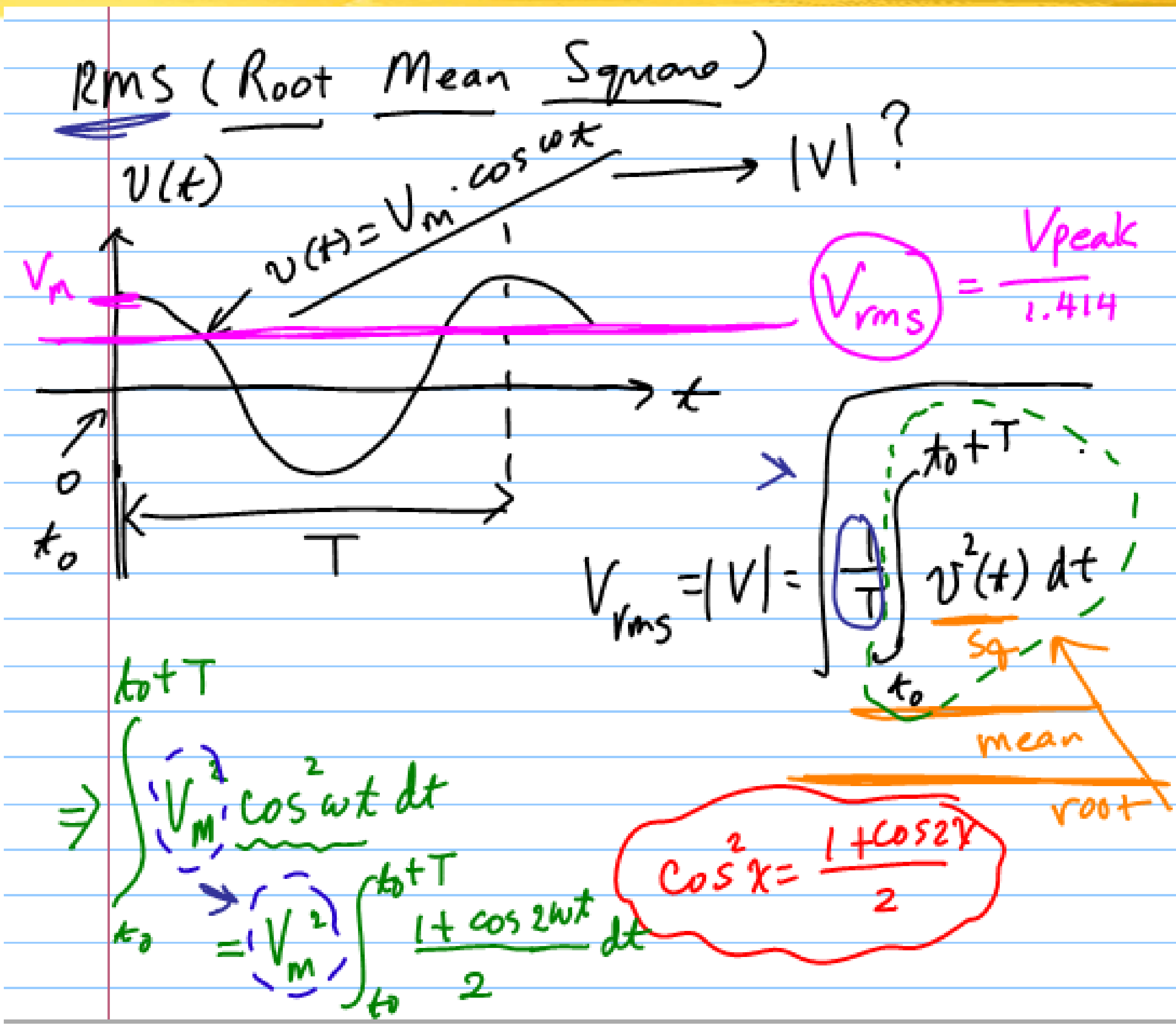
$$\omega = \frac{2\pi}{T}$$

- ANSWER

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2 \omega t}$$

RMS value

- RMS value of a sinusoidal voltage



RMS value

⌘ RMS value of a sinusoidal voltage

$$\cos(x+y) = \cos x \cdot \cos y - \sin x \cdot \sin y$$

$$\cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y$$

$$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y$$

$$\sin(x-y) = \sin x \cdot \cos y - \cos x \cdot \sin y$$

$\sin^2 x \rightarrow$

$$\cos(x+x) = \cos x \cdot \cos x - \sin x \cdot \sin x$$

$$= \cos^2 x - \sin^2 x$$

$$= \underbrace{\cos^2 x + \sin^2 x}_{= 1} - 2 \cdot \sin^2 x$$

$$\cos 2x = 1 - 2 \cdot \sin^2 x$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

RMS value

⌘ RMS value of a sinus

$$\Rightarrow \frac{1}{T}\int_{t_0}^{t_0+T} V_M^2 \cos^2\omega t\, dt$$

$$= \frac{V_M^2}{T}\int_{t_0}^{t_0+T} \frac{1+\cos 2\omega t}{2}\, dt$$

$$= \int_{t_0}^{t_0+T} \frac{1}{2}\, dt + \int_{t_0}^{t_0+T} \frac{\cos 2\omega t}{2}\, dt$$

$$= \frac{1}{2}t\Big]_{t_0}^{t_0+T} + \frac{1}{4\omega}\Big[\sin 2\omega t\Big]_{t_0}^{t_0+T}$$

$$= \frac{T}{2} + \frac{1}{4\omega}\Big[\cancel{\sin[2\omega t_0 + 2\omega T]} - \sin 2\omega t_0\Big]$$

$\sin 2\omega t_0$

$2\cdot 2\pi f\cdot T = 4\pi$

$f = \frac{1}{T}$

$$= \frac{T}{2}$$

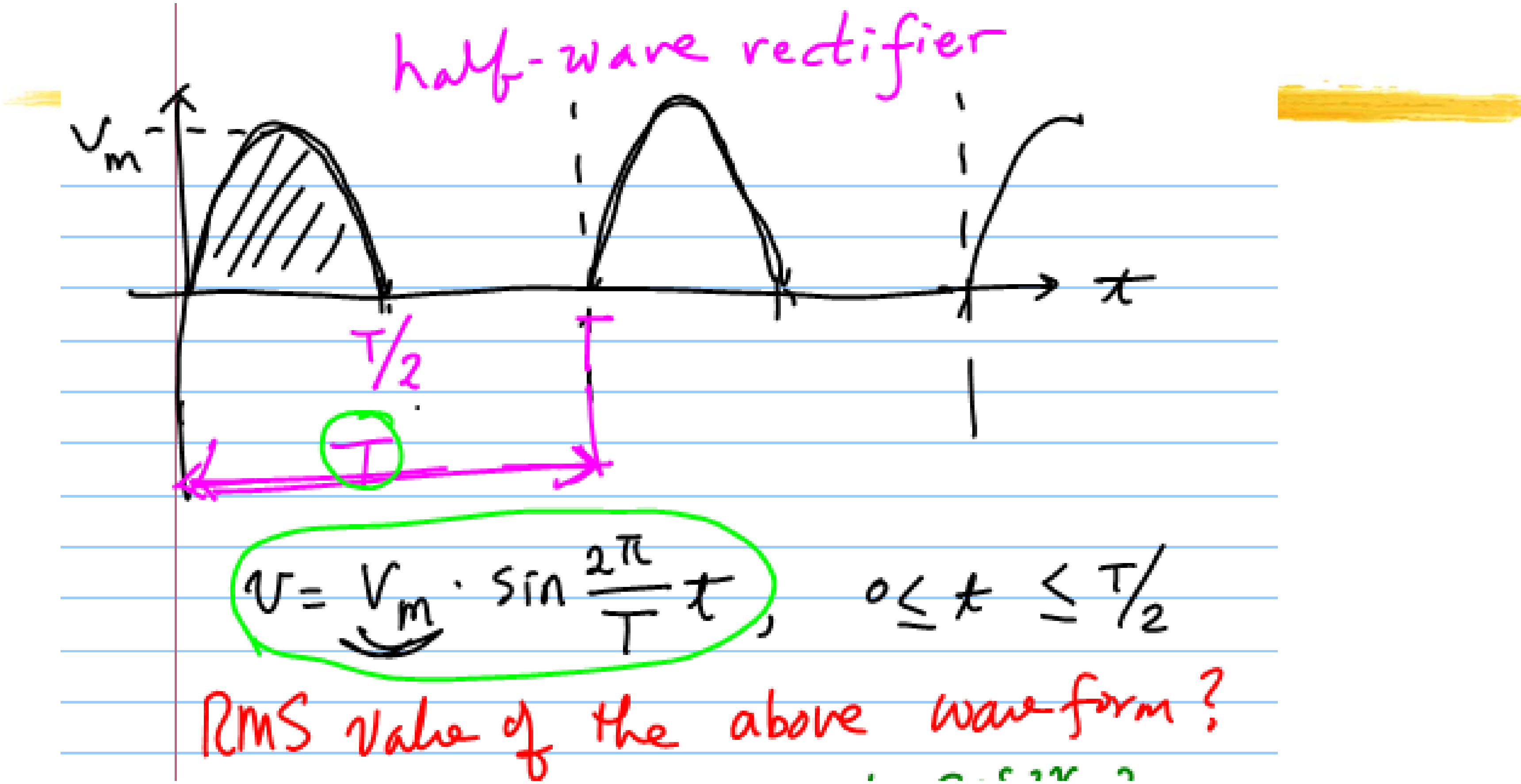
$\sin(x + 4\pi) = \sin x$

$$V_{rms} = \sqrt{\frac{1}{T} V_m^2 \cdot \frac{T}{2}} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

$$\text{Rms} = \frac{\text{Peak Value}}{\sqrt{2}}$$

$$V_{\text{rms}} = V_m\sqrt{\frac{1}{2}} = \frac{V_m}{\sqrt{2}}$$

RMS value of a half-wave



cf. $\sin^2 x = \frac{1 - \cos 2x}{2}$

RMS & Magnitude

- Magnitude is usually an RMS value as in
  "We have 120 V 60-Hz available in the outlets"
  120 → RMS → V_{rms}
- Then, what is the instantaneous voltage equation? $v(t)$

$$\underbrace{v = V_m}_{\text{peak}} \cos \omega t$$

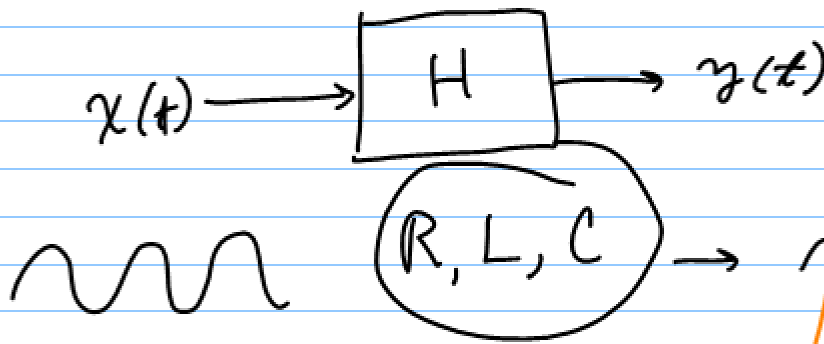
$$V_m = \sqrt{2} V_{\text{rms}} = 120\sqrt{2} = 169.7 \text{ V}$$

$$\omega = 2\pi f = 2\pi 60 = 377 \text{ rad/s}$$

$$v = 169.7 \cos 377t$$

Phasor

⌘ Phasor



$x(t) = A\cos(\omega t + 0^\circ)$

$y(t) = B\cos(\omega t + \theta)$

$X = A\angle 0^\circ$

① $|y| \propto |x|$

② ωt ω $\rightarrow$ $Y = B\angle\theta$

③ θ

Phasor

$v(t) = V_{max} \cos(\omega t + \phi)$

$i(t) = I_{max} \cos \omega t$

lower case ; instantaneous value

$V_{rms} = \frac{V_{max}}{\sqrt{2}}$

V, I magnitude, Amplitude $|V|$ $|I|$

$\bar{V}$ complex phasor

$\bar{V} = 2 + j2$ or $\bar{V} = 2\sqrt{3}\angle 45^\circ$ rms

phasor = rms $\angle\theta$

$v = 141.4 \cos(\omega t + 30^\circ)$

$i = 7.07 \cos(\omega t + 0^\circ)$

$\bar{V} = \frac{141.4}{\sqrt{2}}\angle 30^\circ = \underbrace{100}_{V_{rms}}\ \underbrace{\angle 30^\circ}_{\phi_v}$

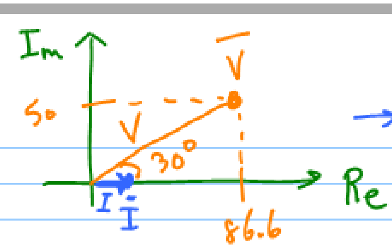
$\bar{I} = \frac{7.07}{\sqrt{2}}\angle 0^\circ = \underbrace{5}_{I_{rms}}\ \underbrace{\angle 0^\circ}_{\phi_i}$

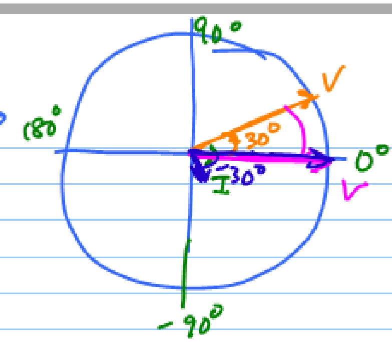
$\phi = \phi_v - \phi_i = 30 - 0 = 30^\circ$

$\bar{V} = 100\angle 30^\circ = 100(\cos 30^\circ + j \sin 30^\circ)$

$= 100\frac{\sqrt{3}}{2} + j100 \cdot \frac{1}{2} = 86.6 + j50$

$\bar{I} = 5\angle 0^\circ = 5(\cos 0^\circ + j \sin 0^\circ) = 5 + j\phi$



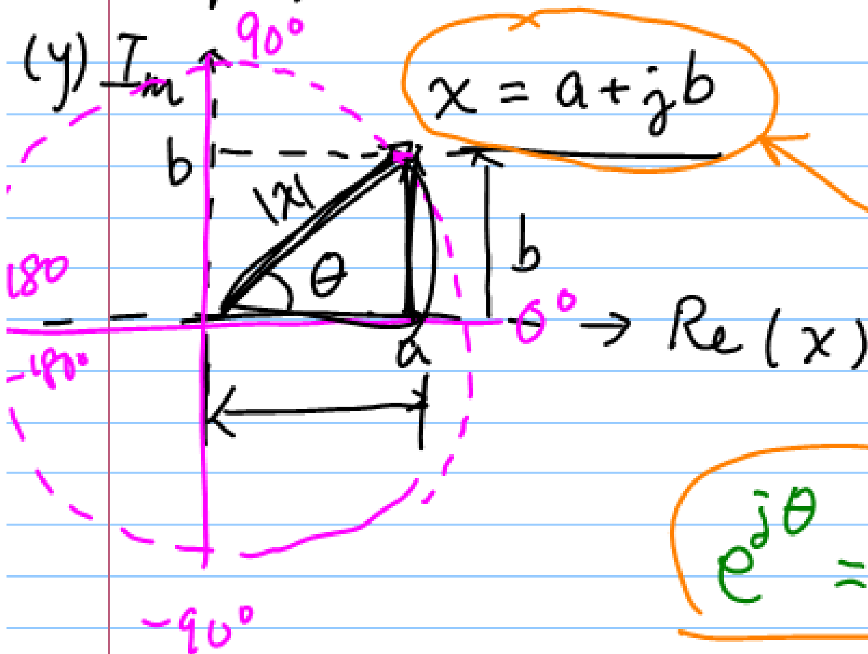


V leads I (Ref) by 30°

I Lags V (Ref) by 30° ← Convention

⌘Complex Numbers

Complex Number



$$x = a + jb$$

$$|x| = \sqrt{a^2 + b^2}$$

$$a = |x| \cos\theta$$

$$b = |x| \sin\theta$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\bar{x} = a + jb = |x| e^{j\theta} = |x| (\cos\theta + j\sin\theta)$$

Phasor

$$\bar{x} = 8 + j10 \longrightarrow \bar{x} + \bar{y} = 13 + j6$$

$$\bar{y} = 5 - j4 \qquad \bar{x} \cdot \bar{y} = (8 + j10)(5 - j4)$$

$$= 8 \cdot 5 + 8(-j4) + 5(j10) - j^2 40$$

$$= 40 - j32 + j50 + 40 = (80 + j16) = \Box \angle \theta$$

$$x = a \angle \theta_1 \qquad y = b \angle \theta_2$$

$$\sqrt{80^2 + 16^2} \angle \tan^{-1} \frac{16}{80}$$

$$82 \angle 12.68^\circ$$

$$xy = ab \angle \theta_1 + \theta_2$$

$$\frac{x}{y} = \frac{a \angle \theta_1}{b \angle \theta_2} = \frac{a}{b} \angle \theta_1 - \theta_2$$

Instantaneous Power *p(t)*

Power in Single-phase ("1φ") circuit

$$v(t) = V_m \cos \omega t$$

$$i(t) = I_m \cos(\omega t - \theta)$$

$$p(t) = v(t)\, i(t) = V_m I_m \cos \underbrace{\omega t}_{\alpha} \cdot \cos(\underbrace{\omega t - \theta}_{\beta})$$

$$\cos\alpha \cdot \cos\beta = \tfrac{1}{2}\cos(\alpha - \beta) + \tfrac{1}{2}\cos(\alpha + \beta)$$

$$= \frac{V_m I_m}{2}\left\{\cos\theta + \cos(\underbrace{2\omega t}_{\alpha} - \underbrace{\theta}_{\beta})\right\}$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$= \frac{V_m I_m}{2}\left\{\cos\theta + \cos 2\omega t \cdot \cos\theta + \sin 2\omega t \cdot \sin\theta\right\}$$

$$= \frac{V_m I_m}{2}\cos\theta + \frac{V_m I_m}{2}\cos\theta \cdot \cos 2\omega t + \frac{V_m I_m}{2}\sin\theta \cdot \sin 2\omega t$$

$$V = \frac{V_m}{\sqrt{2}}$$

$$p(t) = VI\cos\theta + VI\cos\theta \cdot \cos 2\omega t + VI\sin\theta\,\sin 2\omega t$$

$$\text{Average}\{p(t)\} = VI\cos\theta$$

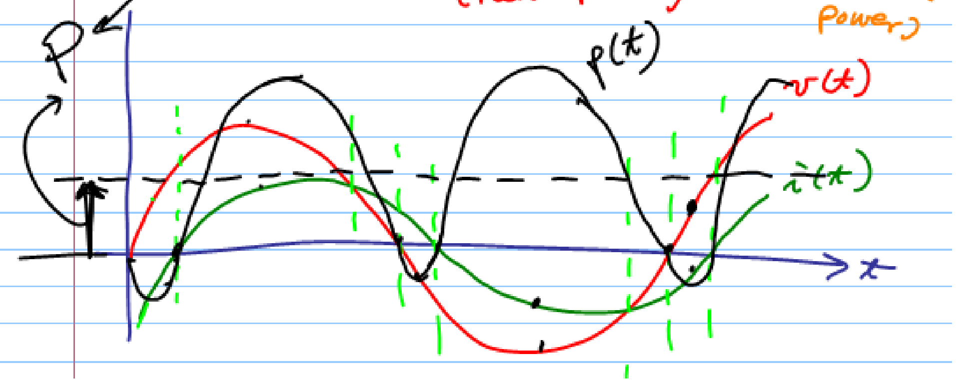
$$\frac{1}{T}\int_0^T \cos\omega t\, dt = 0$$

$$x(t) = A \cdot \cos\omega t \longrightarrow \{x(t)\}_{Avg} = 0$$

T

$$P = VI\cos\theta \longleftarrow \text{Average Power (Real Power)}$$

Q (reactive power)

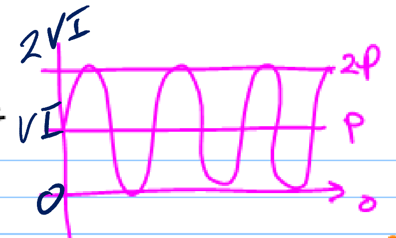


$p(t)$ and S {Complex Power in Phasor}

$\theta = 0^\circ$, $\cos\theta = 1$ $\sin\theta = 0$

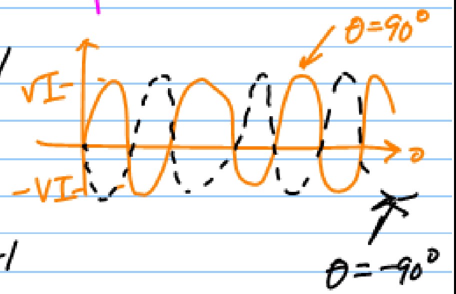
$p(t) = VI + VI\cos 2\omega t$

$= VI(1 + \cos 2\omega t)$



$\theta = 90^\circ$, $\cos\theta = 0$, $\sin\theta = 1$

$p(t) = VI(\sin 2\omega t)$

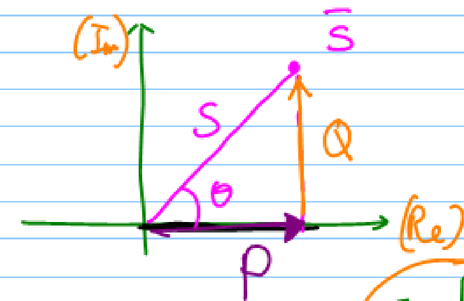


$\theta = -90^\circ$, $\cos\theta = 0$, $\sin\theta = -1$

$p(t) = -VI\sin 2\omega t$

$P = VI\cos\theta$ [W] $\quad Q = VI\sin\theta$ [Var]

$P + jQ = VI\{\cos\theta + j\sin\theta\} = VI\angle\theta = \bar{S}$ Complex power



S: Apparent power

$\boxed{S = \sqrt{P^2 + Q^2} = VI}$

$P = VI\cos\theta$, $Q = VI\sin\theta$ $\quad (VI)^2\cos^2\theta + (VI)^2\sin^2\theta$

$= (VI)^2$

real power

P

$p(t) = VI\cos\theta + VI\cos\theta \cdot \cos 2\omega t$

$+ VI\sin\theta \cdot \sin 2\omega t$

Q reactive power

Circuit Elements in Phasor

$F(s)=\int_0^\infty f(t)e^{-st}\,dt$

$v(t) \xrightarrow{P} V$

$i(t) \xrightarrow{P} I$

✓ $R \xrightarrow{P} R$ $\qquad V = IR$

✓ $L \xrightarrow{P} j\omega L = \omega L\angle 90^\circ$

✓ $C \xrightarrow{P} \dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C}$

$f'(t)$

$\Downarrow \mathcal{L}$

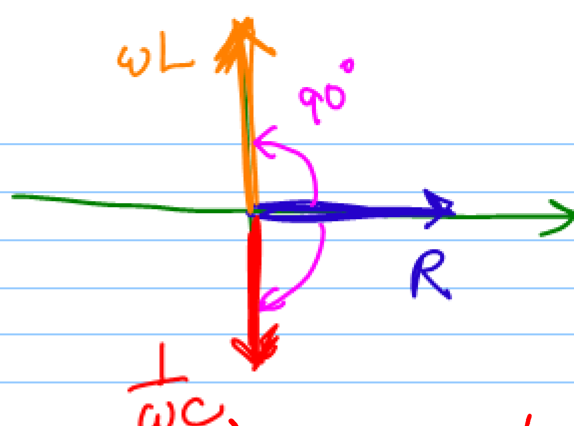
$sF(s)-f(0)$

$\Downarrow$

$s \leftarrow j\omega$

$v = iR$

$v = L\dfrac{di}{dt}$



$v = L\dfrac{di}{dt}$

$\cos \rightarrow \dfrac{di}{dt} : \sin$

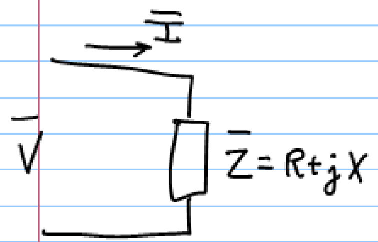
$V(s) = L(sI(s) - i(0)) \longrightarrow sL \longrightarrow j\omega L$

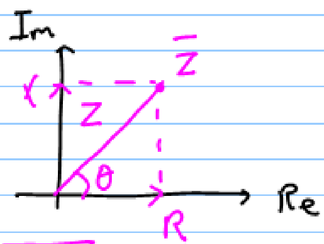
Phasor and Power Factor (pf)

$\cos\theta = \dfrac{P}{\sqrt{P^2+Q^2}} = \dfrac{P}{VI}$ ← power factor

$pf = \cos\theta = 1$ R, L, C

$\cos\theta > .95$





$Z = \sqrt{R^2 + X^2}$

$\left\{ \begin{array}{l} R = Z \cdot \cos\theta \\ X = Z \cdot \sin\theta \end{array} \right\}$

$P = VI\cos\theta$

$\bar{V} = \bar{I}\,\bar{Z}$ $\bar{I} = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{V\angle 0^\circ}{Z\angle\theta} = \dfrac{V}{Z}\angle{-\theta}$

$P = VI\cos\theta = I^2 \cdot \underbrace{Z\cdot\cos\theta} = I^2 R$

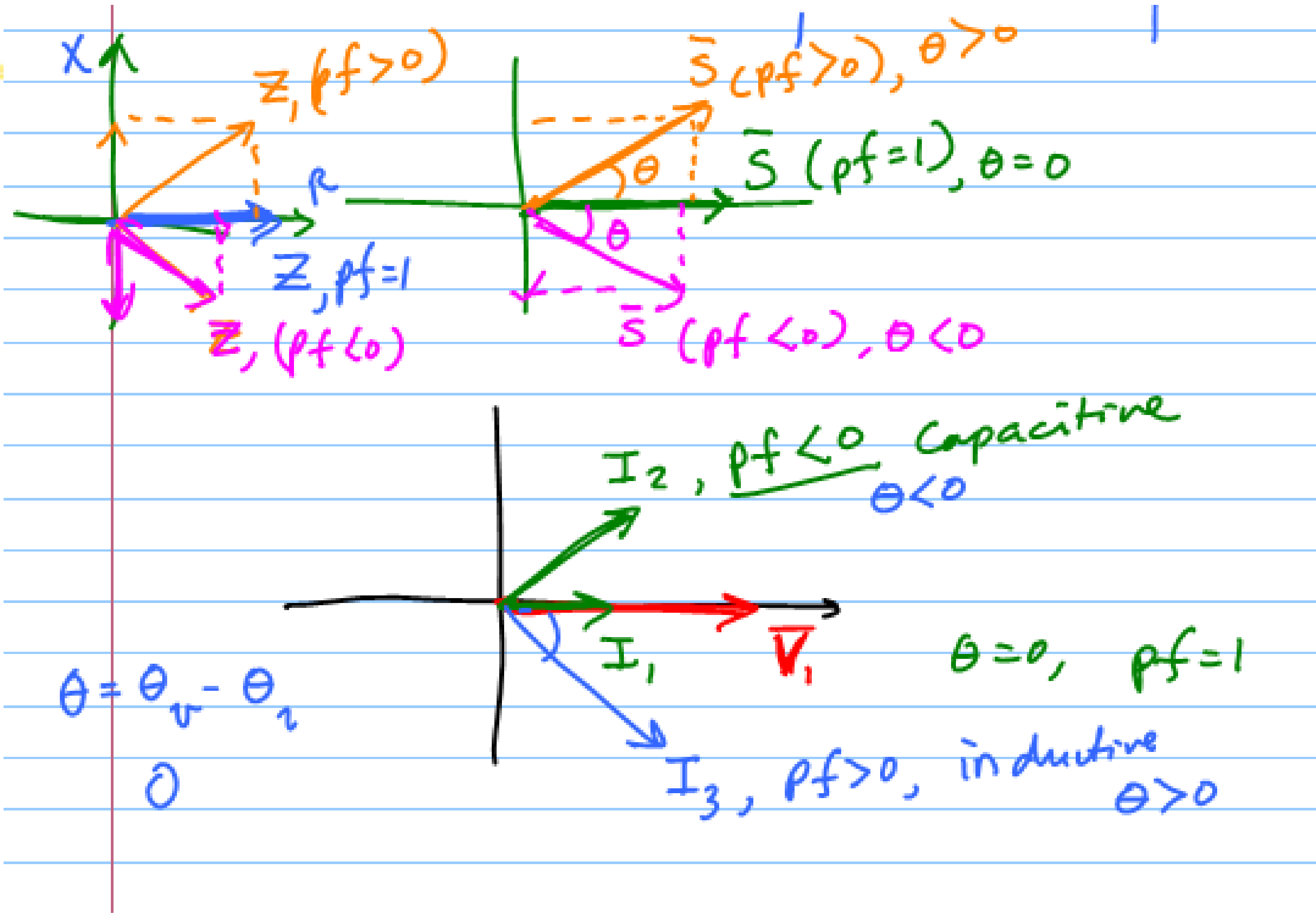
$Q = VI\sin\theta = I^2 \cdot \underbrace{Z\cdot\sin\theta} = I^2 X$

$\cos\theta = \dfrac{R}{Z}$ ← power factor

$pf = \cos\theta$

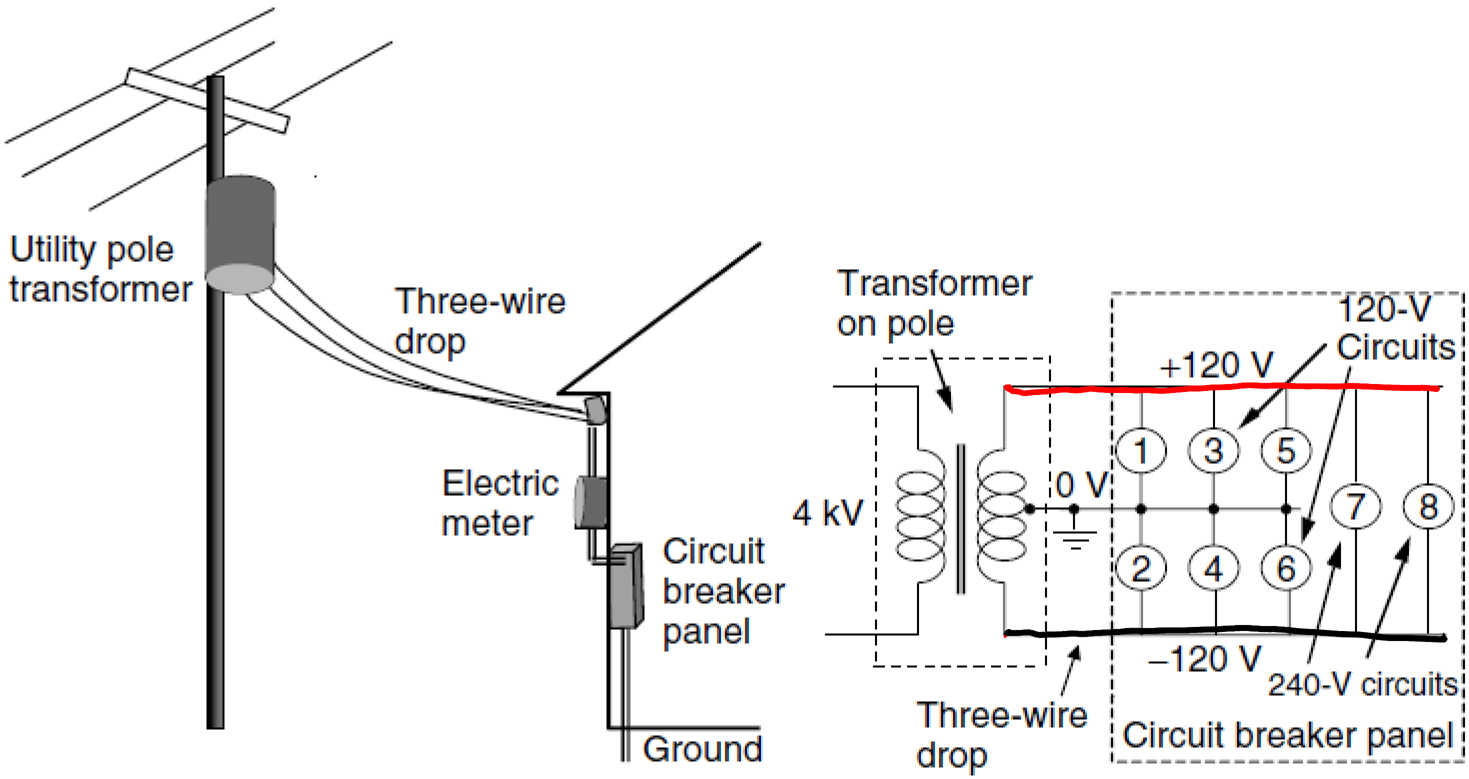
		P, Q	Z
$pf = 1$	$\cos\theta = 1$, $\sin\theta = 0$,	P, $Q = 0$	R only
$pf > 0$	$\cos\theta > 0$, $\sin\theta > 0$,	P, Q	R & L
$pf < 0$	$\cos\theta > 0$, $\sin\theta < 0$	P, $Q < 0$	R & C

Phasor

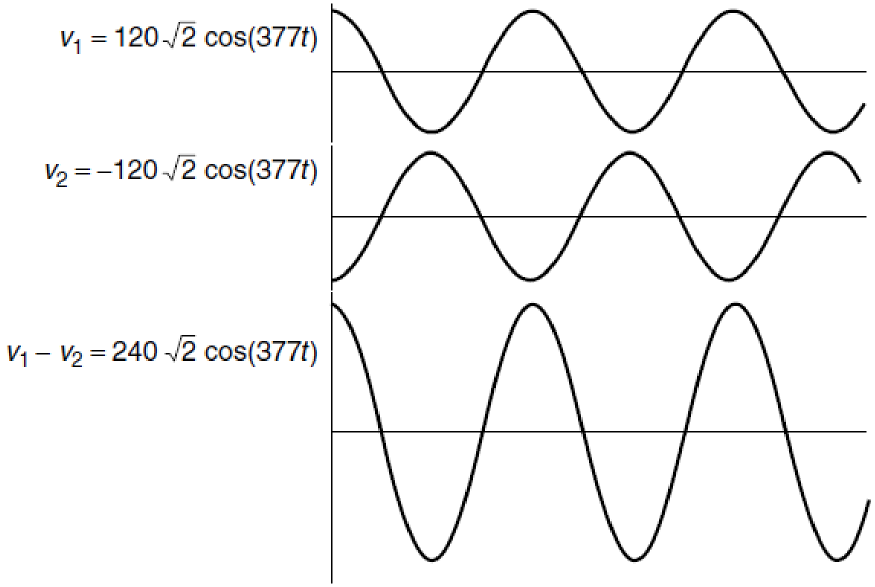

X
R
Z, (pf>0)
Z, pf=1
Z, (pf<0)
S̄ (pf>0), θ>0
S̄ (pf=1), θ=0
S̄ (pf<0), θ<0
θ
I₂, pf<0 capacitive
θ<0
I₁
V̄₁
θ=0, pf=1
I₃, pf>0, inductive
θ>0
θ = θv - θi

3-wire single-phase residential wiring

- Two “hot” sides (red and black)
- Center-tapped ground (Neutral, white)



- Two "hot" sides (red and black)
- Center-tapped ground (Neutral, white)



$$v_1 = 120\sqrt{2}\cos(2\pi \cdot 60t) = 120\sqrt{2}\cos 377t$$

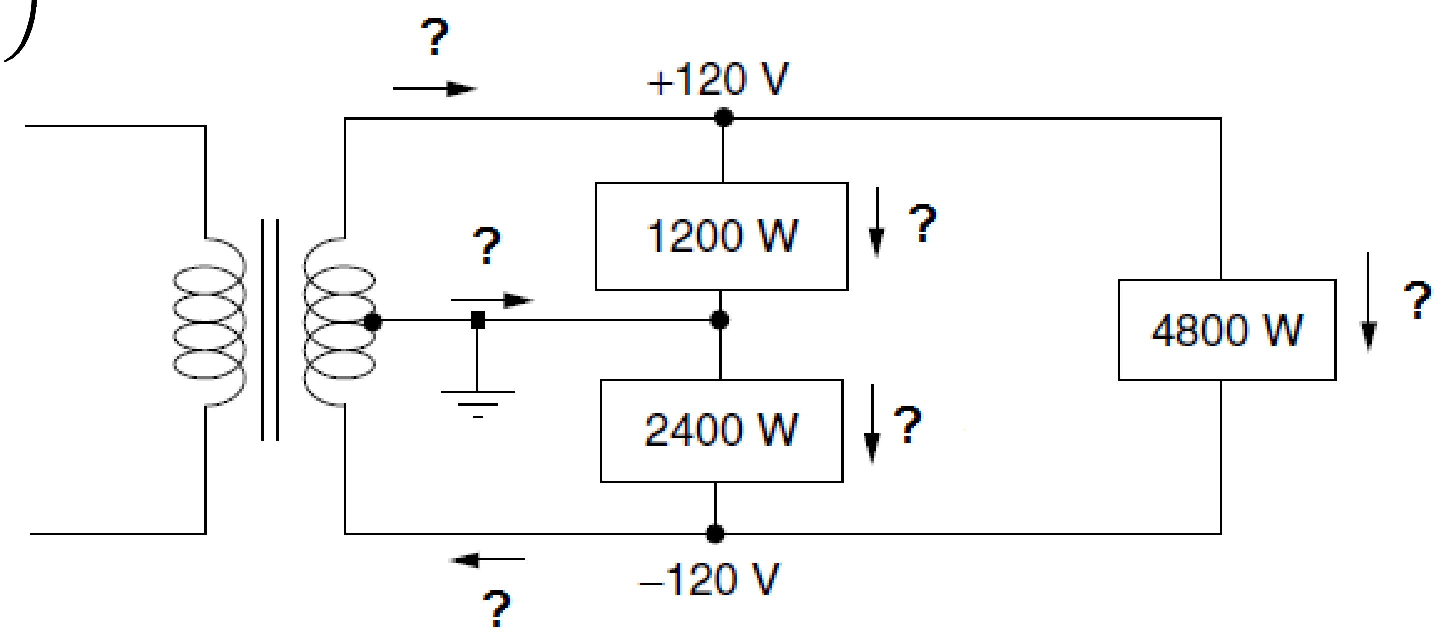
$$v_2 = 120\sqrt{2}\cos(377t + \pi) = -120\sqrt{2}\cos 377t$$

$$v_1 - v_2 = 240\sqrt{2}\cos 377t$$

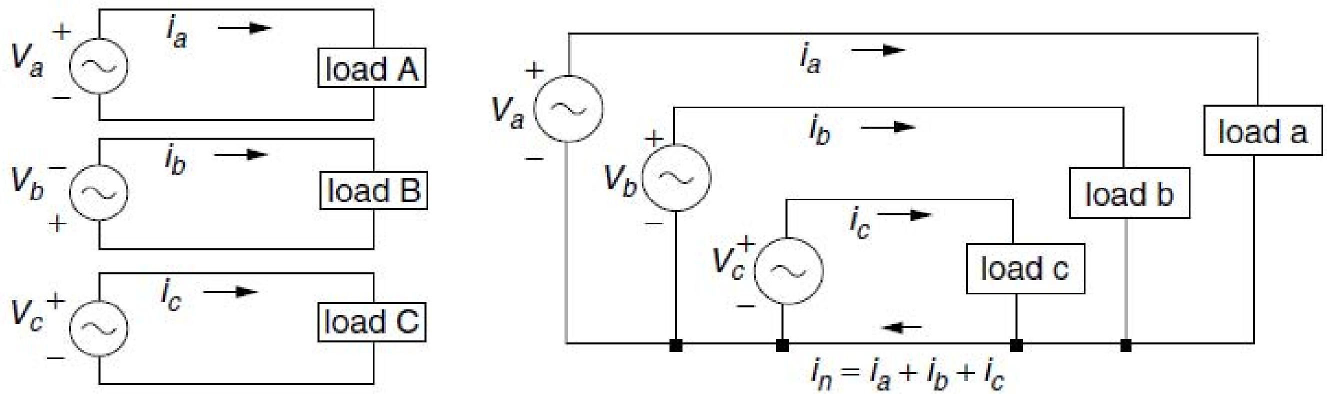
3-wire single-phase residential wiring - Example

⌘Find the current in each of the three legs?

RMS

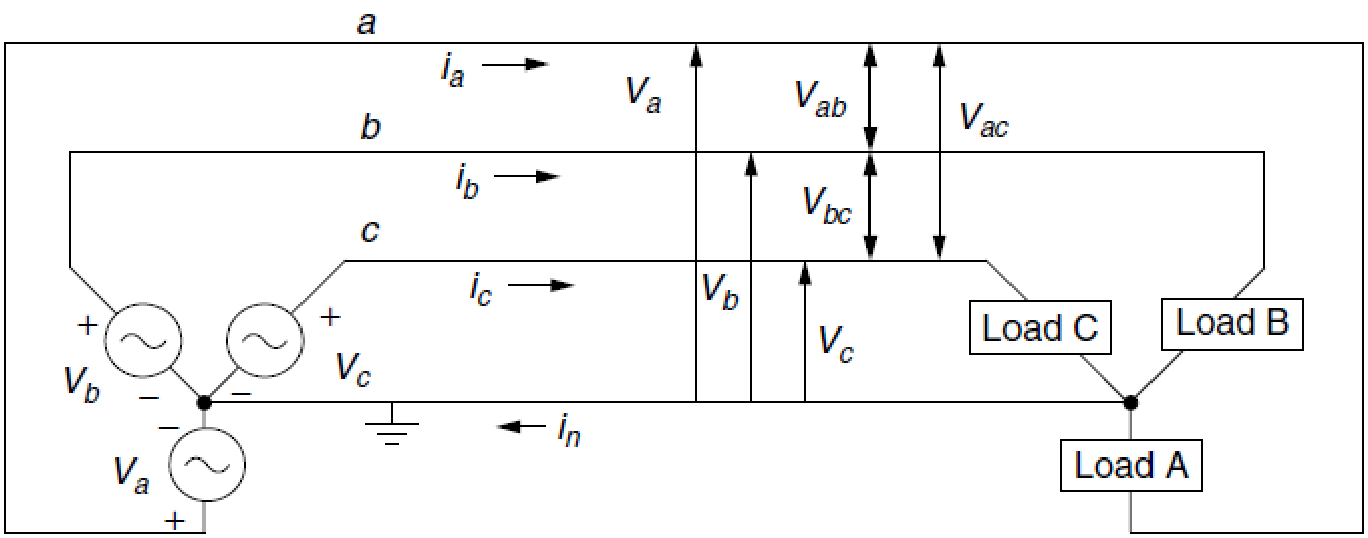


3-phase power system



(a) Three separate circuits

(b) Combined use of the neutral line

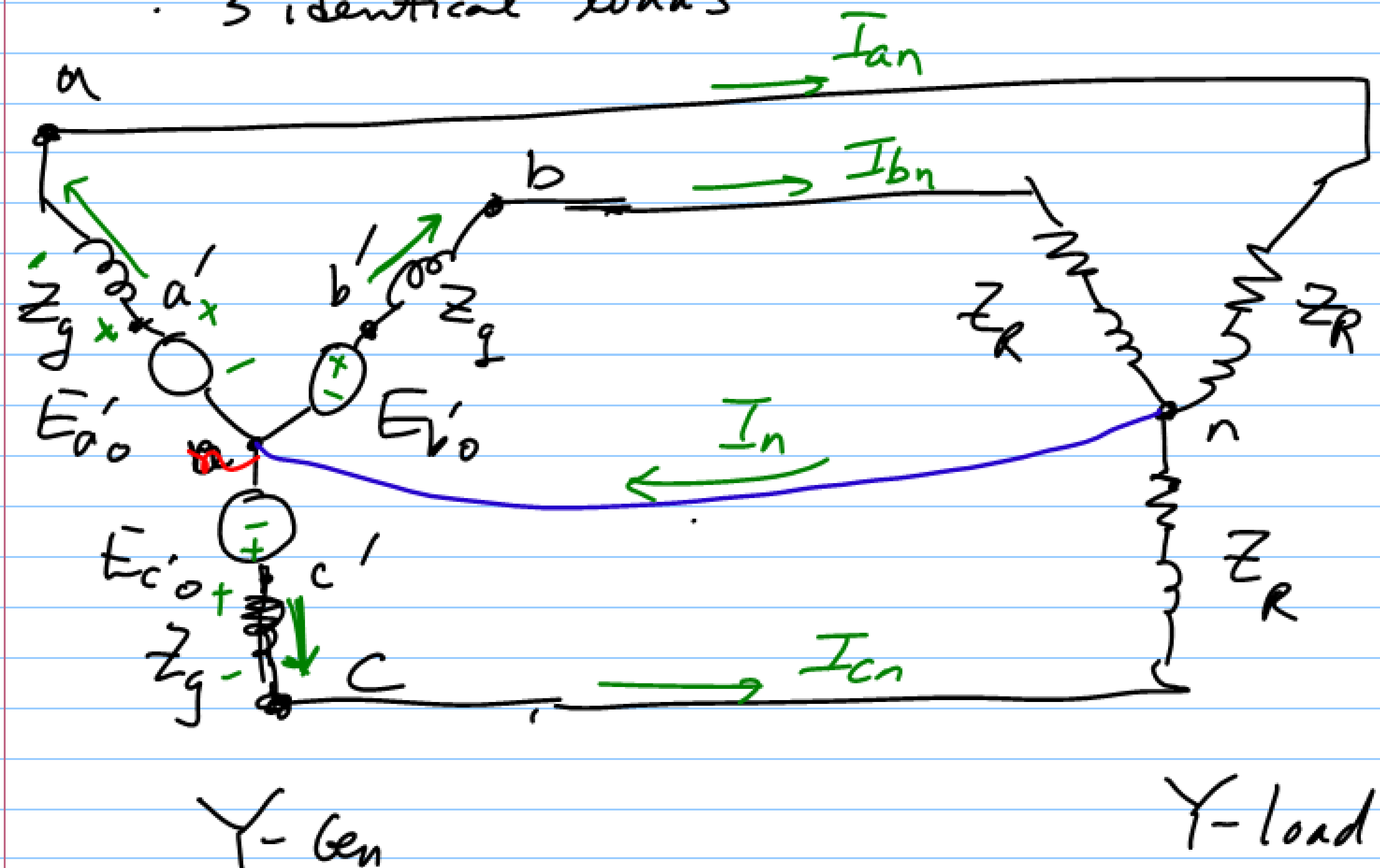


3-phase power systems

"Balanced 3-ϕ system"

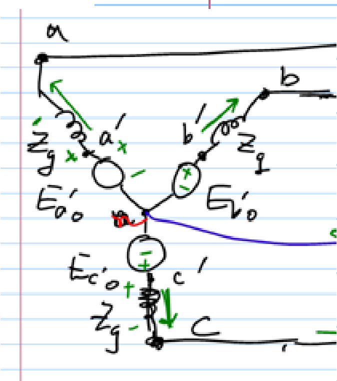
: 3ϕ generator

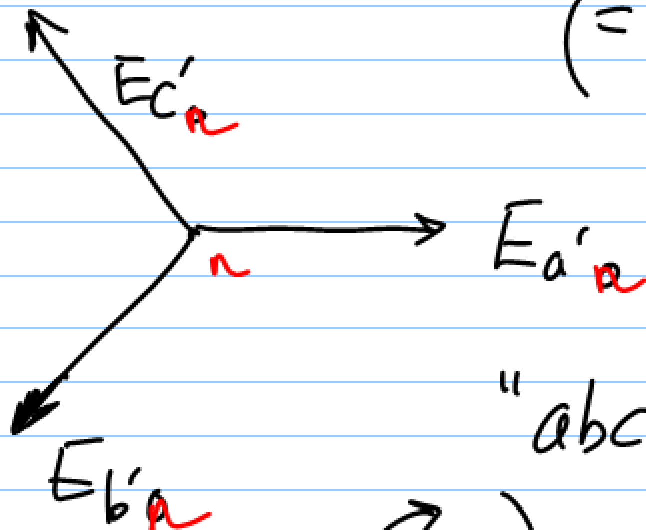
: 3 identical loads



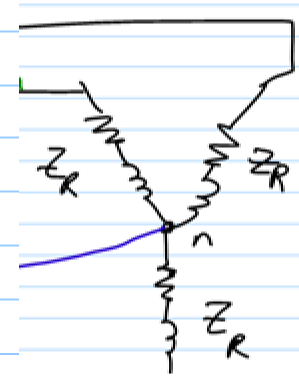
$E_{a'n} = E\angle 0^\circ$ $\quad E_{b'n} = E\angle -120^\circ$ $\quad E_{c'n} = E\angle 120^\circ$

$(= E\angle 240^\circ)$





"abc-Sequence"



Gen

Load

V_{an} =

Gen	Load
$V_{an} = E_{a'n} - I_a Z_g$	$V_{an} = I_a \cdot Z_R \rightarrow I_{an} = \dfrac{V_{an}}{Z_R}$
$V_{bn} = V_{bn} = E_{b'n} - I_b Z_g$	$V_{bn} = I_b Z_R \rightarrow I_b = \dfrac{V_{bn}}{Z_R}$
$V_{cn} = V_{cn} = E_{c'n} - I_c Z_g$	$V_{cn} = I_c Z_R \rightarrow I_{cn} = \dfrac{V_{cn}}{Z_R}$

3-phase power systems

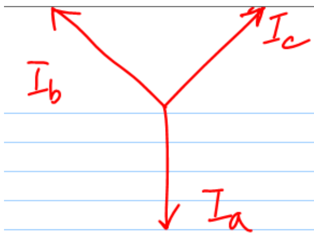
$$E_{a'n} = I_a (Z_g + Z_R) \rightarrow I_a = \frac{E_{a'n}}{Z_g + Z_R} = \frac{V_{an}}{Z_R}$$

$$E_{b'n} = I_b (Z_g + Z_R) \rightarrow I_b = \frac{E_{b'n}}{Z_g + Z_R} = \frac{V_{bn}}{Z_R}$$

$$E_{c'n} = I_c (Z_g + Z_R) \rightarrow I_c = \frac{E_{c'n}}{Z_g + Z_R} = \frac{V_{cn}}{Z_R}$$

Since $E_{a'n}$, $E_{b'n}$, and $E_{c'n}$ are balanced,

I_a, I_b, & I_c are also balanced.



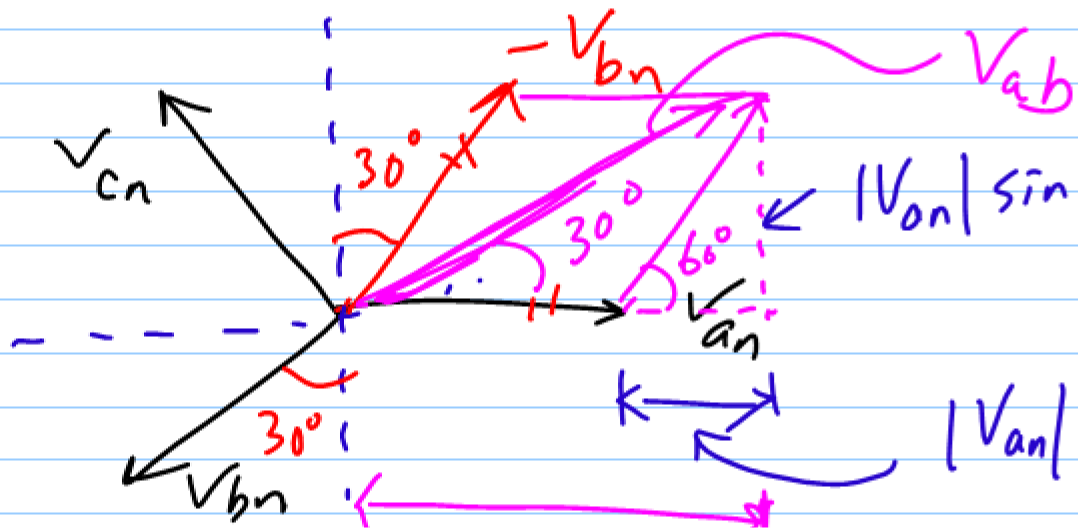
and $I_a + I_b + I_c = 0$

$\rightarrow I_n = 0$ No current flow back to generator.

3-phase power systems

Line-to-Line Voltage

$$\bar{V}_{ab} = \bar{V}_{an} + \bar{V}_{nb} = \bar{V}_{an} - \bar{V}_{bn}$$



$$|V_{an}| \sin 60^\circ = \frac{\sqrt{3}}{2} |V_{an}|$$

$$|V_{an}| \cos 60^\circ = \frac{|V_{an}|}{2}$$

$$|V_{ab}| = \sqrt{\left(\frac{\sqrt{3}}{2}|V_{an}|\right)^2 + \left(\frac{3}{2}|V_{an}|\right)^2} = \sqrt{\frac{12}{4}|V_{an}|^2} = \sqrt{3}\,|V_{an}|$$

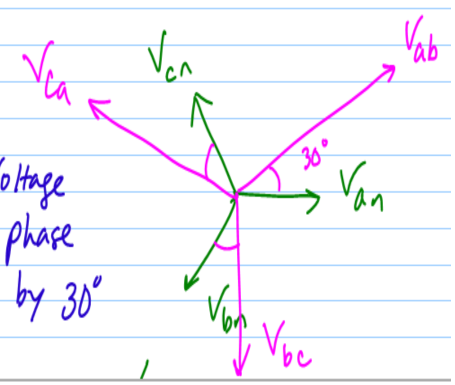
3

$$\Longrightarrow \bar{V}_{ab} = \sqrt{3}\,\bar{V}_{an} \angle 30^\circ$$

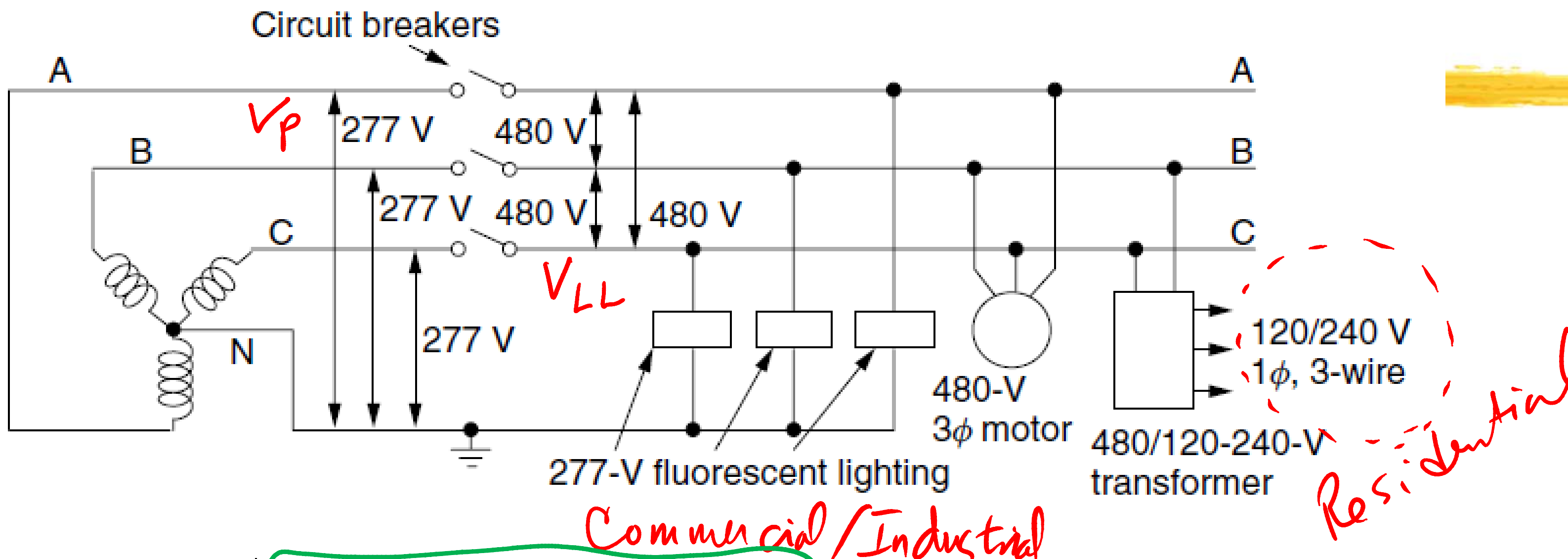
$$\Longrightarrow \bar{V}_{bc} = \sqrt{3}\,\bar{V}_{bn} \angle 30^\circ$$

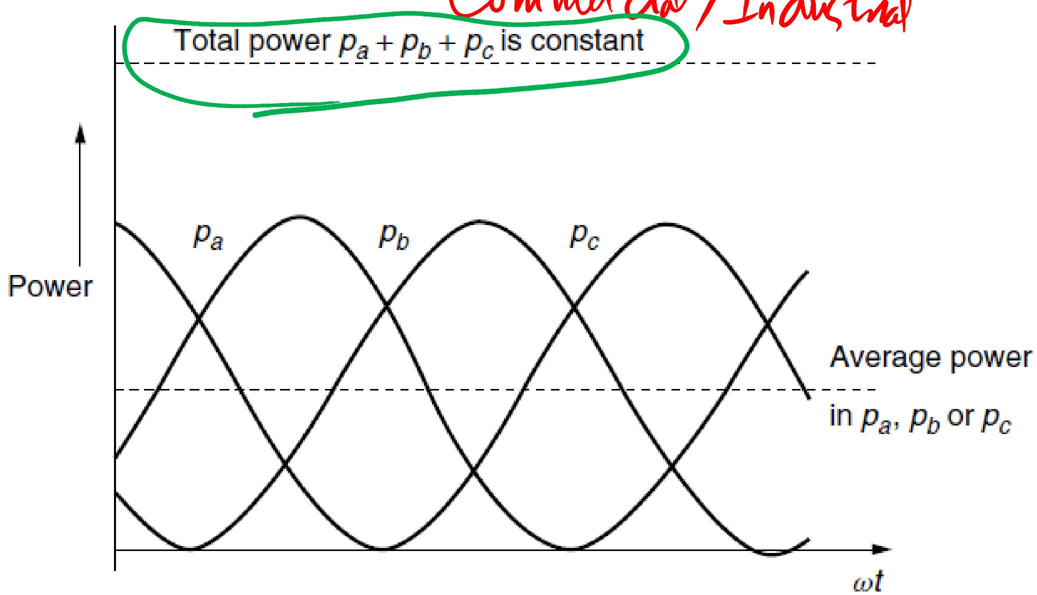
$$\Rightarrow \bar{V}_{ca} = \sqrt{3}\,\bar{V}_{cn} \angle 30^\circ$$

Line Voltage leads phase Voltage by 30°



3-phase power systems





3-phase power systems

3ϕ instantaneous power $P_3(t)$

$v_a = V_m \cos \omega t$

$v_b = V_m \cos(\omega t - 120°)$

$v_c = V_m \cos(\omega t + 120°)$

$i_a = I_m \cos(\omega t - \theta)$

$i_b = I_m \cos(\omega t - \theta - 120°)$

$i_c = I_m \cos(\omega t - \theta + 120°)$

$$
\begin{aligned}
P_{3\phi} &= P_a(t) + P_b(t) + P_c(t) \\
&= V_m I_m \cos \omega t \cdot \cos(\omega t - \theta) \\
&+ V_m I_m \cos(\omega t - 120°) \cdot \cos(\omega t - 120° - \theta) \\
&+ V_m I_m \cos(\omega t + 120°) \cdot \cos(\omega t + 120° - \theta)
\end{aligned}
$$

3-phase power systems

From, 1ϕ instantaneous power $p(t)$

Phase a:

$$p_a(t) = \frac{V_m I_m}{2}\cos\theta + \frac{V_m I_m}{2}\cos\theta \cdot \cos 2\omega t + \frac{V_m I_m}{2}\sin\theta \cdot \sin 2\omega t$$

Phase b:

$$p_b(t) = \frac{V_m I_m}{2}\cos\theta + \frac{V_m I_m}{2}\cos\theta \cdot \cos\left(2\omega t - 240^\circ\right)$$

$$\frac{V_m I_m}{2}\sin\theta \cdot \sin\left(2\omega t - 240^\circ\right)$$

Phase C:

$$p_c(t) = \frac{V_m I_m}{2}\cos\theta + \frac{V_m I_m}{2}\cos\theta \cdot \cos(2\omega t + 240^\circ)$$

$$\frac{V_m I_m}{2}\sin\theta \cdot \sin(2\omega t + 240^\circ)$$

$$P_3(t) = P_a(t) + P_b(t) + P_c(t)$$

$$= 3\left(\frac{V_m I_m}{2}\cos\theta\right) + \frac{V_m I_m}{2}\cos\theta\,\underbrace{\left\{\cos 2\omega t + \cos(2\omega t - 240^\circ) + \cos(2\omega t + 240^\circ)\right\}}_{0}$$

$$+ \frac{V_m I_m}{2}\sin\theta\,\underbrace{\left\{\sin 2\omega t + \sin(2\omega t - 240) + \sin(2\omega t + 240^\circ)\right\}}_{0}$$

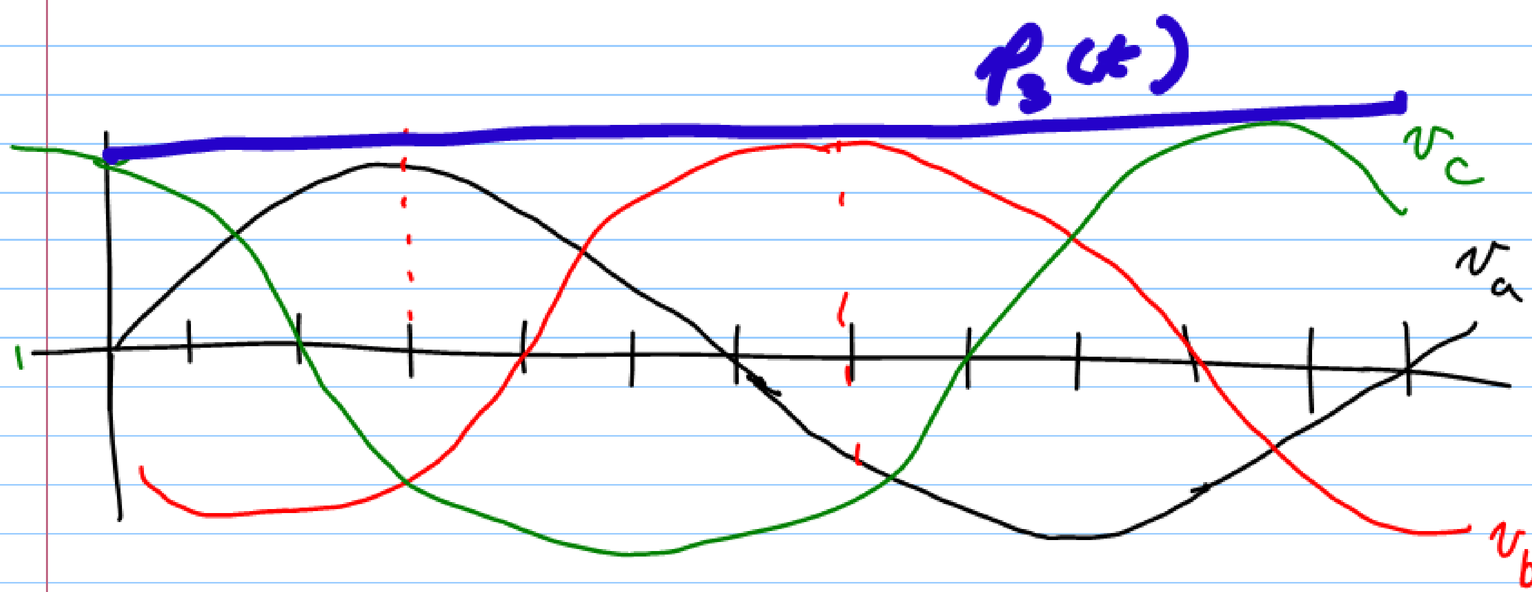
3-phase power systems

$$= 3 \frac{V_m I_m}{2} \cos\theta$$

$$= 3 |V||I| \cos\theta$$

Constant !!

$P_3(t) =$ Constant



Power in Balanced 3-ϕ Circuits

$$P_{3\phi} = P_a + P_b + P_c$$

$$= 3 P_a \quad \text{(in Balanced system)}$$

V_p: phase voltage magnitude to neutral

$$V_p = |V_{an}| = |V_{bn}| = |V_{cn}|$$

I_p: phase current magnitude for Y-Load

$$I_p = |I_{an}| = |I_{bn}| = |I_{cn}|$$

θ_p: phase angle between $\bar{V}_p$ & $\bar{I}_p$
: angle of the impedance $\longrightarrow \theta_Z$

3-phase power systems

$$\Rightarrow P_{3\phi} = 3 V_p I_p \cos\theta_p$$

V_L: Line-to-Line Voltage magnitude

or 1ϕ equ

$$V_p = \frac{V_L}{\sqrt{3}} \quad \text{and} \quad I_p = I_L \quad (\text{Y- Load case})$$

current over a Line

$I_L = I_p\sqrt{3}$

Δ Load case

$$\Rightarrow P_{3\phi} = 3 V_p I_p \cos\theta_p$$

$$= 3 \frac{V_L}{\sqrt{3}} I_L \cos\theta_p = \sqrt{3}\, V_L I_L \cos\theta_p$$

& $Q_{3\phi} = 3 V_p I_p \sin\theta_p = \sqrt{3}\, V_L I_L \sin\theta_p$

3-phase power systems

Further,

$$S_{3\phi} = P_{3\phi} + jQ_{3\phi}$$

$$|S_{3\phi}| = \sqrt{P_{3\phi}^2 + Q_{3\phi}^2} = \sqrt{3}\, V_L I_L$$

$$= 3\, V_p I_p$$

LAB

www.mwftr.com/325S18.html

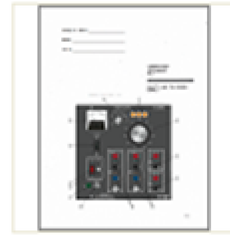
Lab Manual/Handout:
1. Check the web for Lab manual
2. Printout and Read before coming to the lab
3. Report --- Write on the Lab manual & Submit 1 week after

EECE325 Fundamentals of Energy Systems + Lab (EECE326)

Course Introduction (for 325 and 326)

EECE325 - Lecture	EECE326 - Lab
Syllabus	Syllabus
Subject 1: Background	Lab 1 -- Safety and Power Supply
Chapter 2	Lab 2 -- Phase Sequence
	Lab 3 -- Real and Reactive Power
Subject 2: Power Industry and Distributed Generation	Lab 4 -- Power Flow
Chapter 3	Lab 5 -- Phase Angle and Voltage Drop
Chapter 4	Lab 6 -- Syhchnronous Machine
Subject 3: Wind Electricty Generation	Lab 7 -- Wind Power Generation
Chapter 6	Lab 8 --Power Inverter
Subject 4: Phovoltaic Electricty Generation	
Chapter 7	Lab 9 -- PV Systems
Chapter 8	Lab 10 -- Battery Systems
Chapter 9	Lab 11 -- Additional Lab

LAB1



326Lab1_Safety and Power Su...